Quicksort

Based on
Kruse and Ryba

Quicksort Algorithm

Given an array of *n* elements (e.g., integers):

- IF array only contains one element, return
- ELSE
  - pick one element to use as *pivot.*
  - move the pivot to right position (the position of the pivot as if sorted) ⇔ partition the input array into two sub-array:
    - ArrayL: elements on the left of the pivot, each is smaller than the pivot
    - ArrayR: elements on the right of the pivot, each is greater than the pivot.
  - Do quicksort (recursively) for ArrayL and ArrayR.
  - Return the result.

Example

We are given array of n integers to sort:

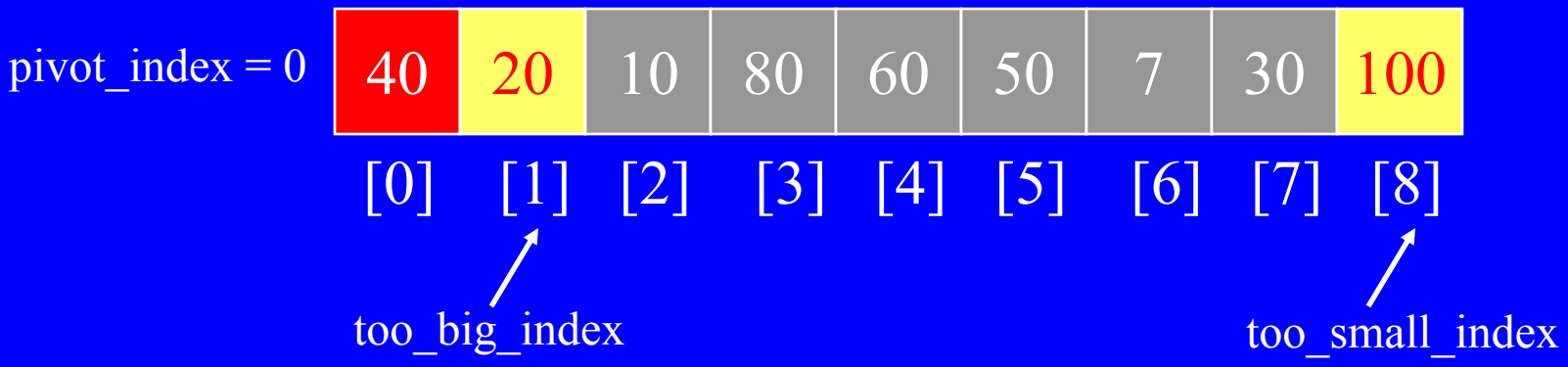
60	20	10	80	40	50	7	30	100

PASS 1: move one element to right position

Pickup a pivot: 40 is selected, random!
Swap 40 with the first before doing next

40	20	10	80	60	50	7	30	100

See step-by-step on next slides


pivot_index = 0
40
20
10
80
60
50
7
30
100
[0]
[1]
[2]
[3]
[4]
[5]
[6]
[7]
[8]
too_big_index
too_small_index

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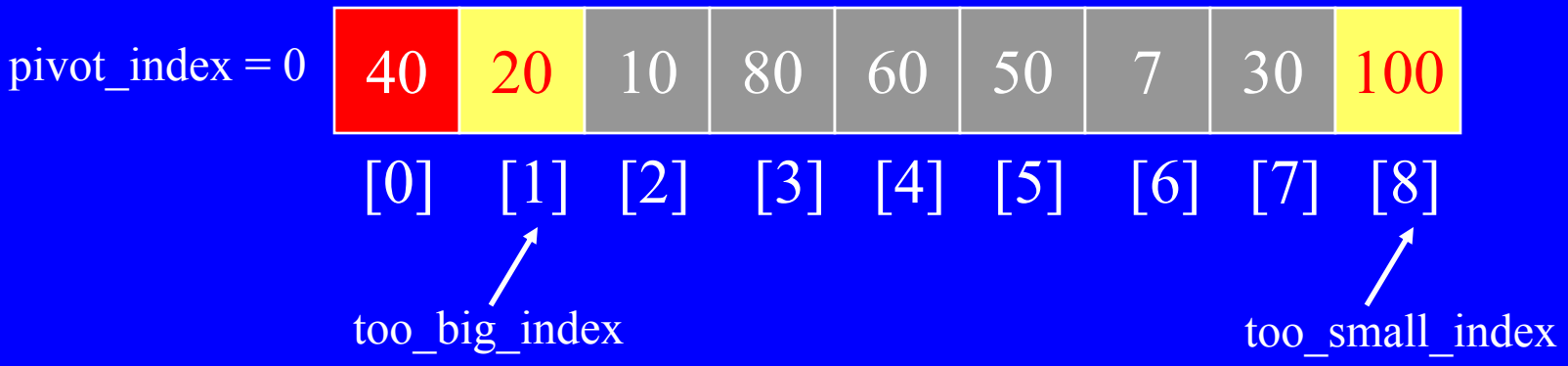

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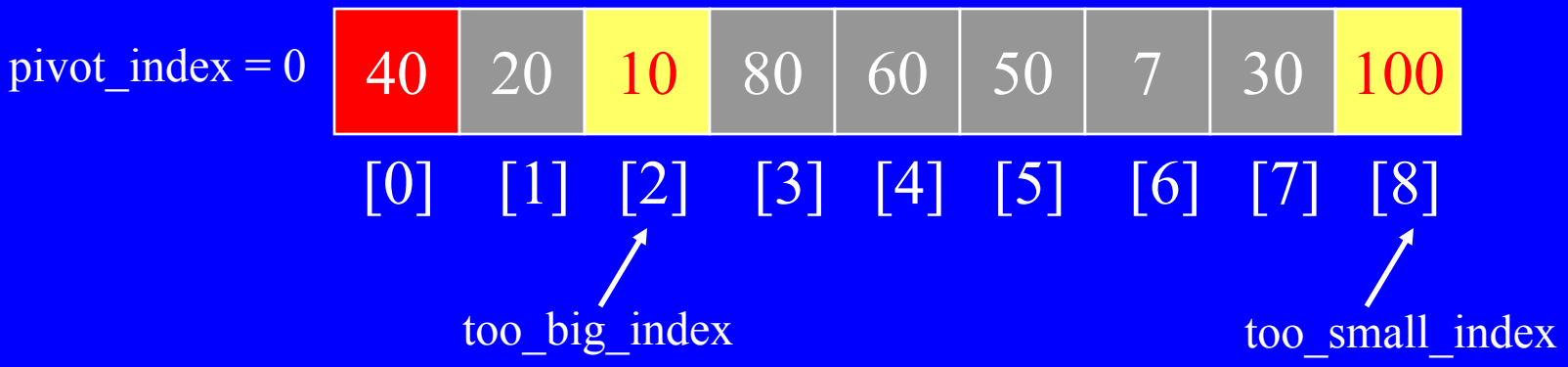

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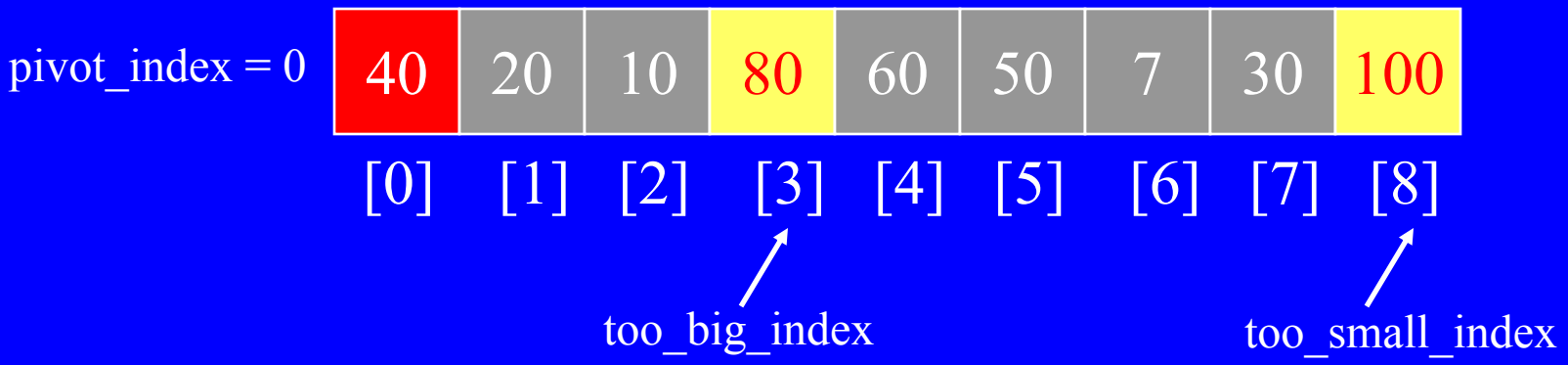

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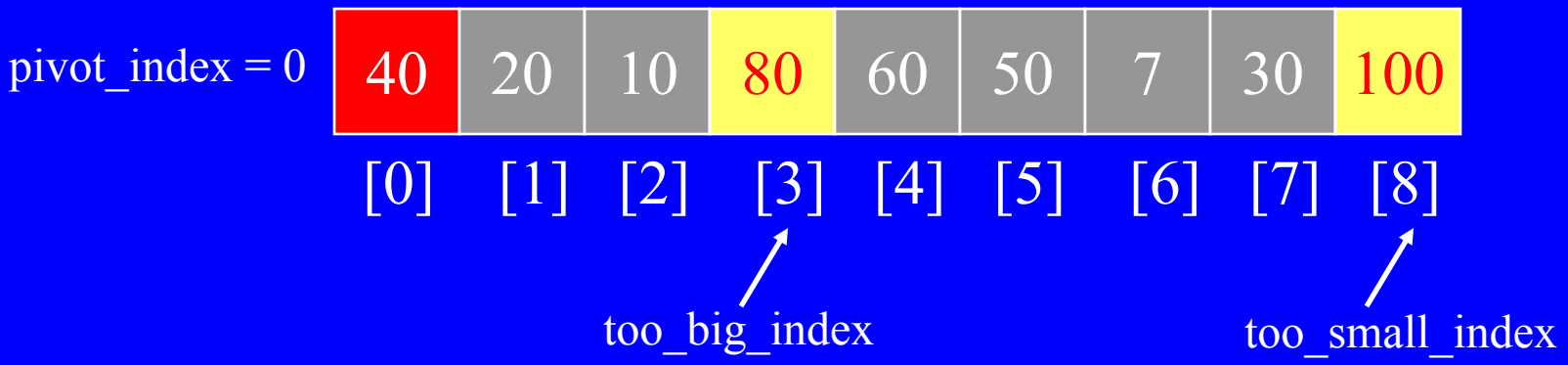

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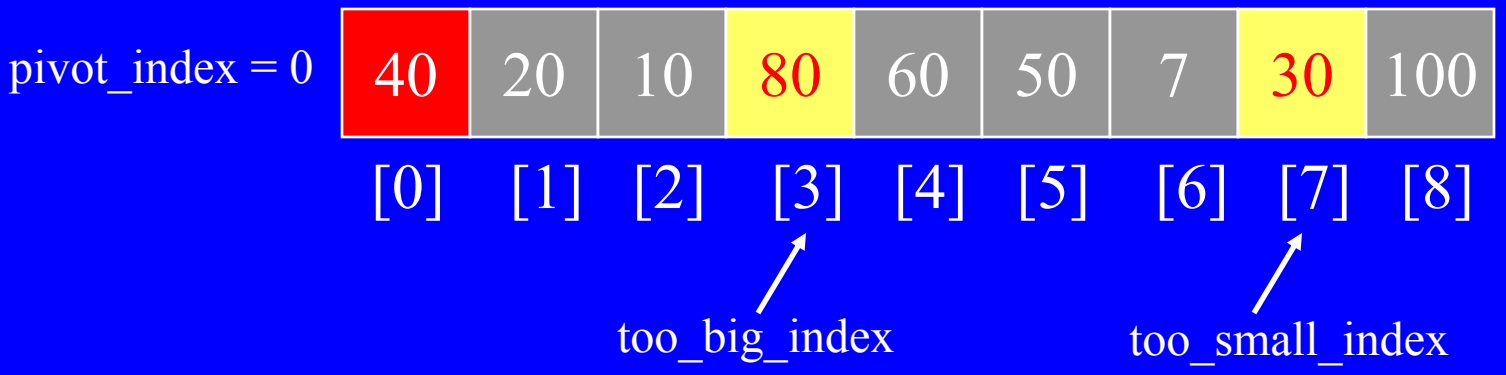

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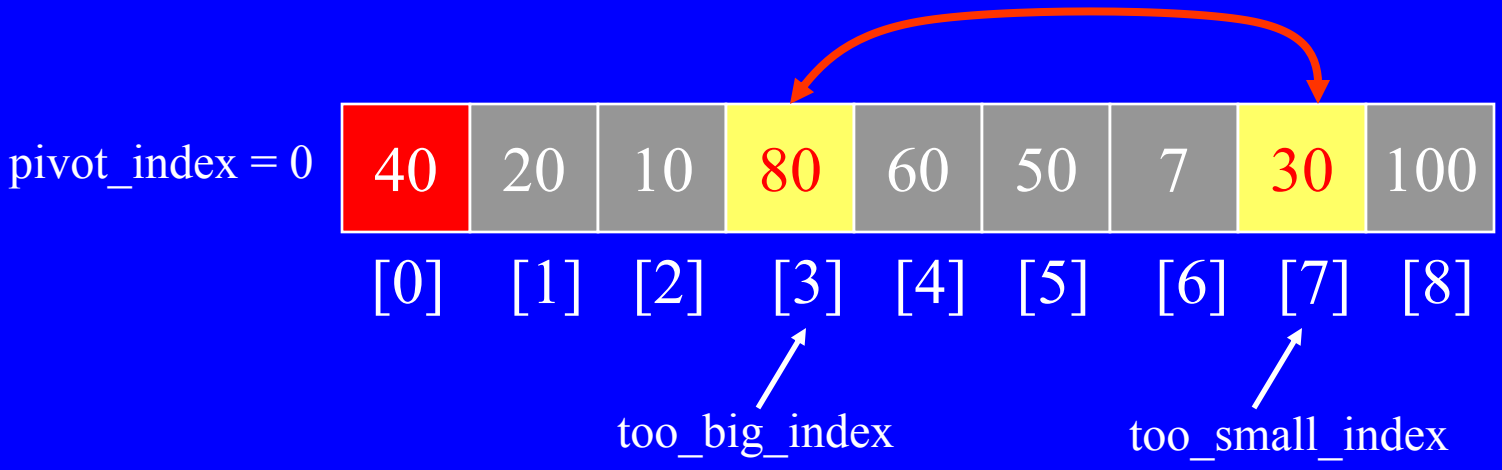

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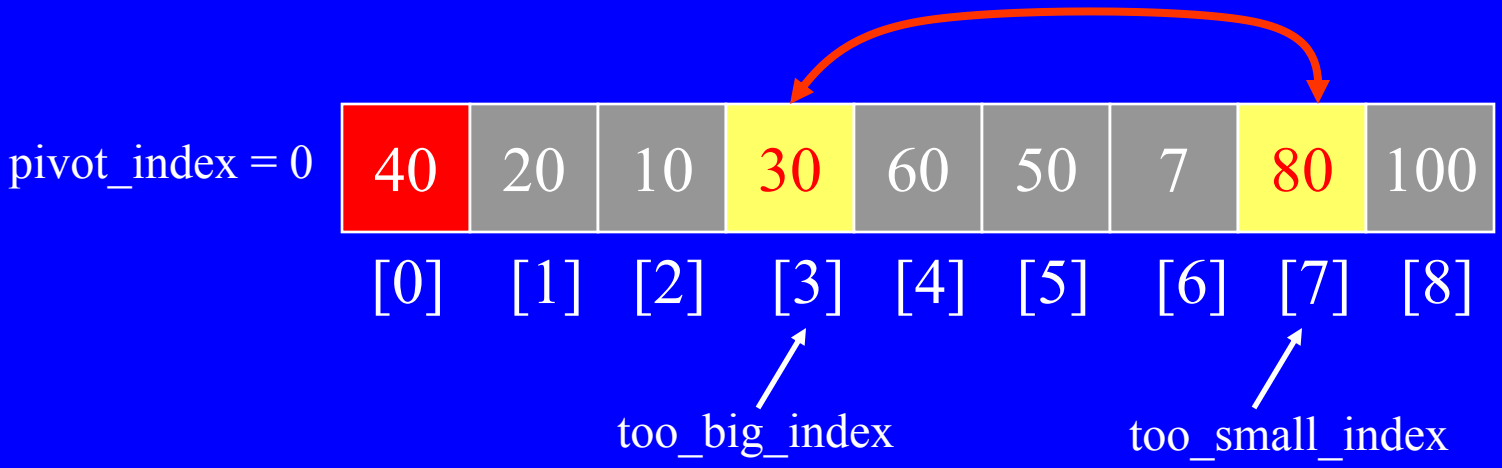

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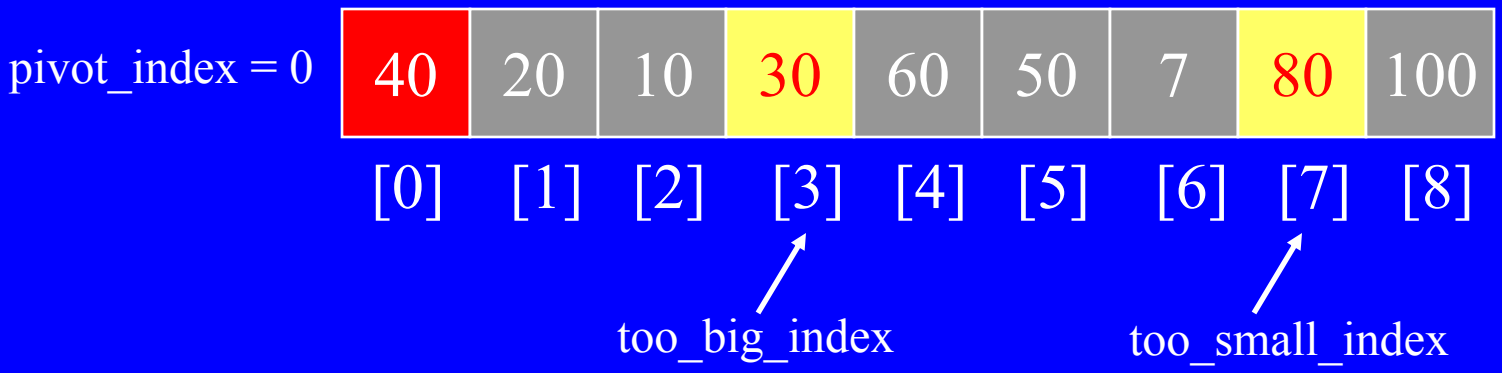

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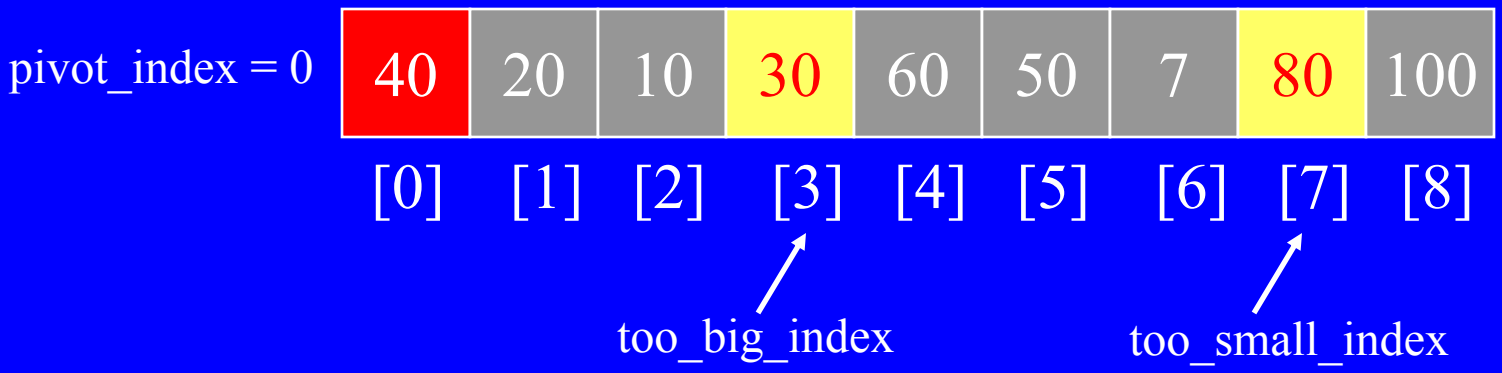

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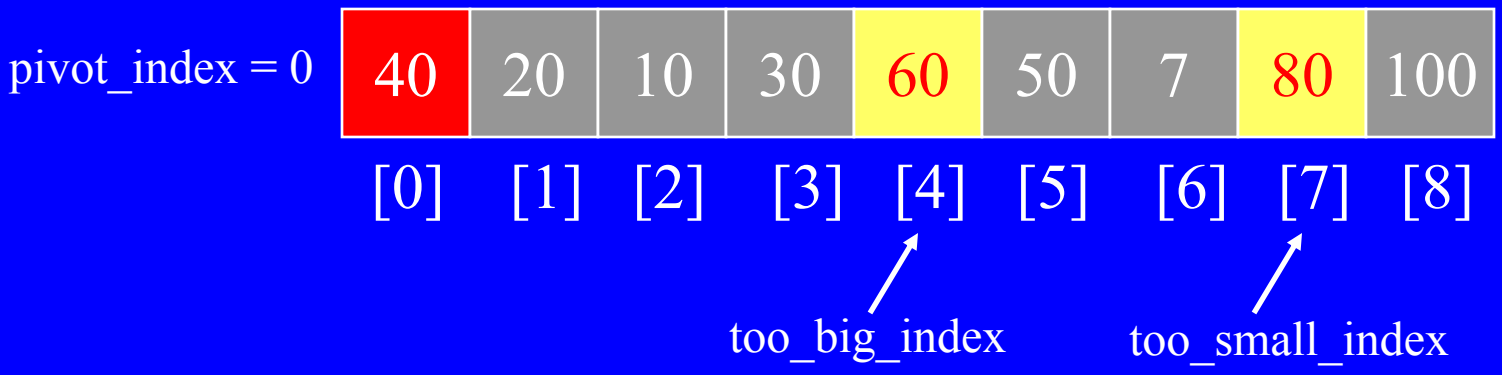

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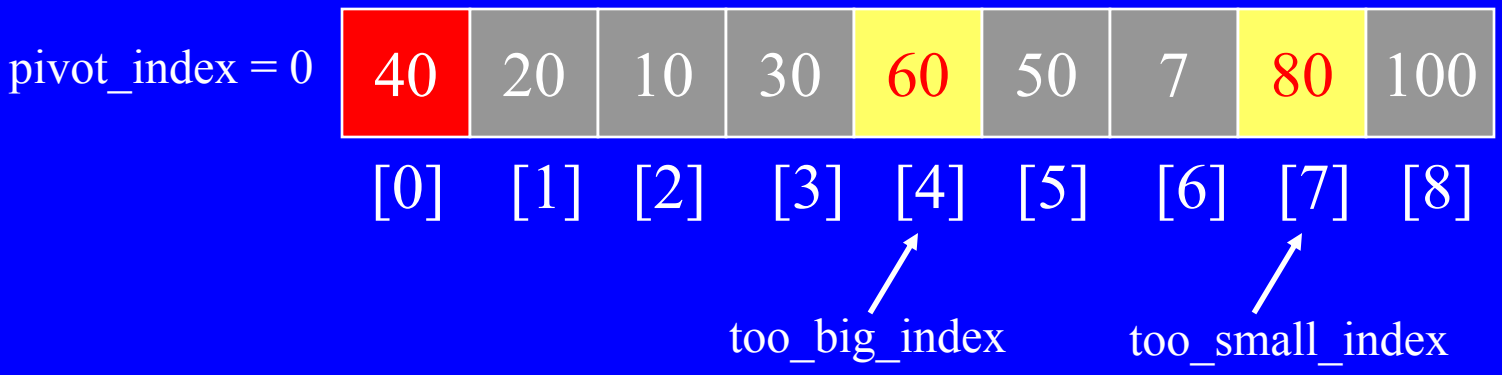

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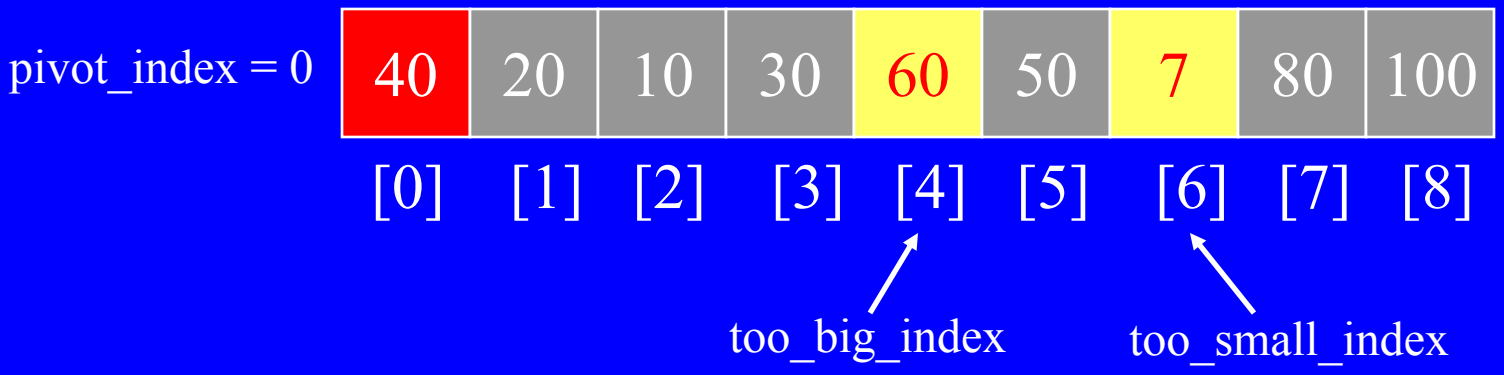

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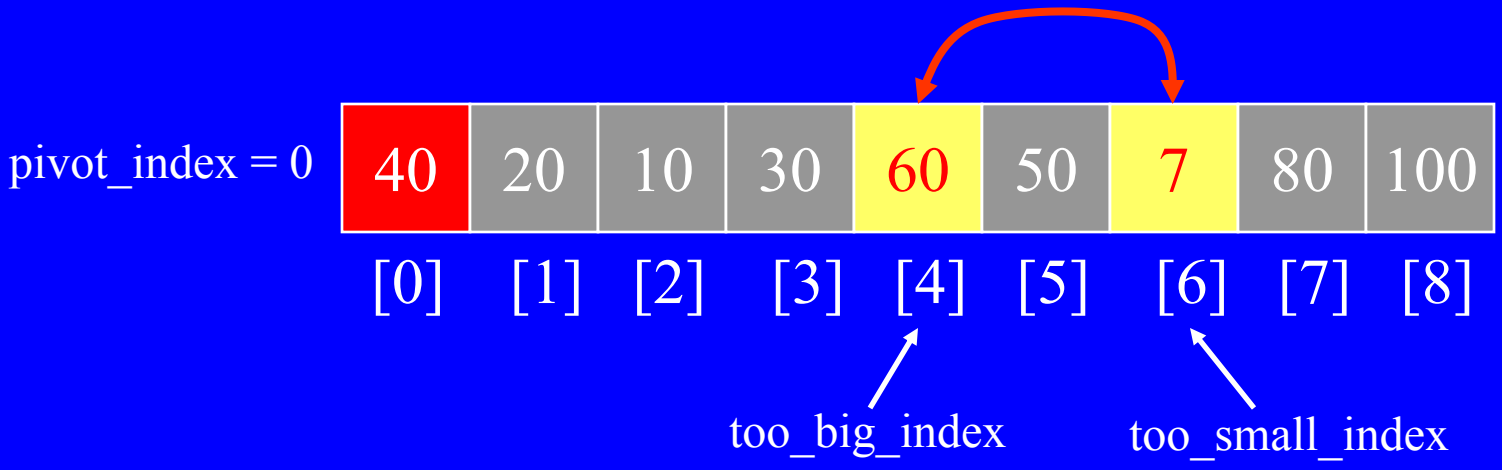

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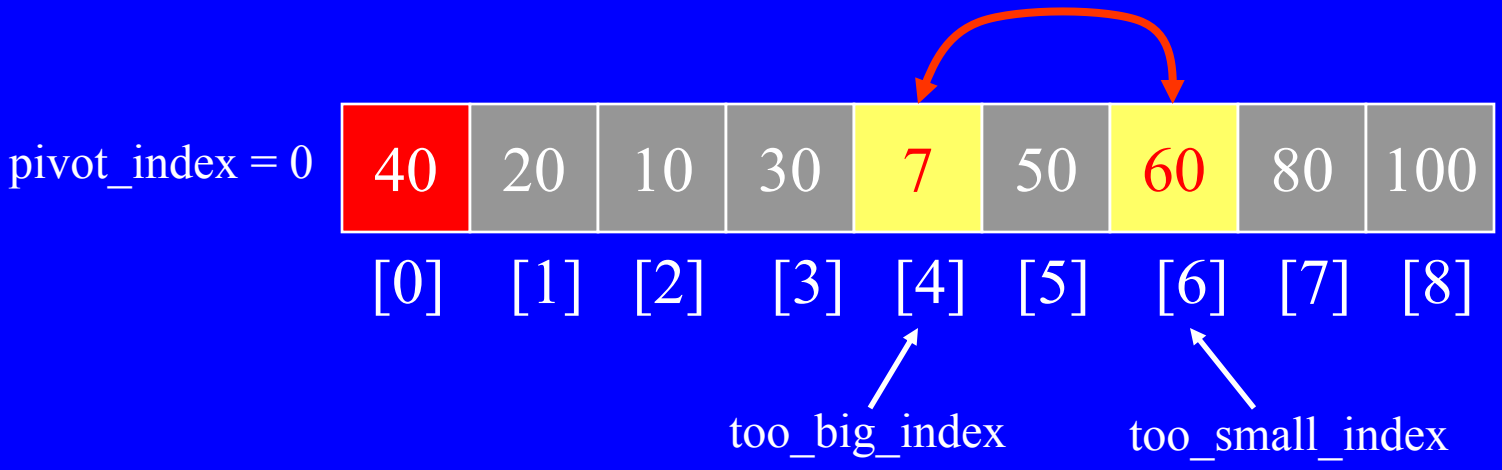

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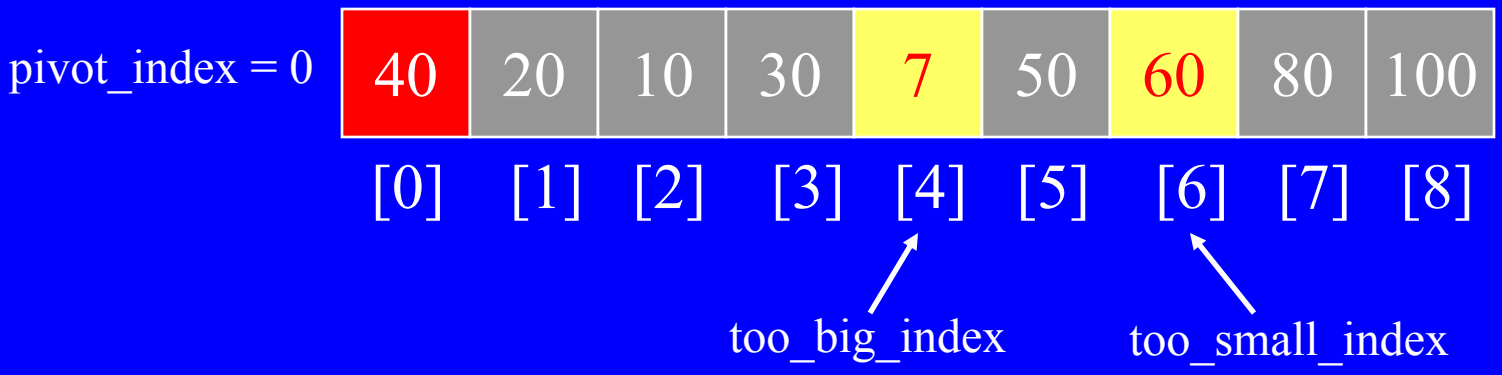

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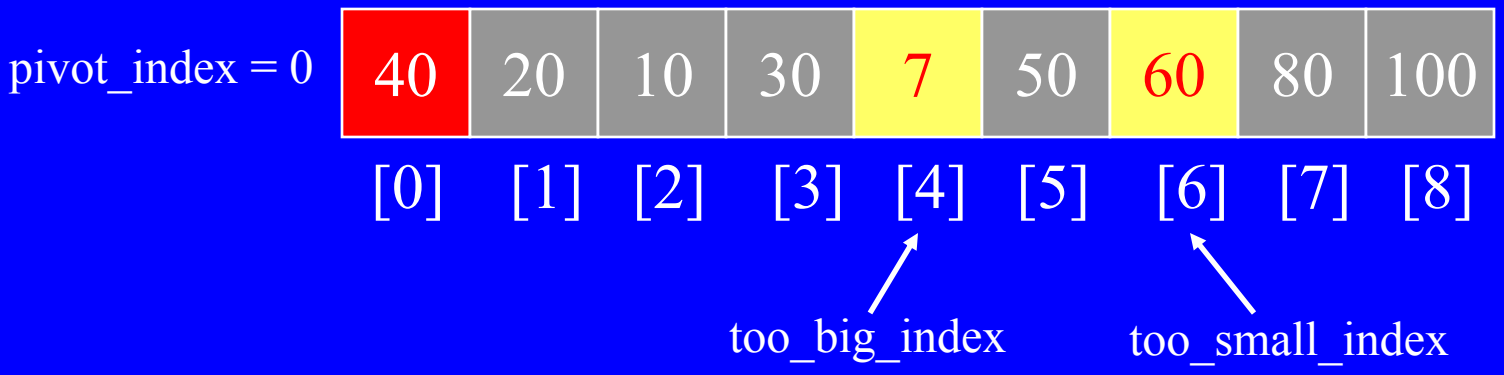

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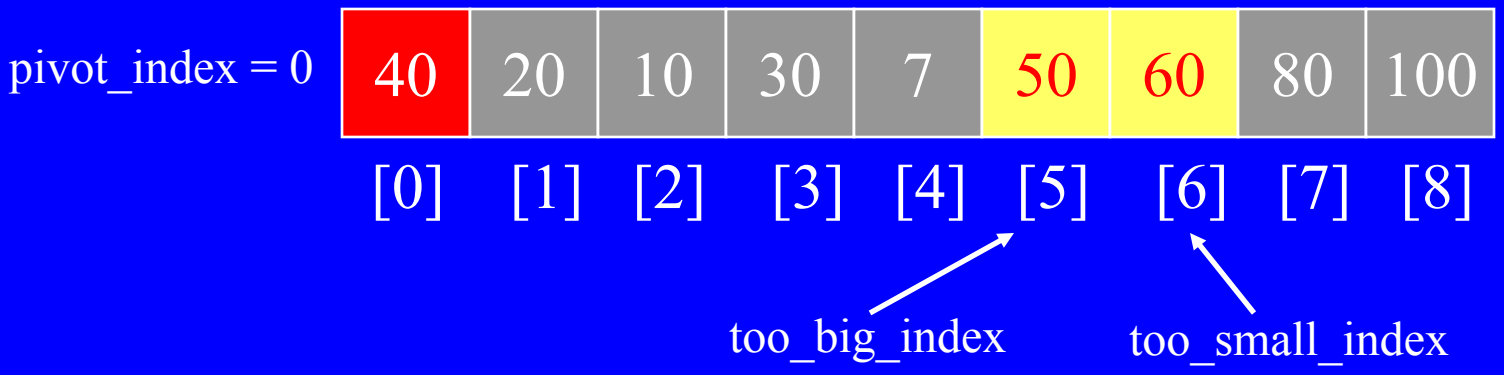

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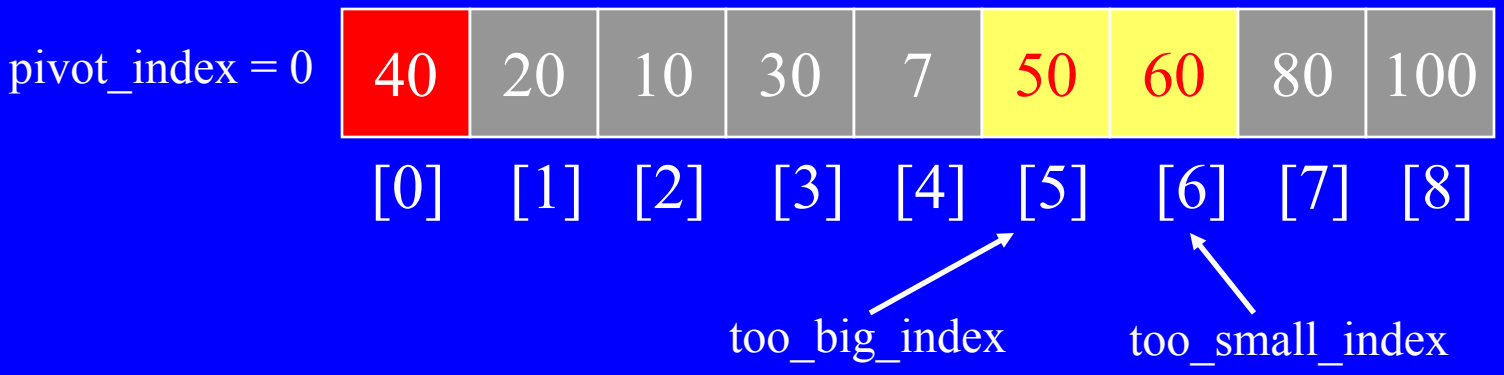

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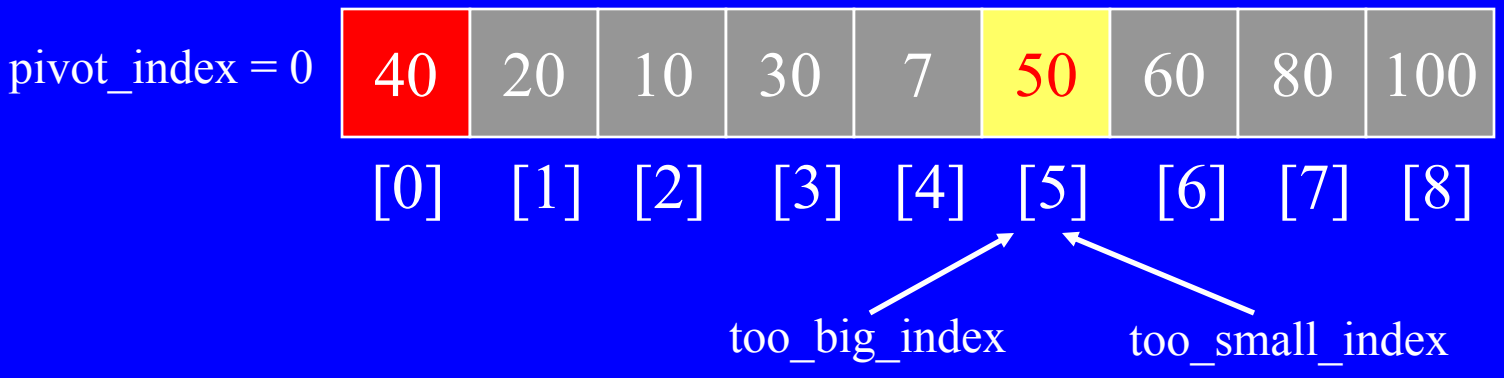

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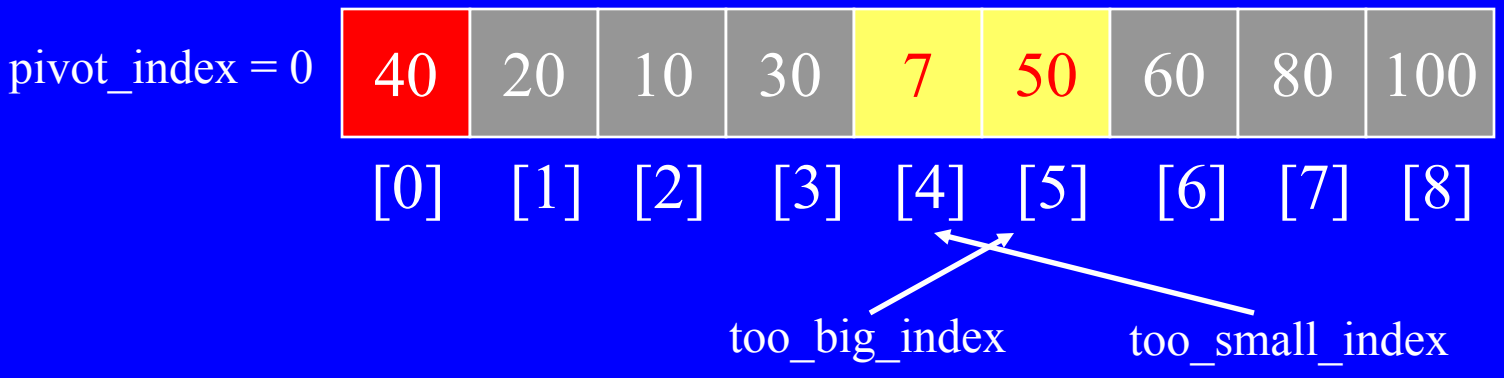

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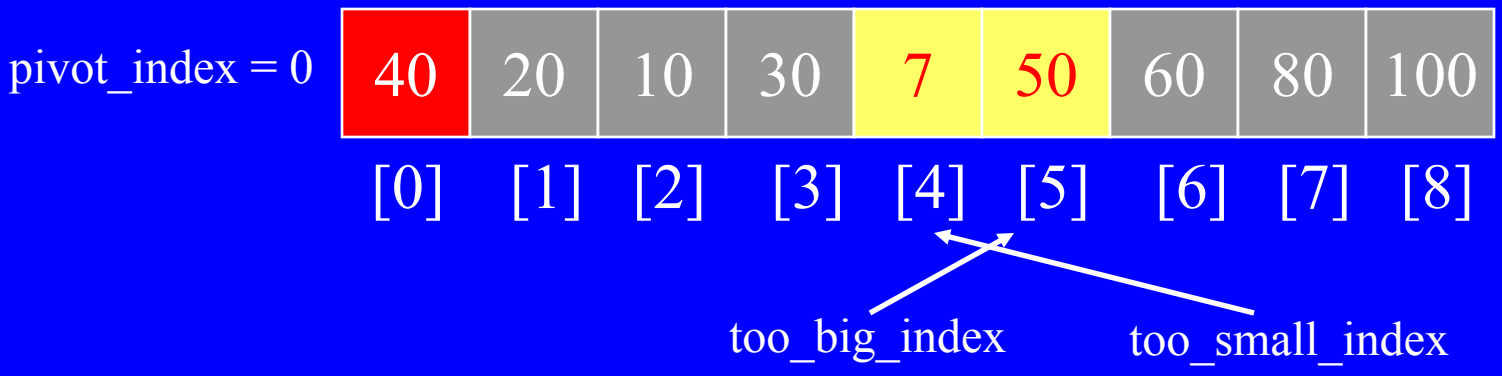

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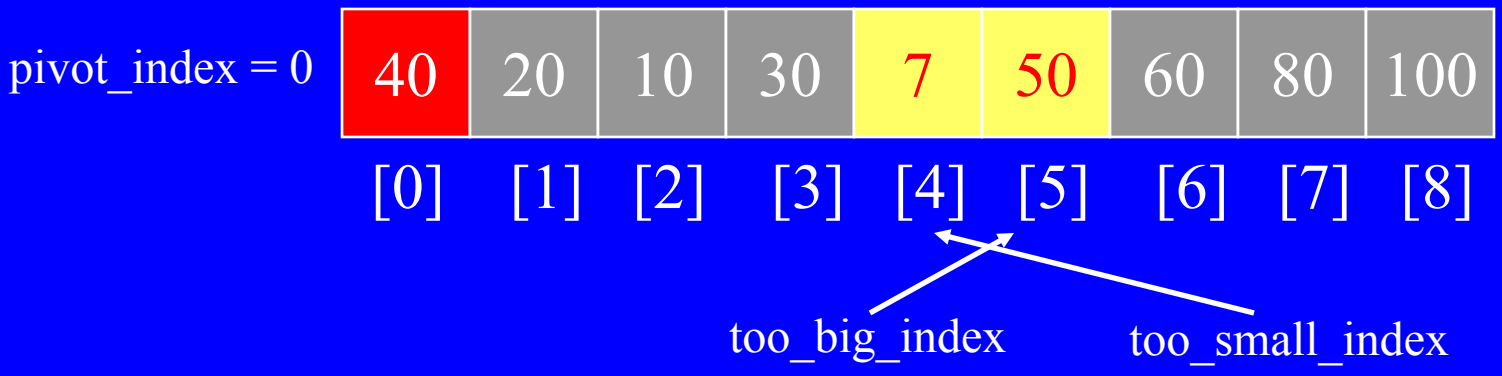

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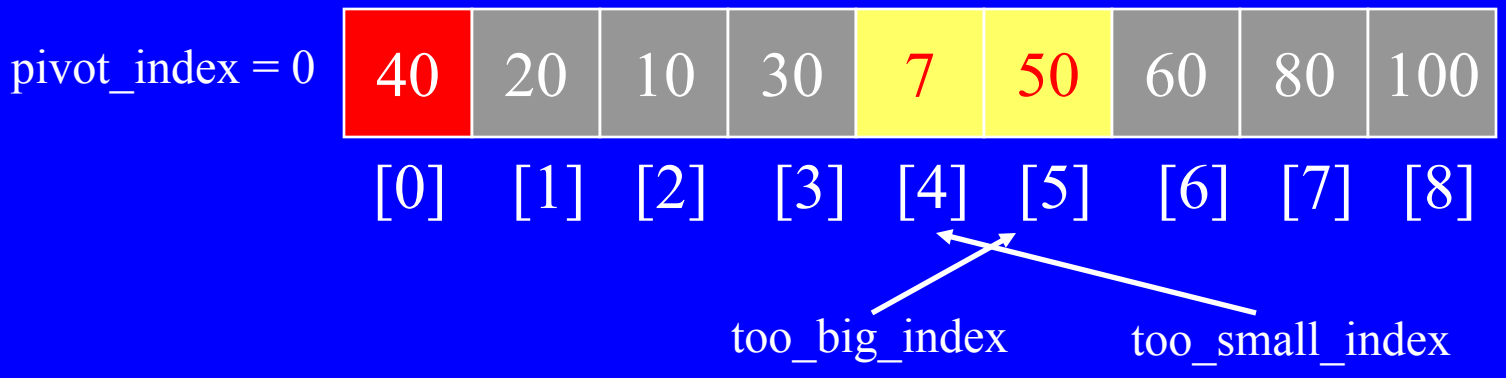

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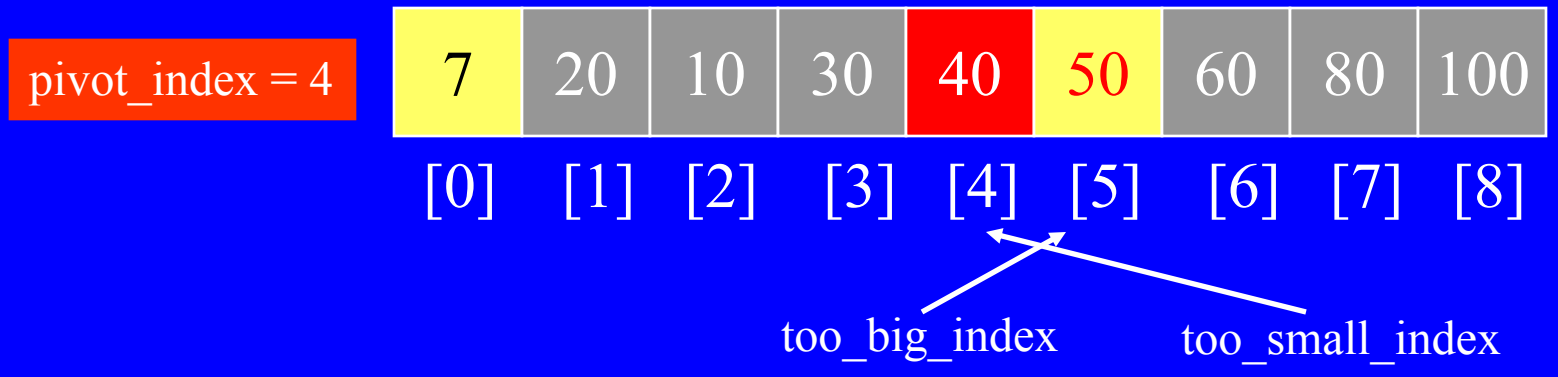
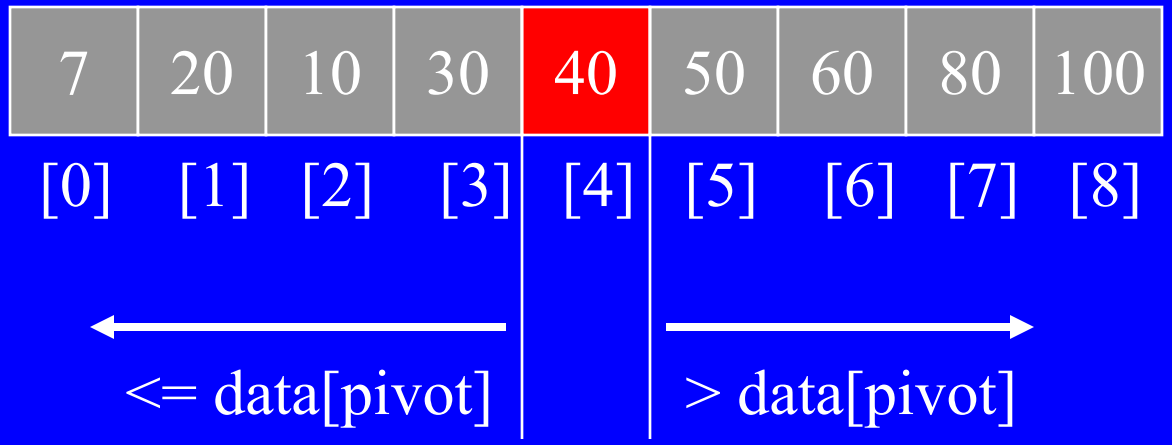
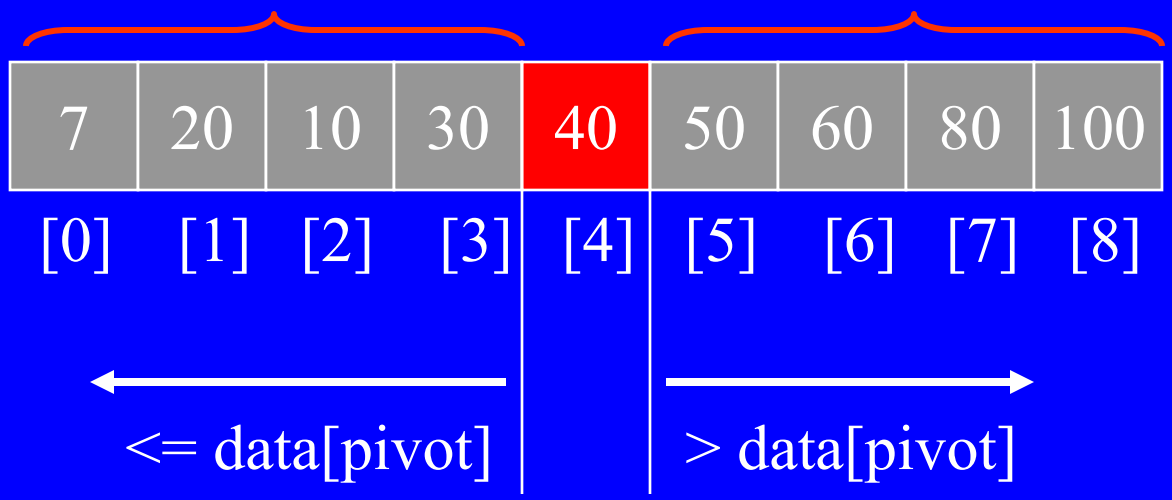


Result of PASS1



Recursion: Quicksort Sub-arrays



Next PASS: call quicksort recursively

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- What is best case running time?

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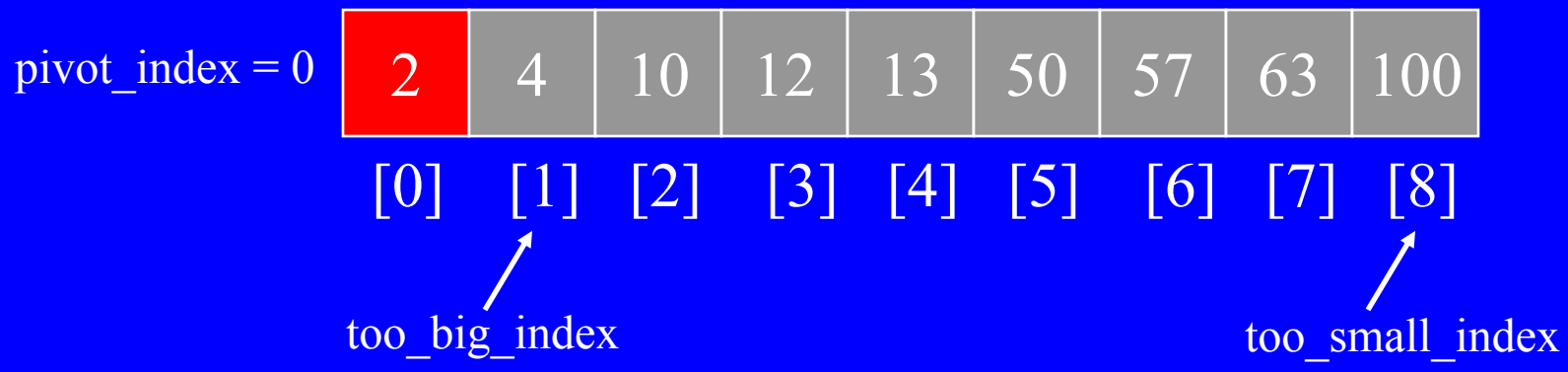
- Worst case running time?

Quicksort: Worst Case

- Assume first element is chosen as pivot.
- Assume we get array that is already in order:



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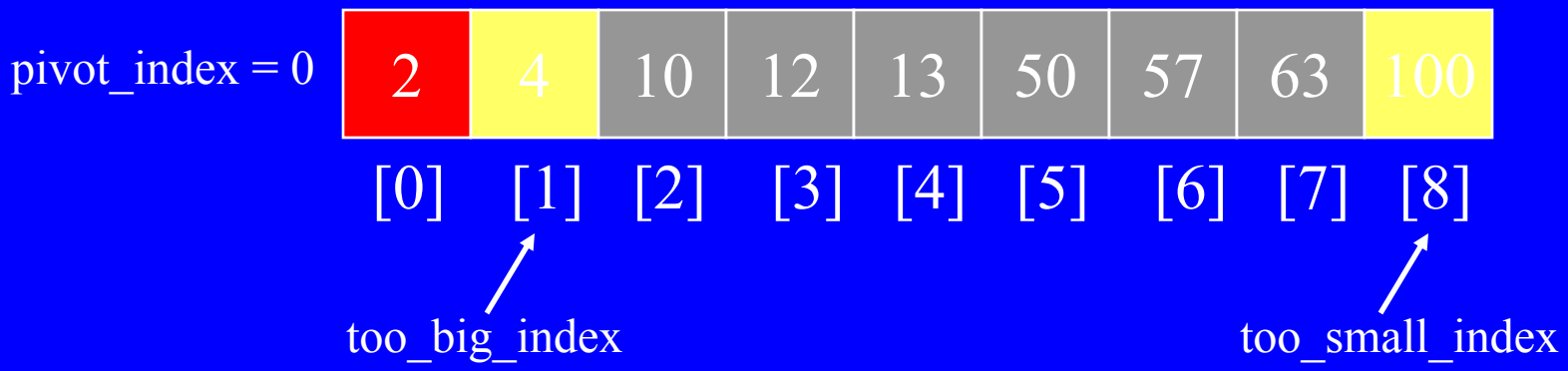

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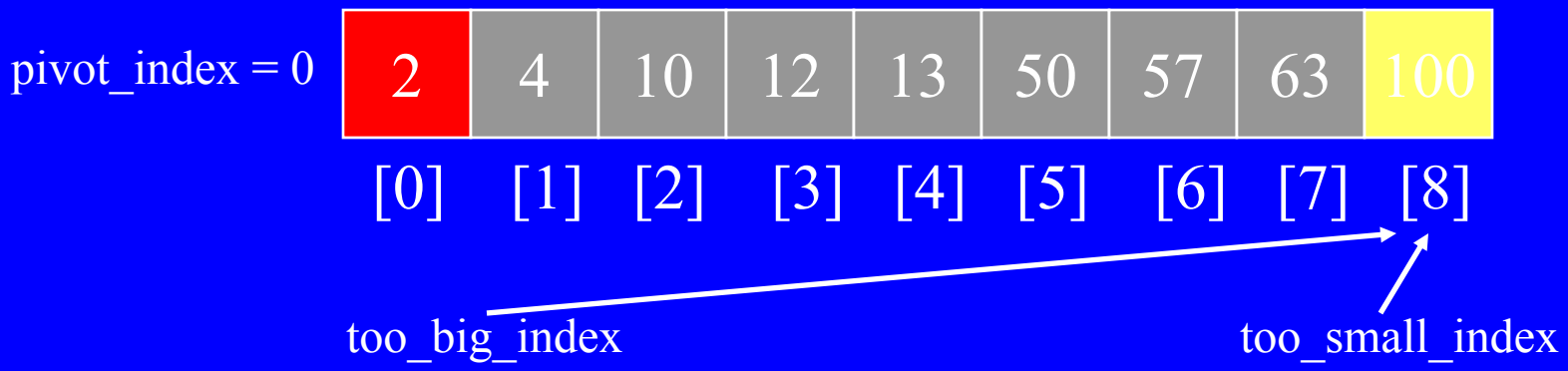

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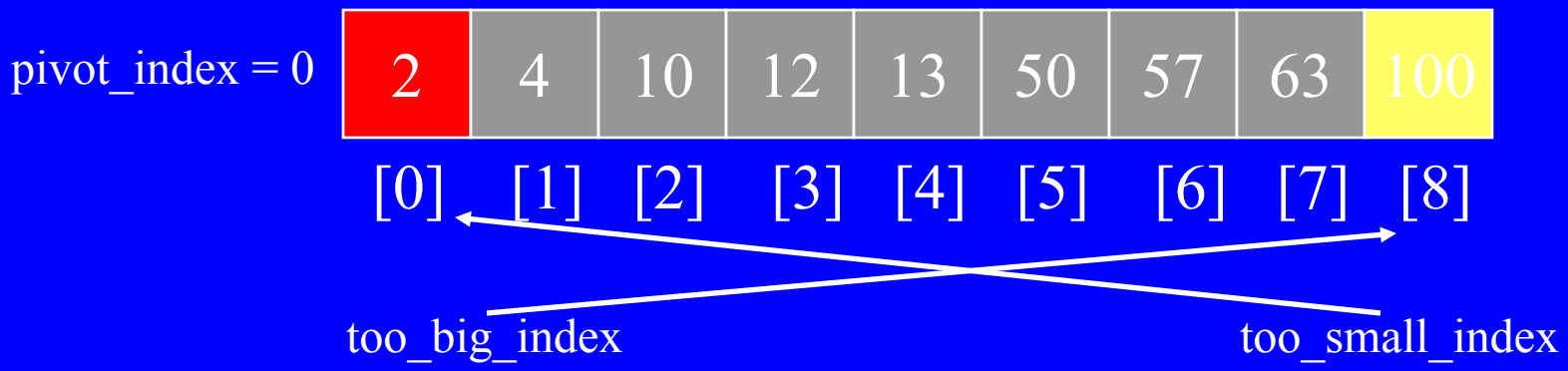

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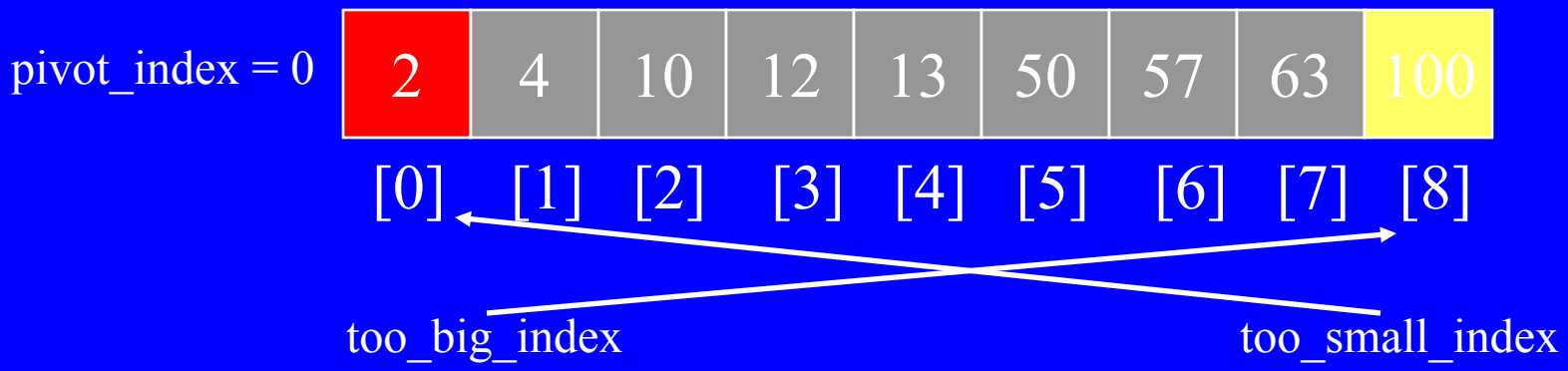

1. While data[too_big_index] <= data[pivot]
   ++too_big_index
2. While data[too_small_index] > data[pivot]
   --too_small_index
3. If too_big_index < too_small_index
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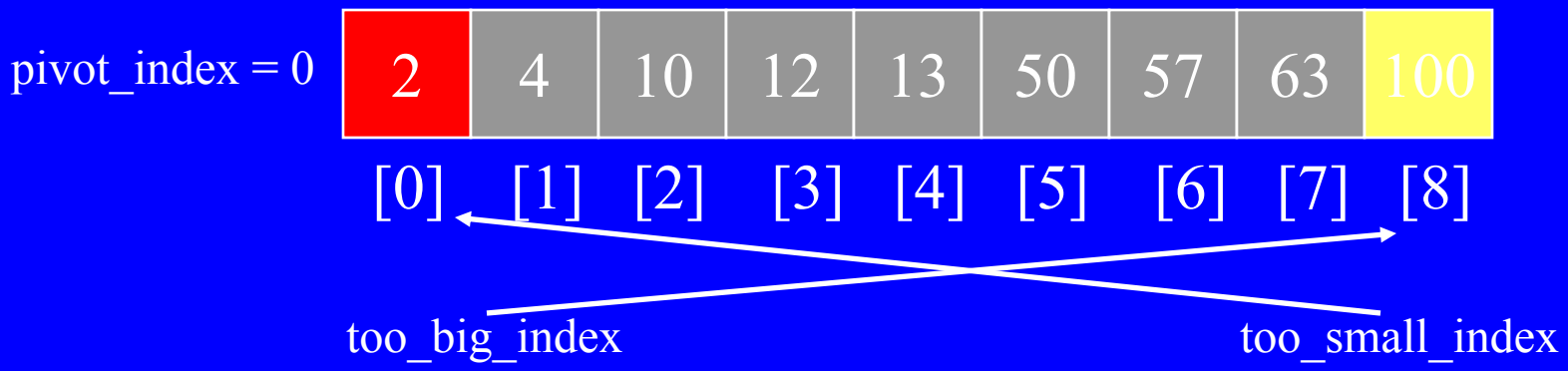

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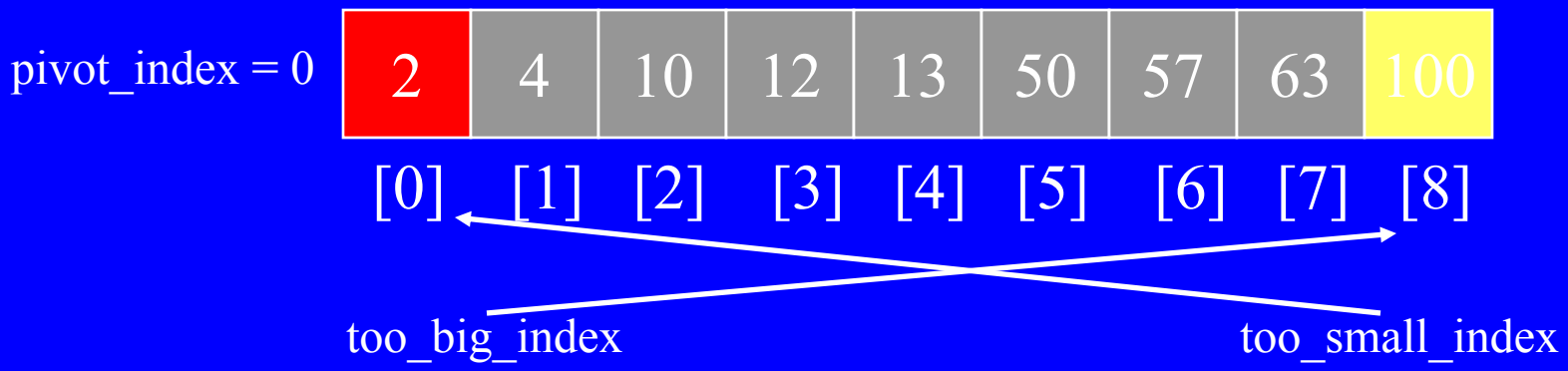

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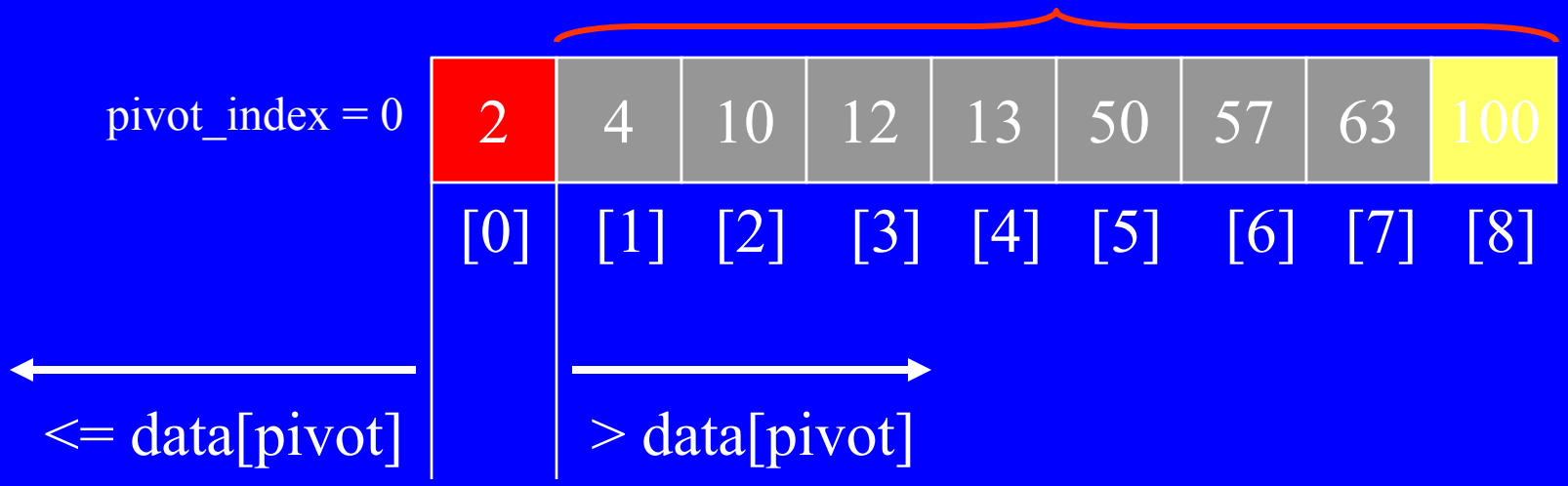

Quicksort Analysis

- Assume that keys are random, uniformly distributed.
- Best case running time: $O(n \log_2 n)$
- Worst case running time?
  - Recursion:
    1. Partition splits array in two sub-arrays:
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       - the other sub-array of size n-1
    2. Quicksort each sub-array
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- What can we do to avoid worst case?

Improved Pivot Selection

Pick median value of three elements from data array:

data[0], data[n/2], and data[n-1].

Use this median value as pivot.

Improving Performance of Quicksort

- Improved selection of pivot.
- For sub-arrays of size 3 or less, apply brute force search:
  - Sub-array of size 1: trivial
  - Sub-array of size 2:
    - if(data[first] > data[second]) swap them
  - Sub-array of size 3: left as an exercise.