Chapter 9
Hash

Data Structures and Algorithms

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Outcomes

- **L.O.5.1** - Depict the following concepts: hashing table, key, collision, and collision resolution.
- **L.O.5.2** - Describe hashing functions using pseudocode and give examples to show their algorithms.
- **L.O.5.3** - Describe collision resolution methods using pseudocode and give examples to show their algorithms.
- **L.O.5.4** - Implement hashing tables using C/C++.
- **L.O.5.5** - Analyze the complexity and develop experiment (program) to evaluate methods supplied for hashing tables.
- **L.O.1.2** - Analyze algorithms and use Big-O notation to characterize the computational complexity of algorithms composed by using the following control structures: sequence, branching, and iteration (not recursion).

Contents

Basic concepts

Basic concepts

- Sequential search: $O(n)$
- Binary search: $O(\log_2 n)$

$\rightarrow$ Requiring several key comparisons before the target is found.

Basic concepts

Search complexity:

Size	Binary	Sequential (Average)	Sequential (Worst Case)
16	4	8	16
50	6	25	50
256	8	128	256
1,000	10	500	1,000
10,000	14	5,000	10,000
100,000	17	50,000	100,000
1,000,000	20	500,000	1,000,000

Basic concepts

Is there a search algorithm whose complexity is $O(1)$?

Basic concepts

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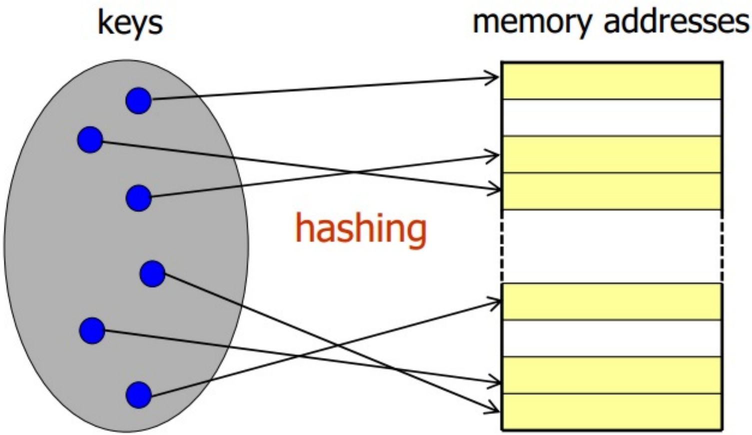
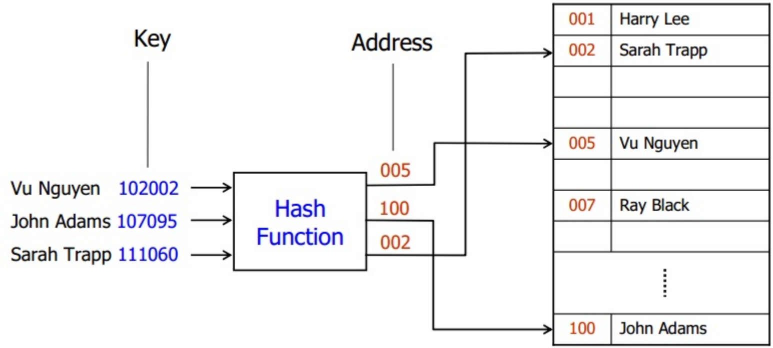
YES

Basic concepts



Hình: Each key has only one address

Basic concepts



Basic concepts

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Basic concepts

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- Prime area: memory that contains all the home addresses.
- Synonyms: a set of keys that hash to the same location.
- Collision: the location of the data to be inserted is already occupied by the synonym data.
- Ideal hashing:
  - No location collision
  - Compact address space

Basic concepts

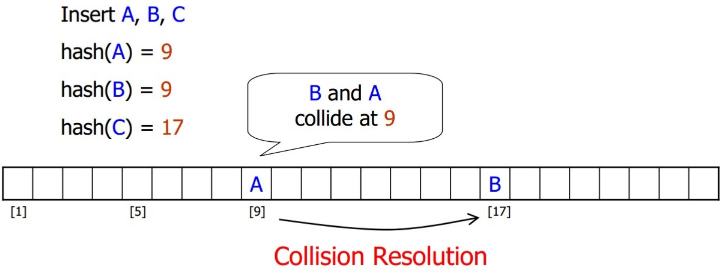
Insert A, B, C

hash(A) = 9

hash(B) = 9

hash(C) = 17

Basic concepts



Basic concepts

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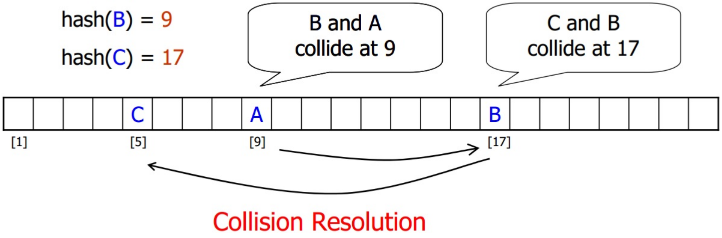


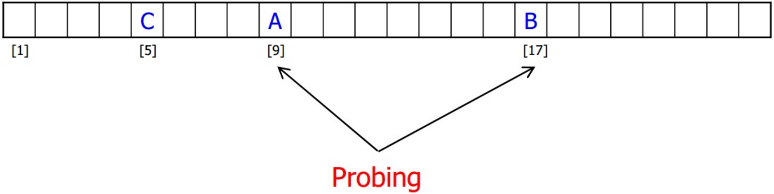
Basic concepts

Searh for B

hash(A) = 9

hash(B) = 9

hash(C) = 17



Hash functions

Hash functions

- Direct hashing
- Modulo division
- Digit extraction
- Mid-square
- Folding
- Rotation
- Pseudo-random

Direct Hashing

The address is the key itself:
$hash(Key) = Key$

Direct Hashing

- Advantage: there is no collision.
- Disadvantage: the address space (storage size) is as large as the key space.

Modulo division

$$Address = Key \ mod \ listSize$$

- Fewer collisions if $listSize$ is a prime number.
- Example:
  Numbering system to handle 1,000,000 employees
  Data space to store up to 300 employees
  $hash(121267) = 121267 \ mod \ 307 = 2$

Digit extraction

$$Address = selected\ digits\ from\ Key$$

Example:

379452→394

121267→112

378845→388

160252→102

045128→051

Mid-square

$$Address = middle\ digits\ of\ Key^2$$

Example:

9452 * 9452 = 89340304 → 3403

Mid-square

- Disadvantage: the size of the Key^2 is too large.
- Variations: use only a portion of the key.
  Example:
  379452: 379 * 379 = 143641 $\rightarrow$ 364
  121267: 121 * 121 = 014641 $\rightarrow$ 464
  045128: 045 * 045 = 002025 $\rightarrow$ 202

Folding

The key is divided into parts whose size matches the address size.

Example:
Key $= 123|456|789$
fold shift
$123 + 456 + 789 = 1368$
$\rightarrow 368$

Folding

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Example:

Key $= 123|456|789$

fold shift

$123 + 456 + 789 = 1368$

$\rightarrow 368$

fold boundary

$321 + 456 + 987 = 1764$

$\rightarrow 764$

Rotation

- Hashing keys that are identical except for the last character may create synonyms.
- The key is rotated before hashing.

original key	rotated key
600101	160010
600102	260010
600103	360010
600104	460010
600105	560010

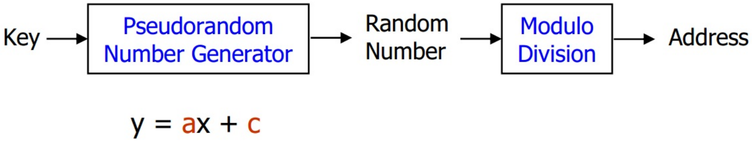
Rotation

- Used in combination with fold shift.

original key	rotated key
600101 → 62	160010 → 26
600102 → 63	260010 → 36
600103 → 64	360010 → 46
600104 → 65	460010 → 56
600105 → 66	560010 → 66

Spreading the data more evenly across the address space.

Pseudo-random



For maximum efficiency, a and c should be prime numbers.

Pseudo-random

Example:

Key = 121267

a = 17

c = 7

listSize = 307

Address = ((17*121267 + 7) mod 307

= (2061539 + 7) mod 307

= 2061546 mod 307

= 41

Collision resolution

Collision resolution

- Except for the direct hashing, none of the others are one-to-one mapping
  $\rightarrow$ Requiring collision resolution methods
- Each collision resolution method can be used independently with each hash function

Collision resolution

- Open addressing
- Linked list resolution
- Bucket hashing

Open addressing

When a collision occurs, an unoccupied element is searched for placing the new element in.

Open addressing

Hash function:

$$h : U \rightarrow \{0, 1, 2, ..., m-1\}$$

set of keys addresses

Open addressing

Hash and probe function:

$$hp : U \times \{0, 1, 2, ..., m-1\} \rightarrow \{0, 1, 2, ..., m-1\}$$

set of keys   probe numbers   addresses

Open Addressing

Algorithm hashInsert(ref T <array>, val k <key>)
Inserts key k into table T.

```
i = 0
while i < m do
    j = hp(k, i)
    if T[j] = nil then
        T[j] = k
        return j
    else
        i = i + 1
    end
end
return error: "hash table overflow"
```

End hashInsert

Open Addressing

Algorithm hashSearch(val T <array>, val k <key>)
Searches for key k in table T.

```
i = 0
while i < m do
    j = hp(k, i)
    if T[j] = k then
        return j
    else if T[j] = nil then
        return nil
    else
        i = i + 1
    end
end
return nil
End hashSearch
```

Open Addressing

There are different methods:

- Linear probing
- Quadratic probing
- Double hashing
- Key offset

Linear Probing

- When a home address is occupied, go to the next address (the current address + 1): $hp(k, i) = (h(k) + i)\ mod\ m$

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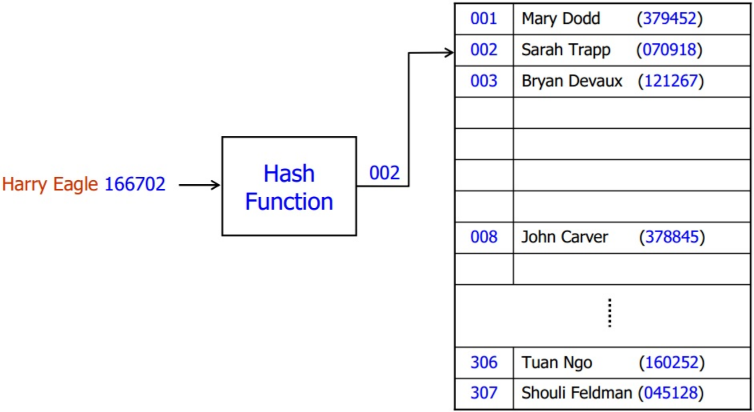
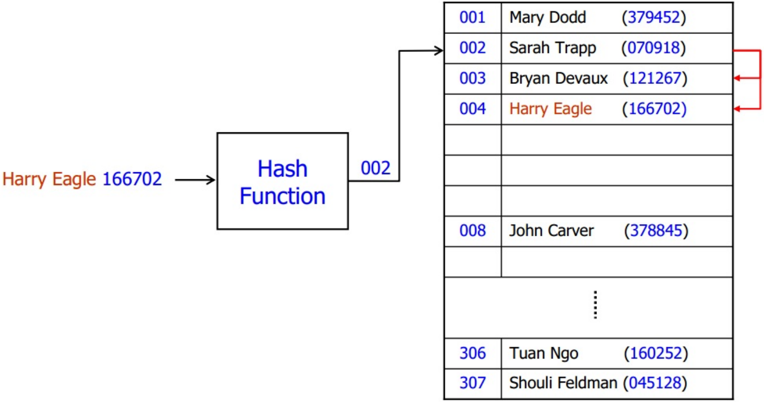


Linear Probing



Linear Probing

- Advantages:
  - quite simple to implement
  - data tend to remain near their home address (significant for disk addresses)
- Disadvantages:
  - produces primary clustering

Quadratic Probing

- The address increment is the collision probe number squared:
  $hp(k, i) = (h(k) + i^2) \; mod \; m$

Quadratic Probing

- Advantages:
  - works much better than linear probing
- Disadvantages:
  - time required to square numbers
  - produces secondary clustering
    $h(k_1) = h(k_2) \rightarrow hp(k_1, i) = hp(k_2, i)$

Double Hashing

- Using two hash functions:
  $hp(k, i) = (h_1(k) + ih_2(k))\ mod\ m$

Key Offset

- The new address is a function of the collision address and the key.

$$offset = [key/listSize]$$
$$newAddress = (collisionAddress + offset)\ mod\ listSize$$

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$$newAddress = (collisionAddress + offset) \; mod \; listSize$$

$$hp(k,i) = (hp(k,i-1) + [k/m]) \; mod \; m$$

Open addressing

Hash and probe function:

$$hp : U \times \{0, 1, 2, ..., m-1\} \rightarrow \{0, 1, 2, ..., m-1\}$$

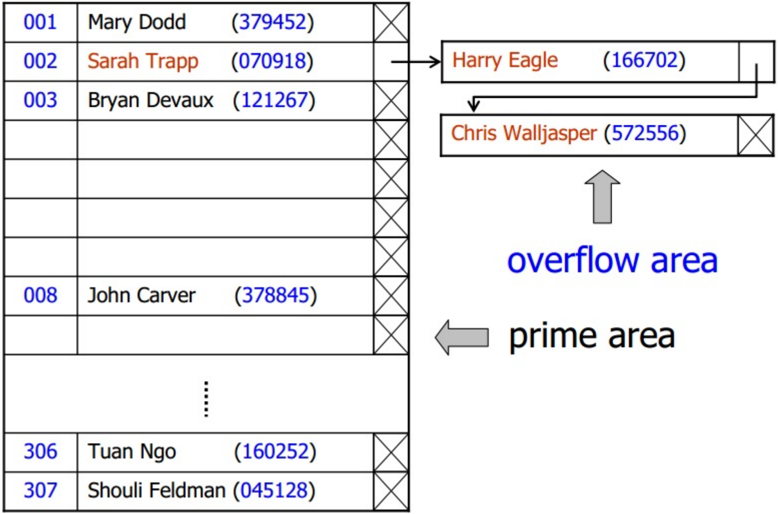
set of keys   probe numbers   addresses

$\{hp(k, 0), hp(k, 1), \ldots, hp(k, m-1)\}$ is a permutation of $\{0, 1, \ldots, m-1\}$

Linked List Resolution

- Major disadvantage of Open Addressing: each collision resolution increases the probability for future collisions.
  $\rightarrow$ use linked lists to store synonyms

Linked list resolution



Bucket hashing

- Hashing data to buckets that can hold multiple pieces of data.
- Each bucket has an address and collisions are postponed until the bucket is full.

Bucket hashing