FUNDAMENTALS OF FLUID MECHANICS

Chapter 8 Pipe Flow

MAIN TOPICS

- General Characteristics of Pipe Flow
- Fully Developed Laminar Flow
- Fully Developed Turbulent Flow
- Dimensional Analysis of Pipe Flow
- Pipe Flow Examples
- Pipe Flowrate Measurement

Introduction

- Flows completely bounded by solid surfaces are called **INTERNAL FLOWS** which include flows through **pipes (**Round cross section**), ducts (NOT** Round cross section**)**, nozzles, diffusers, sudden contractions and expansions, valves, and fittings.
- The basic principles involved are independent of the cross-sectional shape, although the details of the flow may be dependent on it.
- The flow regime (laminar or turbulent) of internal flows is primarily a function of the Reynolds number (->inertial force/viscous force).
  - ⇨Laminar flow: Can be solved analytically.
  - ⇨Turbulent flow: Rely heavily on semi-empirical theories and experimental data.

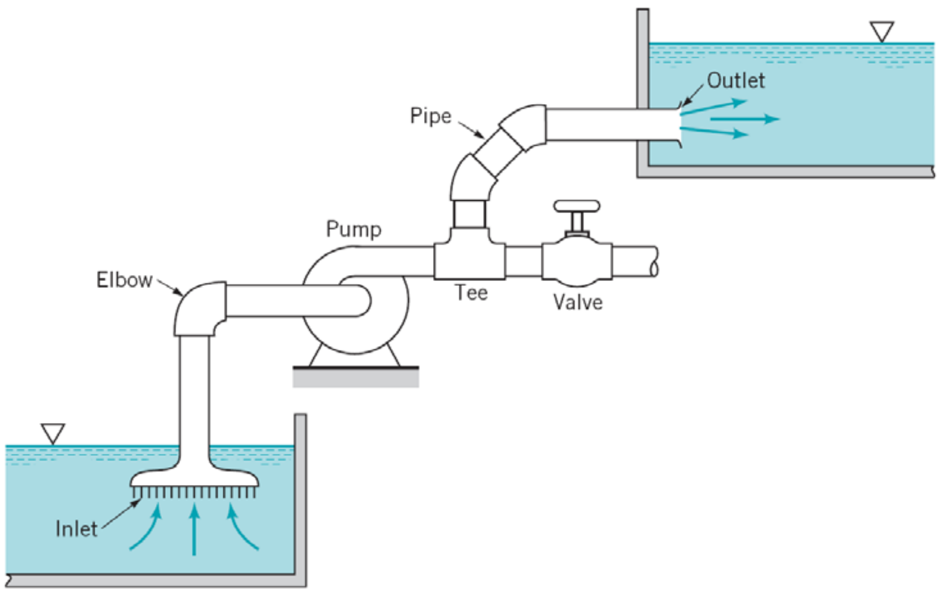
General Characteristics of Pipe Flow

Laminar vs. Turbulent

Entrance Region vs. Fully Developed Flow

Pipe System

❖ A pipe system include the pipes themselves (perhaps of more than one diameter), the various fittings, the flowrate control devices valves) , and the pumps or turbines.



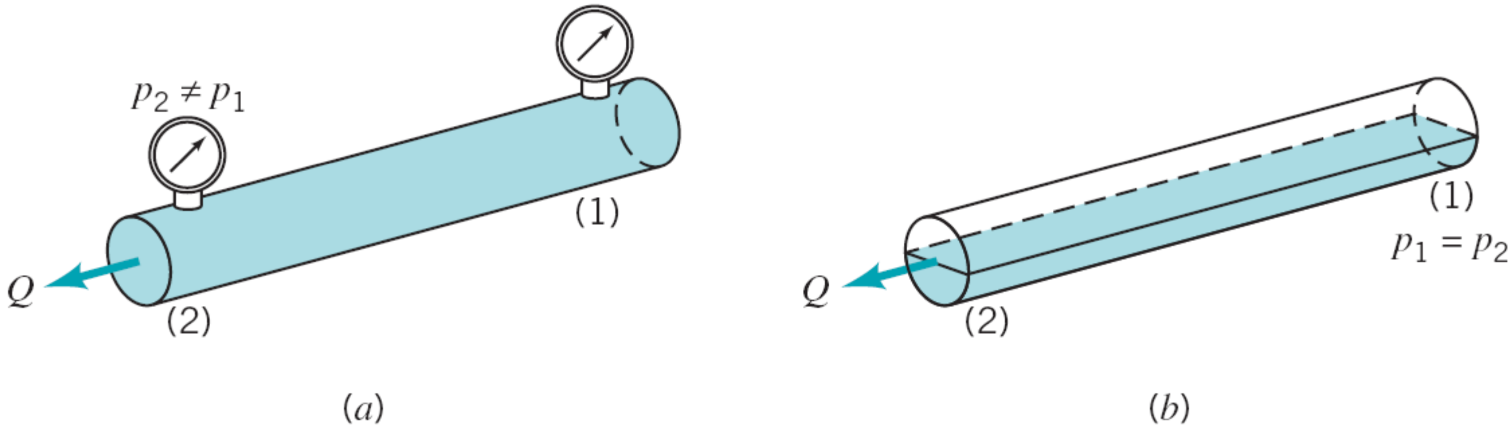
Pipe Flow vs. Open Channel Flow

- Pipe flow: Flows completely filling the pipe. (a)

  The pressure gradient along the pipe is main driving force.

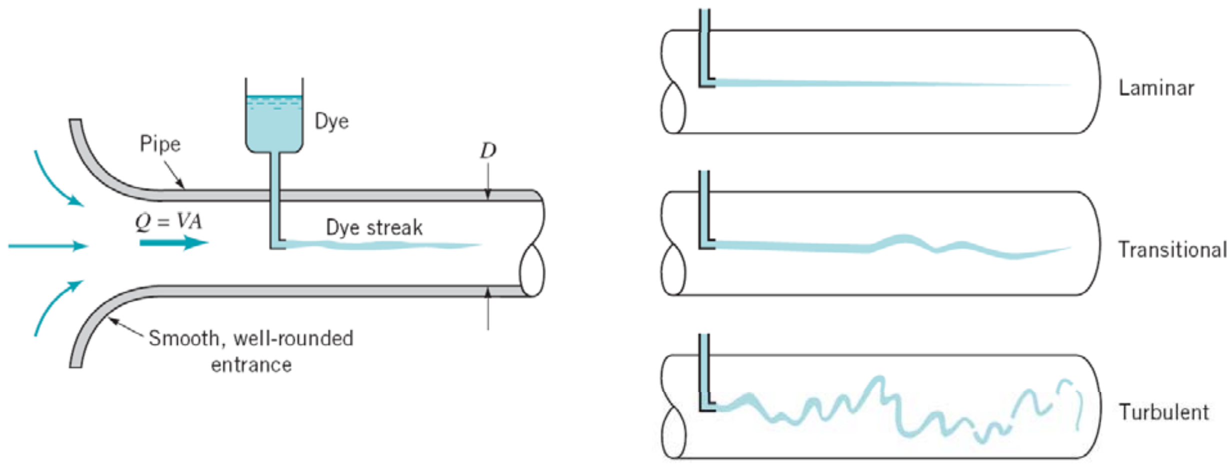
- Open channel flow: Flows without completely filling the pipe. (b)

  The gravity alone is the driving force.



Laminar or Turbulent Flow 1/2

- The flow of a fluid in a pipe may be **Laminar ? Or Turbulent ?**
- **Osborne Reynolds**, a British scientist and mathematician, was the first to distinguish the difference between these classification of flow by using a **simple apparatus** as shown.

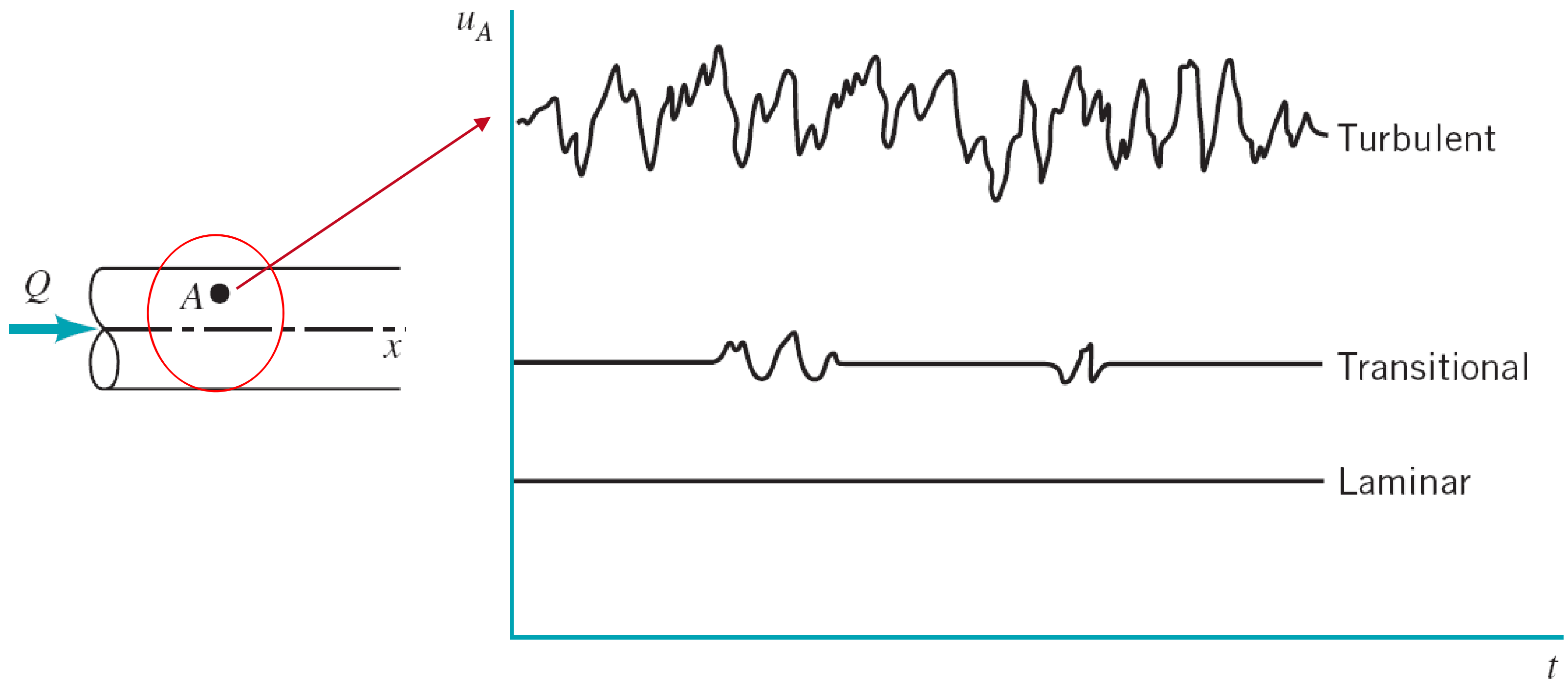


(a) (b)

Laminar or Turbulent Flow 2/2

- ⇨For "**small enough flowrate**" the dye streak will remain as a well-defined line as it flows along, with only slight blurring due to molecular diffusion of the dye into the surrounding water.
- ⇨For a somewhat larger "**intermediate flowrate**" the dye fluctuates in time and space, and intermittent bursts of irregular behavior appear along the streak.
- ⇨For "**large enough flowrate**" the dye streak almost immediately become blurred and spreads across the entire pipe in a random fashion.

Time Dependence of Fluid Velocity at a Point

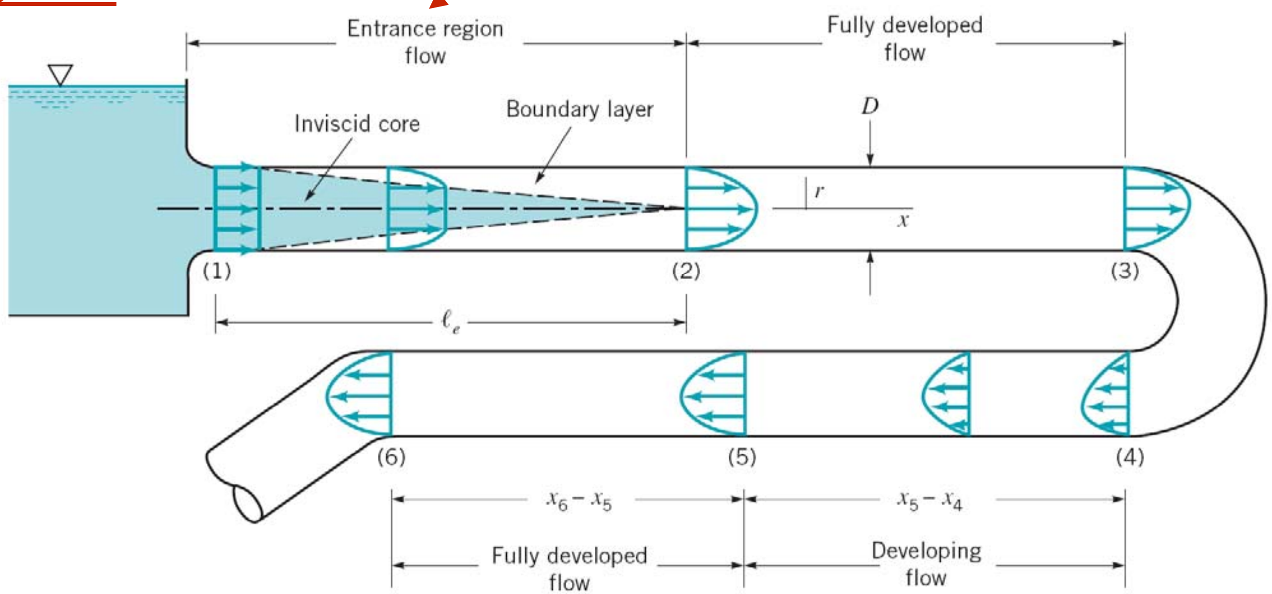


Indication of Laminar or Turbulent Flow

- The term **flowrate** should be replaced by Reynolds number, $R_e = \rho VL / \mu$,where V is the average velocity in the pipe, and L is the characteristic dimension of a flow. L is usually D *(diameter)* in a pipe flow. -> a measure of inertial force to the viscous force.
- It is **not only the fluid velocity** that determines the character of the flow – its density, viscosity, and the pipe size are of equal importance.
- For general engineering purpose, the flow in a **round pipe**
  - ⇨**Laminar** $R_e < 2100$
  - ⇨**Transitional**
  - ⇨**Turbulent** $R_e > 4000$

Entrance Region and Fully Developed Flow 1/5

- Any fluid **flowing in a pipe** had to enter the pipe at some location.
- The region of flow near where the fluid enters the pipe is termed the ***entrance (entry) region or developing flow region***.



Entrance Region and Fully Developed Flow 2/5

- The fluid typically enters the pipe with a nearly uniform velocity profile at section (1).
- As the fluid moves through the pipe, viscous effects cause it to stick to the pipe wall **(the no slip boundary condition)**.

Entrance Region and Fully Developed Flow 3/5

- A boundary layer in which viscous effects are important is produced along the pipe wall such that the initial velocity profile changes with distance along the pipe, x , until the fluid reaches the end of the **entrance length, section (2),** beyond which the velocity profile does not vary with x.
- The boundary layer has grown in thickness to completely fill the pipe.

Entrance Region and Fully Developed Flow 4/5

- Viscous effects are of considerable importance within the boundary layer. Outside the boundary layer, the viscous effects are negligible.
- The shape of the velocity profile in the pipe depends on whether the flow is laminar or turbulent, as does the length of the entrance region, ℓ_ℓ .

For laminar flow

$$\frac{\ell_\ell}{D} = 0.06 R_e$$

Dimensionless entrance length

For turbulent flow

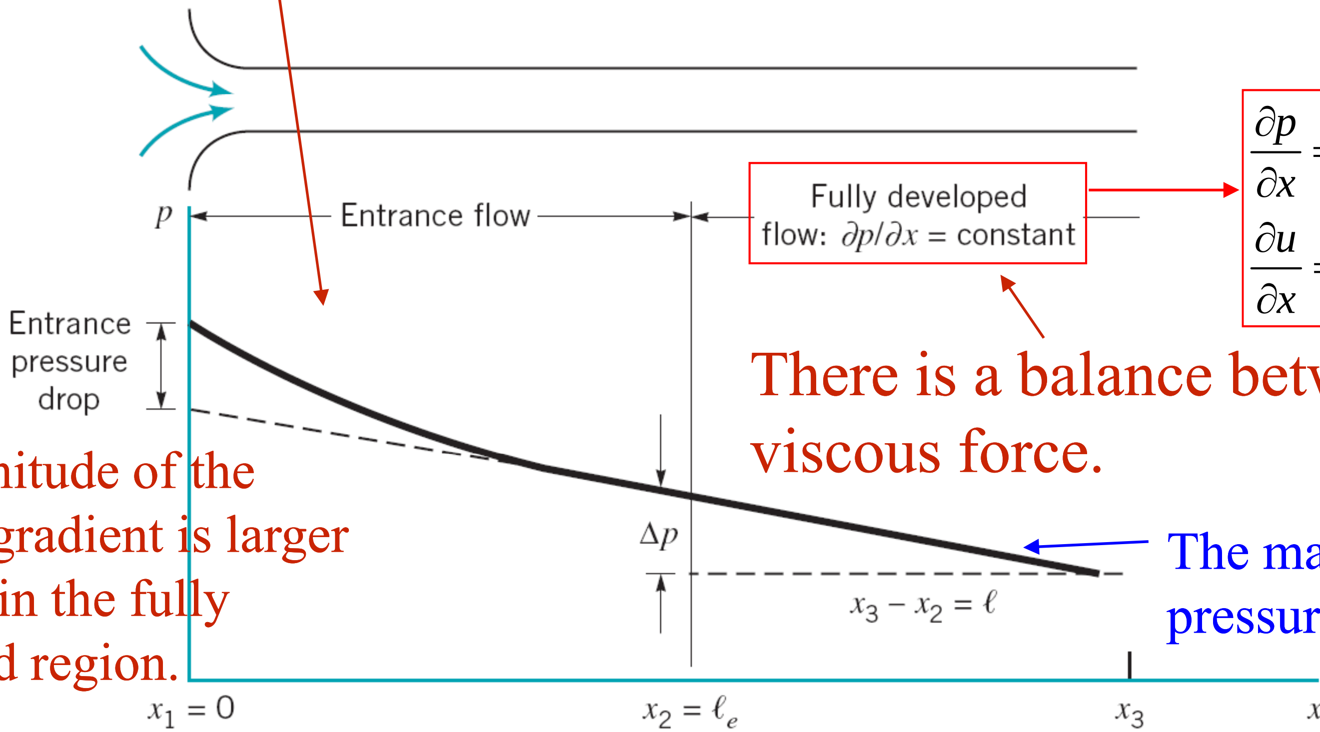
$$\frac{\ell_\ell}{D} = 4.4 R_e^{1/6}$$

Entrance Region and Fully Developed Flow 5/5

- Once the fluid reaches the end of the entrance region, section (2), the flow is simpler to describe because *the velocity is a function of only the distance from the pipe centerline, r, and independent of x*.
- The flow between (2) and (3) is termed **fully developed.**

Pressure Distribution along Pipe

In the entrance region of a pipe, the fluid accelerates or decelerates as it flows. There is a balance between pressure, viscous, and <u>inertia (acceleration) force</u>.



$$\frac{\partial p}{\partial x} = -\frac{\Delta p}{\ell} = \text{constant} < 0$$

$$\frac{\partial u}{\partial x} = 0$$

There is a balance between pressure and viscous force.

The magnitude of the pressure gradient is larger than that in the fully developed region.

The magnitude of the pressure gradient is constant.

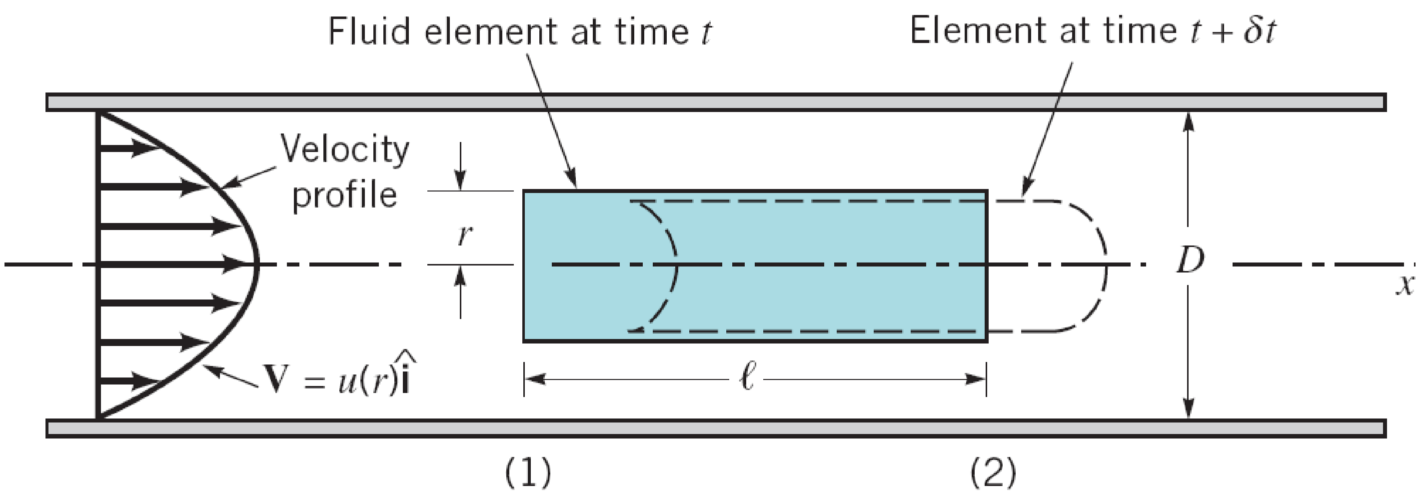
Fully Developed Laminar Flow

There are numerous ways to derive important results pertaining to fully developed laminar flow:

- ⇨From F=ma applied directly to a fluid element.
- ⇨From the Navier-Stokes equations of motion
- ⇨From dimensional analysis methods

From F=ma 1/8

- Considering a fully developed axisymmetric laminar flow in a long, straight, constant diameter section of a pipe.
- **The fluid element** is a circular cylinder of fluid of length *l* and radius *r* centered on the axis of a horizontal pipe of diameter *D*.



From F=ma 2/8

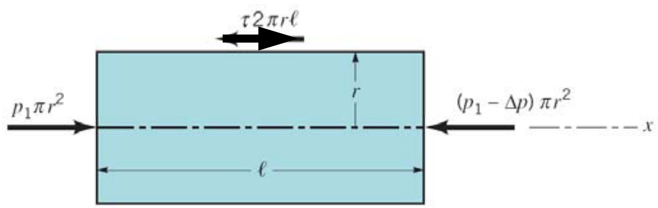
- Because the velocity is not uniform across the pipe, the initially flat end of the cylinder of fluid at time t become distorted at time t+δt when the fluid element has moved to its new location along the pipe.
- If the flow is <u>fully developed and steady</u>, the distortion on each end of the fluid element is the same, and <u>no</u> part of the fluid experiences any <u>acceleration</u> as it flows.

Steady $\dfrac{\partial \vec{V}}{\partial t} = 0$ **Fully developed** $\vec{V} \cdot \nabla \vec{V} = u \dfrac{\partial u}{\partial x} \bar{i} = 0$

From F=ma 3/8

Apply the Newton's second Law to the cylinder of fluid

$$F_x = ma_x$$



The force (pressure & friction) balance

$$p_1\pi r^2 - (p_1 - \Delta p)\pi r^2 + \tau_{rx}\ell(2\pi r) = 0 \Rightarrow -\frac{\Delta p}{\ell} = \frac{2\tau_{rx}}{r} \qquad \Delta p = p_1 - p_2$$

Basic balance in forces needed to drive each fluid particle along the pipe with constant velocity

Not function of r

$$-\frac{\Delta p}{\ell} = \frac{2\tau_{rx}}{r}$$

Not function of r

Independent of r

$$\tau ? \Rightarrow \tau_{rx} = Cr$$

B.C. $r=0$ $\tau_{rx}=0$

$r=D/2$ $\tau_{rx}=\tau_w < 0$

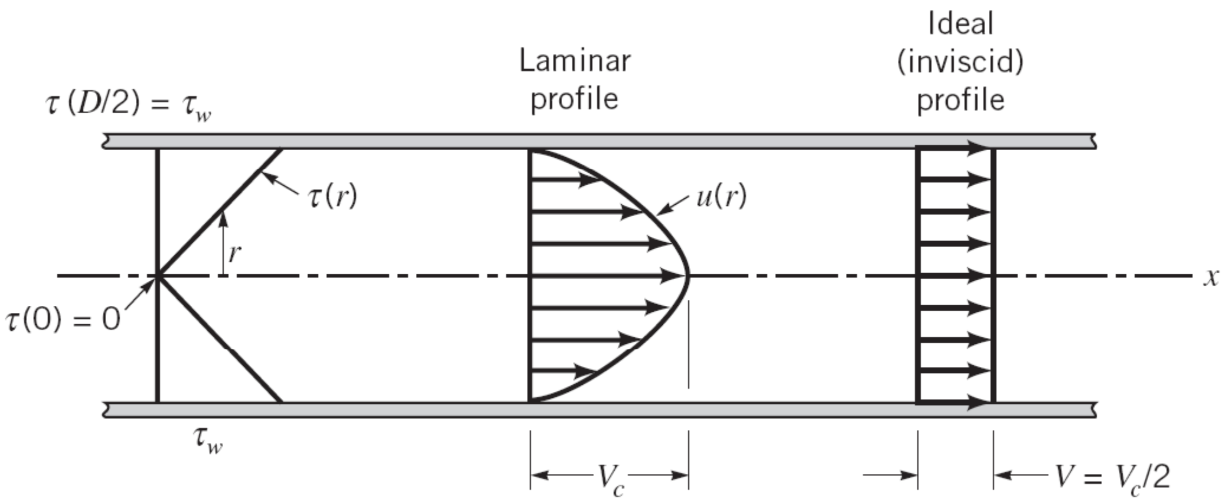
$$\tau_{rx} = \frac{2\tau_w r}{D}$$

From F=ma 4/8

The pressure drop and wall shear stress are related by

$$\tau_{rx} = \frac{2\tau_w r}{D} \quad \oplus \quad -\frac{\Delta p}{\ell} = \frac{2\tau_{rx}}{r} \quad \Rightarrow \quad \Delta p = -\frac{4\ell\tau_w}{D}$$

Valid for both laminar and turbulent flow.



Laminar

$$\tau_{rx} = \mu\frac{du}{dr}$$

From F=ma 5/8

Since $\tau = \mu \dfrac{du}{dr}$ $\Rightarrow$ $\dfrac{du}{dr} = -\left(\dfrac{\Delta p}{2\mu\ell}\right) r$ $\qquad -\dfrac{\Delta p}{\ell} = \dfrac{2\tau_{rx}}{r}$

Laminar

$$\int du = -\frac{\Delta p}{2\mu\ell}\int r\,dr \Rightarrow u = -\left(\frac{\Delta p}{4\mu\ell}\right) r^2 + C_1$$

With the boundary conditions:

u=0 at r=D/2

$$C_1 = \frac{\Delta p D^2}{16\mu\ell}$$

Velocity distribution

$$u(r) = \frac{\Delta p D^2}{16\mu\ell}\left[1-\left(\frac{2r}{D}\right)^2\right] = V_C\left[1-\left(\frac{2r}{D}\right)^2\right]$$

$$\Delta p = -\frac{4\ell\tau_w}{D}$$

$$u(r) = -\frac{\tau_w D}{4\mu}\left(1-\left(\frac{r}{R}\right)^2\right)$$

From F=ma 6/8

- The shear stress distribution

$$\tau_{rx} = \mu \frac{du}{dr} = -\frac{r\Delta p}{2\ell}$$

- Volume flowrate

$$u(r) = \frac{\Delta p D^2}{16\mu\ell}\left[1-\left(\frac{2r}{D}\right)^2\right] = V_C\left[1-\left(\frac{2r}{D}\right)^2\right]$$

$$\mathrm{Q} = \int_{\mathrm{A}} \mathrm{u} \cdot d\vec{A} = \int_0^R u(r) 2\pi r dr = \ldots\ldots = \frac{\pi R^2 V_C}{2}$$

$$Q = \frac{\pi D^4 \Delta p}{128\mu\ell}$$

Poiseuille's Law

Valid for Laminar flow only

From F=ma 7/8

- Average velocity

$$V_{average} \equiv \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{\Delta p D^2}{32\,\mu\ell} \qquad Q = \frac{\pi D^4 \Delta p}{128\mu\ell}$$

- Point of maximum velocity

$$\frac{\mathrm{du}}{\mathrm{dr}} = 0 \quad \text{at r=0}$$

$$u(r) = \frac{\Delta p D^2}{16\mu\ell}\left[1-\left(\frac{2r}{D}\right)^2\right] = V_C\left[1-\left(\frac{2r}{D}\right)^2\right]$$

$$u = u_{\max} = U = \frac{R^2 \Delta p}{4\,\mu\ell} = 2V_{average}$$

From F=ma 8/8

- Making adjustment to account for nonhorizontal pipes

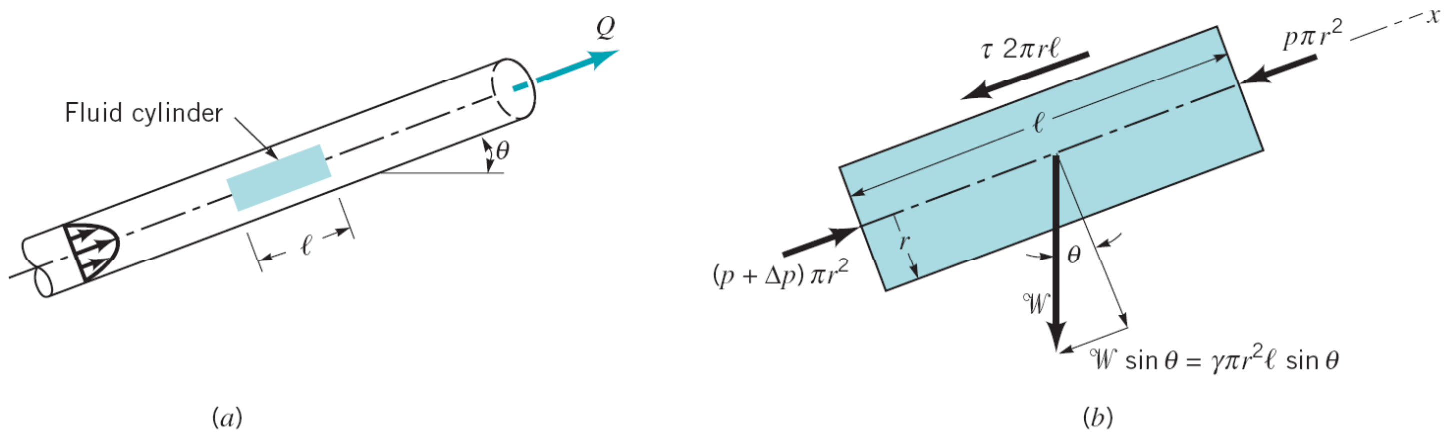
$$\Delta p \rightarrow \Delta p - \gamma \ell \sin \theta$$

θ>0 if the flow is uphill
θ<0 if the flow is downhill

$$p\pi r^2 - (p - \Delta p)\pi r^2 + \tau_{rx}\ell(2\pi r) - \rho g \pi r^2 \ell \sin\theta = 0 \Rightarrow -\frac{(\Delta p - \rho g \ell \sin\theta)}{\ell} = \frac{2\tau_{rx}}{r}$$

$$Q = \frac{\pi(\Delta p - \gamma \ell \sin\theta)D^4}{128\mu\ell} \qquad V_{average} = \frac{(\Delta p - \gamma \ell \sin\theta)D^2}{32\mu\ell}, \gamma = \rho g$$

Specific weight



Example 8.2 Laminar Pipe Flow

- An oil with a viscosity of μ= 0.40 N·s/m^2 and density ρ= 900 kg/m^3 flows in a pipe of diameter D= 0.20m .
  - (a) What pressure drop, p_1-p_2, is needed to produce a flowrate of Q=2.0×10^{-5} m^3/s if the pipe is horizontal with x_1=0 and x_2=10 m?
  - (b) How steep a hill, θ,must the pipe be on if the oil is to flow through the pipe at the same rate as in part (a), but with p_1=p_2?
  - (c) For the conditions of part (b), if p_1=200 kPa, what is the pressure at section, x_3=5 m, where x is measured along the pipe?

Example 8.2 Solution[1/2]

$$R_e = \rho VD / \mu = 2.87 < 2100$$

$$V = \frac{Q}{A} = 0.0637\,\text{m/s}$$

The flow is laminar flow

$$\Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4} = \ldots = 20.4\text{kPa}$$

If the pipe is on the hill of angle θ with Δp=0

$$\sin\theta = -\frac{128\mu\ell Q}{\pi\rho g D^4} = \ldots \Rightarrow \theta = -13.34°$$

Example 8.2 Solution$^{2/2}$

With $p_1=p_2$ the length of the pipe, ℓ, does not appear in the flowrate equation

$\Delta p=0$ for all ℓ

$$p_1 = p_2 = p_3 = 200\,\text{kPa}$$

From the Navier-Stokes Equations 1/3

❖ General motion of an incompressible Newtonian fluid is governed by the continuity equation and the momentum equation

Mass conservation $\longrightarrow$ $$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Navier-Stokes Equation in a cylindrical coordinate

r-Direction

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial r} + \rho g_r + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] \quad \text{(a)}$$

θ-Direction

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] \quad \text{(b)}$$

z-Direction

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right)$$

Acceleration

$$= -\frac{\partial p}{\partial z} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] \quad \text{(c)}$$

From the Navier-Stokes Equations 2/3

Simplify the Navier-Stokes equation

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{1}{r}\dfrac{\partial}{\partial r}(\rho r v_r) + \dfrac{1}{r}\dfrac{\partial}{\partial \theta}(\rho v_\theta) + \dfrac{\partial}{\partial z}(\rho v_z) = 0 \\ \rho\left(\dfrac{\partial v_z}{\partial t} + v_r \dfrac{\partial v_z}{\partial r} + \dfrac{v_\theta}{r}\dfrac{\partial v_z}{\partial \theta} + v_z \dfrac{\partial v_z}{\partial z}\right) \\ \qquad = -\dfrac{\partial p}{\partial z} + \rho g_z + \mu\left[\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial v_z}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 v_z}{\partial \theta^2} + \dfrac{\partial^2 v_z}{\partial z^2}\right] \end{cases}$$

axial component: *z*

The flow is governed by a balance of pressure, weight, and viscous forces in the flow direction.

For steady, fully developed flow in a pipe, the velocity contains only an axial component, which is a function of only the radial coordinate $\vec{V} = u(r)\vec{i}$

From the Navier-Stokes Equations [3/3]

axial component: *x*

$$\vec{V} = u(r)\vec{i} \quad \Rightarrow \quad \frac{\partial p}{\partial x} + \rho g \sin\theta = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

Function of, at most, only x

Function of ,at most, only r

$$\frac{\partial p}{\partial x} = \text{const.} \rightarrow \frac{\partial p}{\partial x} = \frac{-\Delta p}{\ell}$$

Integrating ⇒ **Velocity profile u(r)=**

B.C. (1) $r = R$, $u = 0$;

(2) $r = 0$, $u < \infty$ or $\partial u/\partial r = 0$

From Dimensional Analysis 1/3

❖ Assume that the pressure drop in the horizontal pie, Δp, is a function of the average velocity of the fluid in the pipe, V, the length of the pipe, ℓ, the pipe diameter, D, and the viscosity of the fluid, μ.

$$\Delta p = F(V, \ell, D, \mu)$$

Dimensional analysis

$$\frac{D\Delta p}{\mu V} = \phi\left(\frac{\ell}{D}\right)$$

an unknown function of the length to diameter ratio of the pipe.
k-r = 5 (총 변수) -3 (reference dimension) =2 dimensionless group

From Dimensional Analysis 2/3

$$\frac{D\Delta p}{\mu V} = C\frac{\ell}{D}$$ where C is a constant.

$$\Rightarrow \frac{\Delta p}{\ell} = \frac{C\mu V}{D^2} \quad Q = AV = \frac{\pi}{4}D^2\frac{\Delta p}{C\mu\ell}D^2 = \frac{(\pi/4C)\Delta p D^4}{\mu\ell}$$

The value of C must be determined by theory or experiment. For a round pipe, C=32. For duct of other cross-sectional shapes, the value of C is different.

$$Q = \frac{(\pi/4C)D^4}{\mu\ell}\Delta p = \frac{1}{\text{Flow resistance}}\Delta p$$

Analogy: $i = \dfrac{V}{R}$

For a round pipe $\Delta p = \dfrac{32\mu\ell V}{D^2}$

From Dimensional Analysis 3/3

For a round pipe $$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{32\mu\ell V / D^2}{\frac{1}{2}\rho V^2} = 64\frac{\mu}{\rho V D}\frac{\ell}{D} = \frac{64}{\mathrm{Re}}\frac{\ell}{D}$$

Dynamic pressure -> Characteristic pressure

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$$

$$f = \frac{\Delta p \dfrac{D}{\ell}}{\dfrac{\rho V^2}{2}} = \frac{\Delta p \Big/ \left(\dfrac{\rho V^2}{2}\right)}{\dfrac{\ell}{D}} = \frac{\Delta p^*}{\ell^*}$$

f is termed the friction factor, or sometimes the Darcy friction factor -> dimensionless pressure drop for internal flows.

For laminar flow

$$f = \frac{64}{\mathrm{Re}} = \frac{8\tau_w}{\rho V^2}$$

$$\Delta p = \frac{4\ell\tau_w}{D}$$

Energy Consideration 1/3

❖ The energy equation for incompressible, steady flow between two locations

$$\frac{p_1}{\gamma}+\frac{\alpha_1\overline{V}_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{\alpha_2\overline{V}_2^2}{2g}+z_2+h_L$$

$$\frac{\alpha_1\overline{V}_1^2}{2g}=\frac{\alpha_2\overline{V}_2^2}{2g}$$

$$\left(\frac{p_1}{\gamma}+z_1\right)-\left(\frac{p_2}{\gamma}+z_2\right)=h_L=\frac{1}{\gamma}\left(p_1-p_2+\gamma\left(z_1-z_2\right)\right)$$

$$=\frac{1}{\gamma}\left(\Delta p-\rho g\ell\sin\theta\right)=-\frac{2\tau\ell}{\gamma r}=-\frac{4\ell\tau_w}{\gamma D}$$

$$-\frac{\left(\Delta p-\rho g\ell\sin\theta\right)}{\ell}=\frac{2\tau_{rx}}{r}$$

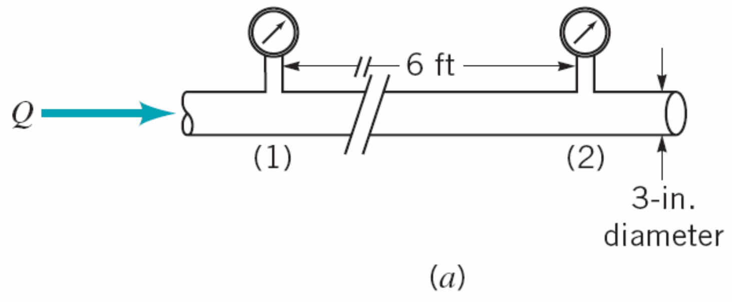
$$\tau=\frac{2\tau_w r}{D}$$

The head loss in a pipe is a result of the viscous shear stress on the wall.

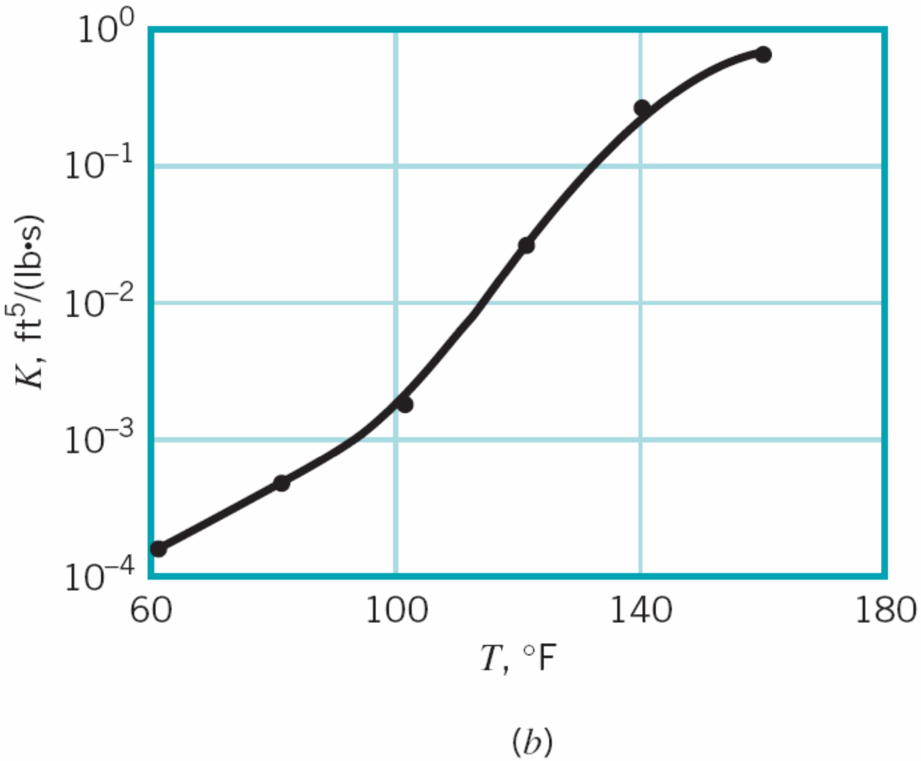
Example 8.3 Laminar Pipe Flow Properties 1/2

- The flowrate, Q, of corn syrup through the horizontal pipe shown in Figure E8.3 is to be monitored by measuring the pressure difference between sections (1) and (2). It is proposed that $Q=K\Delta p$, where the calibration constant, K, is a function of temperature, T, because of the variation of the syrup's viscosity and density with temperature. These variations are given in Table E8.3.
  - (a) Plot K(T) versus T for $60^\circ F \leq T \leq 160^\circ F$.
  - (b) Determine the wall shear stress and the pressure drop, $\Delta p=p_1-p_2$, for $Q=0.5\ ft^3/s$ and $T=100^\circ F$.
  - (c) For the conditions of part (b), determine the nest pressure force.$(\pi D^2/4)\Delta p$, and the nest shear force, $\pi D\ell\tau_w$, on the fluid within the pipe between the sections (1) and (2).

Example 8.3 Laminar Pipe Flow Properties 1/2



■ FIGURE E8.3



■ TABLE E8.3

T (°F)	ρ (slugs/ft^3)	μ (lb · s/ft^2)
60	2.07	4.0×10^{-2}
80	2.06	1.9×10^{-2}
100	2.05	3.8×10^{-3}
120	2.04	4.4×10^{-4}
140	2.03	9.2×10^{-5}
160	2.02	2.3×10^{-5}

Example 8.3 Solution 1/2

If the flow is laminar (-> should be verified)

$$Q = \frac{\pi \Delta p D^4}{128 \mu \ell} = K \Delta p \qquad K = \frac{1.60 \times 10^{-5}}{\mu}$$

For T=100°F, $\mu = 3.8 \times 10^{-3}$ lb·s/ft², Q=0.5ft³/s

$$\Delta p = \frac{128 \mu \ell Q}{\pi D^4} = \ldots = 119 \text{lb}/\text{ft}^2$$

$$V = \frac{Q}{A} = \ldots = 10.2 \text{ft}/\text{s} \qquad R_e = \rho V D / \mu = \ldots = 1380 < 2100$$

$\longrightarrow$ Laminar

$$\Delta p = \frac{4 \ell \tau_w}{D} \Rightarrow \tau_w = \frac{\Delta p D}{4 \ell} = \ldots = 1.24 \text{lb}/\text{ft}^2$$

Example 8.3 Solution$^{2/2}$

The new pressure force and viscous force on the fluid within the pipe between sections (1) and (2) is

$$F_p = \frac{\pi D^2}{4}\Delta p = \ldots = 5.84\text{lb}$$

$$F_v = 2\pi \frac{D}{2}\ell \tau_w = \ldots = 5.84\text{lb}$$

The values of these two forces are the same. The net force is zero; there is no acceleration.

Fully Developed Turbulent Flow

- Turbulent pipe flow is actually more likely to occur than laminar flow in practical situations.
- Turbulent flow is a very complex process.
- Numerous persons have devoted considerable effort in an attempting to understand the variety of baffling aspects of turbulence. Although a considerable amount if knowledge about the topics has been developed, the field of turbulent flow still remains the least understood area of fluid mechanics.

Much remains to be learned about the nature of turbulent flow.

Transition from Laminar to Turbulent Flow in a Pipe 1/2

- For any flow geometry, there is one (or more) dimensionless parameters such as with this parameter value below a particular value the flow is laminar, whereas with the parameter value larger than a certain value the flow is turbulent.

- The important parameters involved and their critical values depend on the specific flow situation involved.

For flow in pipe : Re~4000
For flow along a plate Re_x~500000

→Turbulence initiated.

Consider a long section of pipe that is initially filled with a fluid at rest.

Energy Considerations 1/7 (1-6: option)

❖ Considering the steady flow through the piping system, including a reducing elbow. The basic equation for conservation of energy – the first law of thermodynamics

$$\dot{Q}_{net\ \ in} + \dot{W}_{Shaft\ \ in} + \int_{CS} \sigma_{nn} \vec{V} \cdot \vec{n} dA = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS} e\rho \vec{V} \cdot \vec{n} dA$$

$$\Rightarrow \dot{Q}_{net\ \ in} + \dot{W}_{Shaft\ \ in} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS} e\rho \vec{V} \cdot \vec{n} dA - \int_{CS} \underbrace{\sigma_{nn}}_{-p} \vec{V} \cdot \vec{n} dA$$

Energy equation

$$\frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS} (\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz)\rho \vec{V} \cdot \vec{n} d\vec{A} = \dot{Q}_{net\ in} + \dot{W}_{Shaft\ in}$$

Kinetic energy

$$e = \hat{u} + \frac{V^2}{2} + gz$$

Internal energy

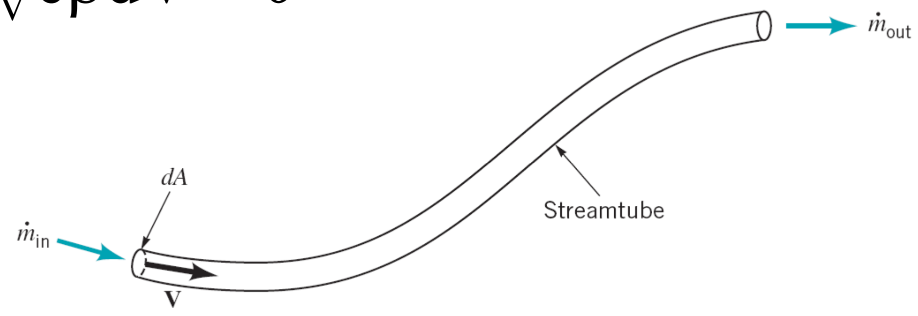
Note: The shear stress power is negligibly small on a control surface.

Energy Considerations 2/7

When the flow is steady $\dfrac{\partial}{\partial t}\int_{CV} e\rho d\forall = 0$

The integral of

$$\int_{CS} \underbrace{\left[\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right]}_{\text{Uniformly distribution}} \rho\vec{V}\cdot\vec{n}dA \quad ???$$



$$\int_{CS}\left[\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right]\rho\vec{V}\cdot\vec{n}dA = \sum_{out}\left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\dot{m} - \sum_{in}\left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\dot{m}$$

Only one stream entering and leaving

$$\int_{CS}\left[\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right]\rho\vec{V}\cdot\vec{n}dA$$

$$= \left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)_{out}\dot{m}_{out} - \left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)_{in}\dot{m}_{in}$$

Energy Considerations 3/7

If shaft work is involved….

$$\dot{m}\left[\hat{u}_{out} - \hat{u}_{in} + \left(\frac{p}{\rho}\right)_{out} - \left(\frac{p}{\rho}\right)_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right]$$

$$= \dot{Q}_{net\ in} + \dot{W}_{shaft\ net\ in}$$

One-dimensional energy equation for steady-in-the-mean flow

Enthalpy $\hat{h} = \hat{u} + \frac{p}{\rho}$

The energy equation is written in terms of enthalpy.

$$\dot{m}\left[\hat{h}_{out} - \hat{h}_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{net/in} + \dot{W}_{shaft\ net/in}$$

Energy Considerations 4/7

For steady, incompressible flow…One-dimensional energy equation

$$\dot{m}\left[\hat{u}_{out}-\hat{u}_{in}+\left(\frac{p_{out}}{\rho}\right)-\left(\frac{p_{in}}{\rho}\right)+\frac{V_{out}^2-V_{in}^2}{2}+g\left(z_{out}-z_{in}\right)\right]=\dot{Q}_{net\ in}$$

$\div\,\dot{m}$ ⇒

$$\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}-\left(\hat{u}_{out}-\hat{u}_{in}-q_{net\ in}\right)$$

where $q_{net\ in}=\dot{Q}_{net\ in}/\dot{m}$

For steady, incompressible, **frictionless flow**… → Examples?

$$p_{out}+\frac{\rho V_{out}^2}{2}+\gamma z_{out}=p_{in}+\frac{\rho V_{in}^2}{2}+\gamma z_{in}$$

Bernoulli equation

⇒ $\hat{u}_{out}-\hat{u}_{in}-q_{net\ in}=0$ **Frictionless flow…**

Energy Considerations 5/7

For steady, incompressible, **frictional flow…**

$$\hat{u}_{out} - \hat{u}_{in} - q_{net\ in} > 0$$ **Frictional flow…**

Defining "useful or available energy"… $\frac{p}{\rho} + \frac{V^2}{2} + gz$

Defining "loss of useful or available energy"… $\hat{u}_{out} - \hat{u}_{in} - q_{net\ in} = \text{loss}$

$$\frac{p_{out}}{\rho} + \frac{V_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{V_{in}^2}{2} + gz_{in} - \text{loss}$$

Energy Considerations 6/7

For steady, incompressible flow with friction and shaft work…

$$\dot{m}\left[\hat{u}_{out}-\hat{u}_{in}+\left(\frac{p_{out}}{\rho}\right)-\left(\frac{p_{in}}{\rho}\right)+\frac{V_{out}^2-V_{in}^2}{2}+g\left(z_{out}-z_{in}\right)\right]=\dot{Q}_{net\ in}+\dot{W}_{shaft\ net\ in}$$

$\div\ \dot{m}$

$$\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}+w_{shaft\ net\ in}-(\hat{u}_{out}-\hat{u}_{in}-q_{net\ in})$$

$$\frac{p_{out}}{\rho}+\frac{V_{out}^2}{2}+gz_{out}=\frac{p_{in}}{\rho}+\frac{V_{in}^2}{2}+gz_{in}+w_{shaft\ net\ in}-\text{loss}$$

$\div\ g$

$$\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}=\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+h_s-h_L$$

Shaft head $h_S=\frac{w_{shaft\ net\ in}}{g}\equiv\frac{\dot{W}_{shaft\ net\ in}}{\dot{m}g}=\frac{\dot{W}_{shaft\ net\ in}}{\gamma Q}$ **Head loss** $h_L=\frac{\text{loss}}{g}$

Energy Considerations 7/7

$$\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}=\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+h_s-h_L$$

- Total head loss , h_L, is regarded as the sum of major losses, **$h_{L\,major}$**, due to frictional effects in **fully developed flow in constant area tubes**, and minor losses, $h_{L\,minor}$, resulting from **entrance, fitting, area changes**, and so on.

$$h_L=h_{L_{major}}+h_{L_{minor}}$$

Major Losses: Friction Factor

❖ The energy equation for steady and incompressible flow with zero shaft work

$$\left(\frac{p_1}{\rho g}+\frac{\alpha_1 V_1^{\ 2}}{2g}+z_1\right)-\left(\frac{p_2}{\rho g}+\frac{\alpha_2 V_2^{\ 2}}{2g}+z_2\right)=h_L$$

$$\alpha=\int_A \frac{\left(\frac{V^2}{2}\right)\rho\vec{V}\cdot\vec{n}dA}{\dot{m}\left(\frac{\overline{V}^2}{2}\right)}\geq 1$$

For fully developed flow through a constant area pipe, $\alpha_1=\alpha_2$, $V_1=V_2$, where α is the kinetic energy coefficient and V is the average velocity. For a uniform velocity, $\alpha=1$.

$$>>> \frac{p_1-p_2}{\rho g}=(z_2-z_1)+h_L$$

For horizontal pipe, $z_2=z_1$

$$>>> \frac{p_1-p_2}{\rho g}=\frac{\Delta p}{\rho g}=h_L$$

Major Losses: Laminar Flow

❖ In fully developed laminar flow in a horizontal pipe, the pressure drop

$$\Delta p = \frac{128\mu\ell Q}{\pi D^4} = \frac{128\mu\ell V\left(\pi D^2/4\right)}{\pi D^4} = 32\frac{\ell}{D}\frac{\mu V}{D}$$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = 64\frac{\mu}{\rho V D}\frac{\ell}{D} = \frac{64}{Re}\frac{\ell}{D}$$

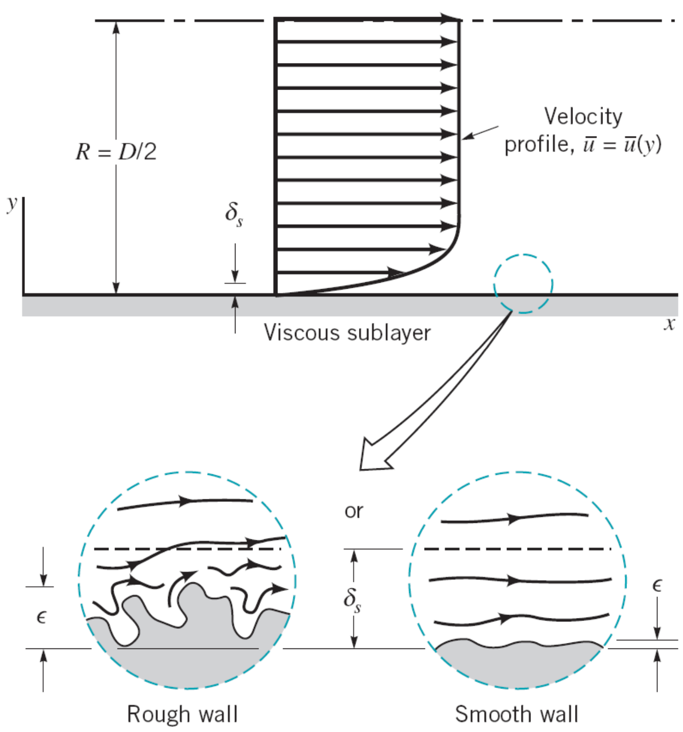
$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2} >> h_L = 32\frac{\ell}{D}\frac{\mu V}{\rho D} = \frac{\ell}{D}\frac{V^2}{2}\left(64\frac{\mu}{\rho V D}\right) = \left(\frac{64}{R_e}\right)\frac{\ell}{D}\frac{V^2}{2}$$

Friction Factor $f = \Delta p\left(D/\ell\right)/\left(\rho V^2/2\right)$

$$f_{\text{laminar}} = \frac{64}{Re}$$

Major Losses: Turbulent Flow 1/3

- In turbulent flow, we cannot evaluate the pressure drop analytically; we must resort to experimental results and use dimensional analysis to correlate the experimental data.
- In fully developed turbulent flow the pressure drop, △p , caused by friction in a horizontal constant-area pipe is known to depend on pipe diameter,D, pipe length, ℓ, pipe roughness,e, average flow velocity, V, fluid densityρ, and fluid viscosity,μ.



$$\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho)$$

Major Losses: Turbulent Flow 2/3

- Applying dimensional analysis, the result were a correlation of the form

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \bar{\phi}\left(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$$

- Experiments show that the nondimensional head loss is directly proportional to ℓ/D. Hence we can write

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\ell}{D}\phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$$

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$$

$$f \equiv \phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$$

Darcy-Weisbach equation

$$h_{L_{major}} \equiv f\frac{\ell}{D}\frac{V^2}{2g}$$

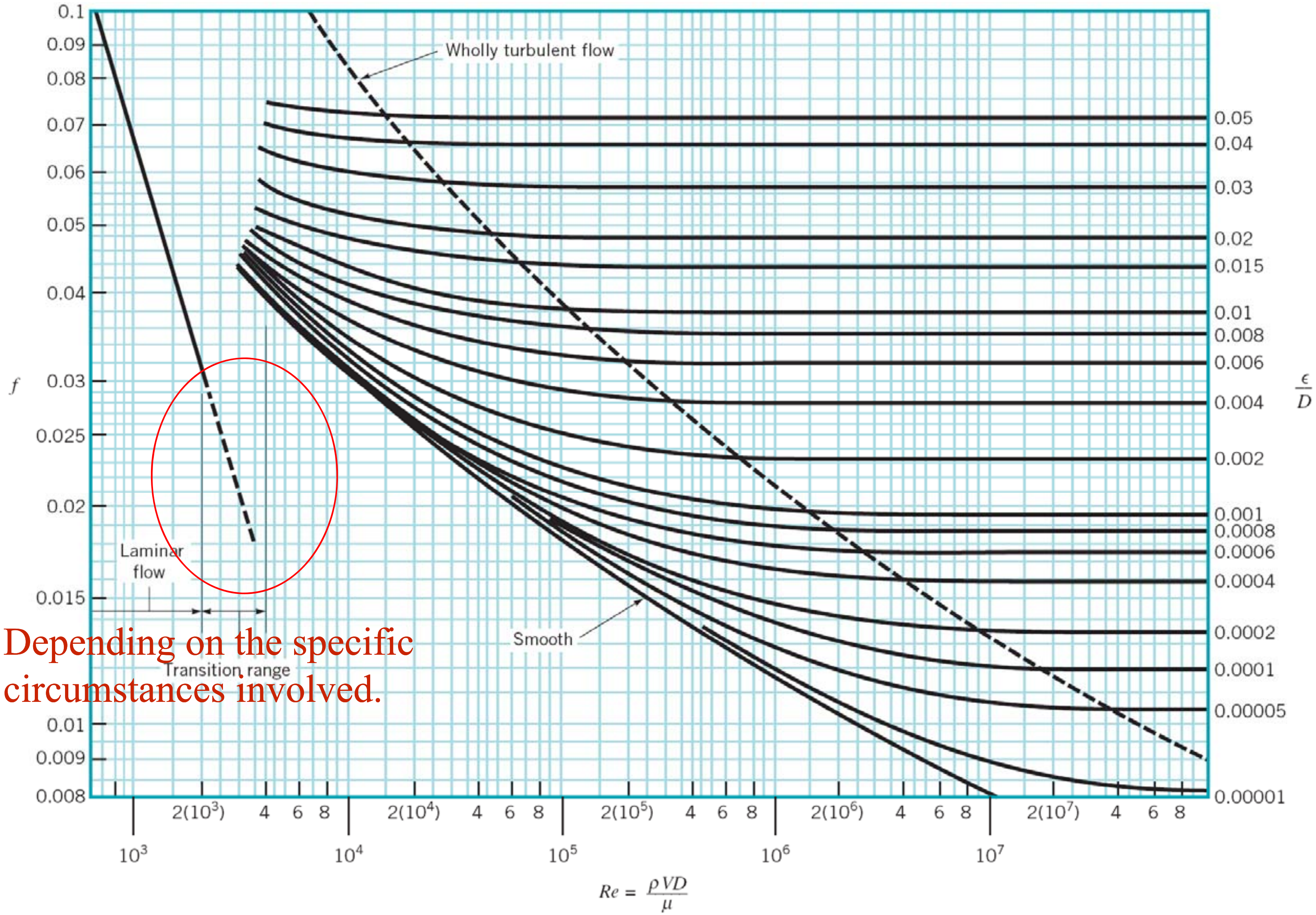
Roughness for Pipes

TABLE 8.1

Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

Moody chart



Depending on the specific circumstances involved.

About Moody Chart

- For laminar flow, $f=64/Re$, which is independent of the relative roughness.
- For very large Reynolds numbers, $f=\Phi(\varepsilon/D)$, which is independent of the Reynolds numbers.
- For flows with very large value of Re, commonly termed completely turbulent flow (or wholly turbulent flow), the laminar sublayer is so thin (its thickness decrease with increasing Re) that the surface roughness completely dominates the character of the flow near the wall.
- For flows with moderate value of Re, the friction factor $f=\Phi(Re,\varepsilon/D)$.

Major Losses: Turbulent Flow 3/3

- **Colebrook** – To avoid having to use a graphical method for obtaining f for turbulent flows.

$$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{\varepsilon / D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right]$$

Valid for the entire nonlaminar range of the Moody chart.

Colebrook formula -> needs iteration.

- **Miler suggests that a single iteration will produce a result within 1 percent if the initial estimate is calculated from**

$$f_0 = 0.25\log\left[\frac{\varepsilon / D}{3.7} + \frac{5.74}{Re^{0.9}}\right]^{-2}$$

Example 8.5 Comparison of Laminar or Turbulent pressure Drop

- Air under standard conditions flows through a 4.0-mm-diameter drawn tubing with an average velocity of V = 50 m/s. For such conditions the flow would normally be turbulent. However, if precautions are taken to eliminate disturbances to the flow (the entrance to the tube is very smooth, the air is dust free, the tube does not vibrate, etc.), it may be possible to maintain laminar flow.
  - (a) Determine the pressure drop in a 0.1-m section of the tube if the flow is laminar.
  - (b) Repeat the calculations if the flow is turbulent.

Example 8.5 Solution$^{1/2}$

Under standard temperature and pressure conditions

P=1.23kg/m^3, μ=1.79×10^{-5}N·s/m

The Reynolds number

$$R_e = \rho VD/\mu = \ldots = 13{,}700 \rightarrow \text{Turbulent flow}$$

If the flow were laminar

f=64/Re=…=0.0467

$$\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2 = \ldots = 0.179\,\text{kPa}$$

Example 8.5 Solution$^{2/2}$

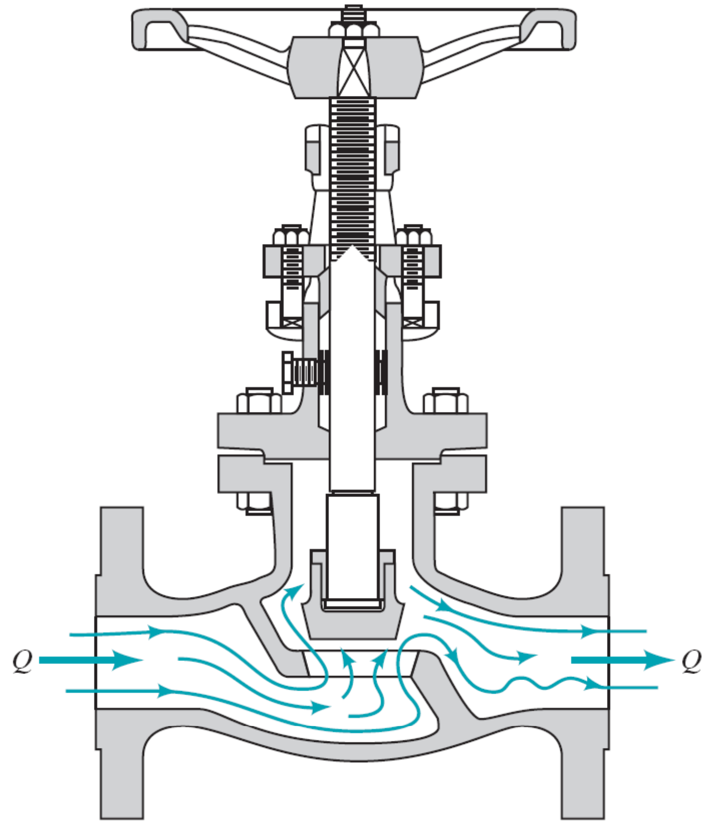
If the flow were turbulent

From Moody chart $f=\Phi(Re,\varepsilon/D) = \ldots 0.028$

$$\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2 = \ldots = 1.076\,\text{kPa}$$

Minor Losses 1/5

- Most pipe systems consist of considerably more than straight pipes. These additional components (valves, bends, tees, and the like) add to the overall head loss of the system.
- Such losses are termed MINOR LOSS. But, it is not minor at all and it may be larger than the major losses.



The flow pattern through a valve

Minor Losses 2/5

- The theoretical analysis to predict the details of flow pattern (through these additional components) is not, as yet, possible.
- The head loss information for essentially all components is given in dimensionless form and based on experimental data. The most common method used to determine these head losses or pressure drops is to specify the loss coefficient, K_L.

Minor Losses 3/5

$$K_L = \frac{h_{L_{minor}}}{V^2/2g} = \frac{\Delta p}{\frac{1}{2}\rho V^2} \Rightarrow \Delta p = K_L \frac{1}{2}\rho V^2 \rightarrow$$

Large K : large pressure drops for given velocities.

Minor losses are sometimes given in terms of an equivalent length ℓ_{eq}

$$\begin{cases} h_{L_{minor}} = K_L \dfrac{V^2}{2g} = f \dfrac{\ell_{eq}}{D} \dfrac{V^2}{2g} \\ \ell_{eq} = K_L \dfrac{D}{f} \end{cases}$$

The actual value of K_L is strongly dependent on the geometry of the component considered. It may also dependent on the fluid properties. That is

$$K_L = \phi(geometry, \mathrm{Re})$$

Minor Losses 4/5

- For many practical applications the Reynolds number is large enough so that the flow through the component is dominated by inertial effects, with viscous effects being of secondary importance.
- In a flow that is dominated by inertia effects rather than viscous effects, it is usually found that pressure drops and head losses correlate directly with the dynamic pressure.
- This is the reason why the friction factor for very large Reynolds number, fully developed pipe flow is independent of the Reynolds number.

Minor Losses 5/5

- This is true for flow through pipe components.
- Thus, in most cases of practical interest the loss coefficients for components are a function of geometry only,

$$K_L = \phi(\text{geometry})$$

Minor Losses Coefficient Entrance flow 1/3

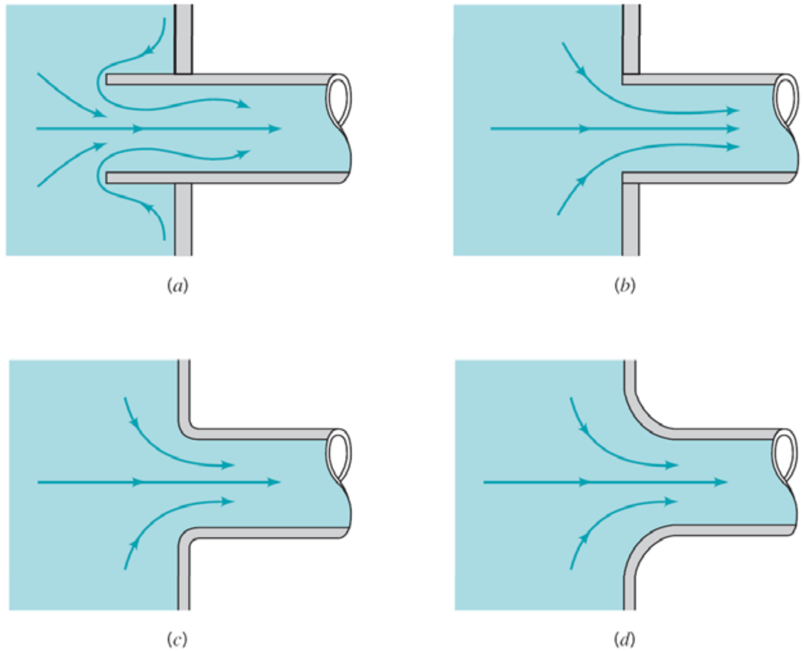
- Entrance flow condition and loss coefficient

  (a) Reentrant, $K_L = 0.8$

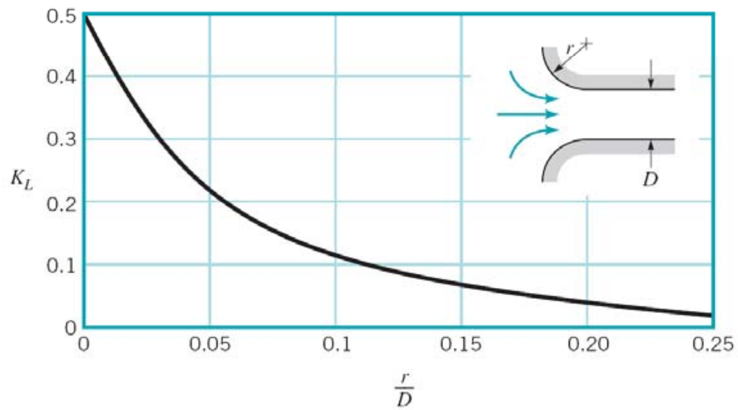
  (b) sharp-edged, $K_L = 0.5$

  (c) slightly rounded, $K_L = 0.2$

  (d) well-rounded, $K_L = 0.04$



K_L = function of rounding of the inlet edge.



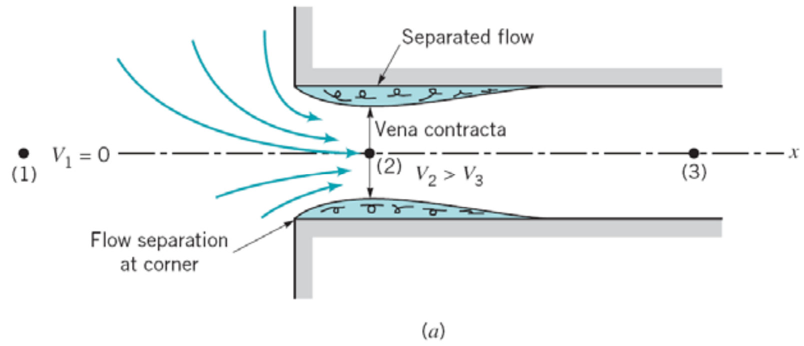
Minor Losses Coefficient Entrance flow 2/3

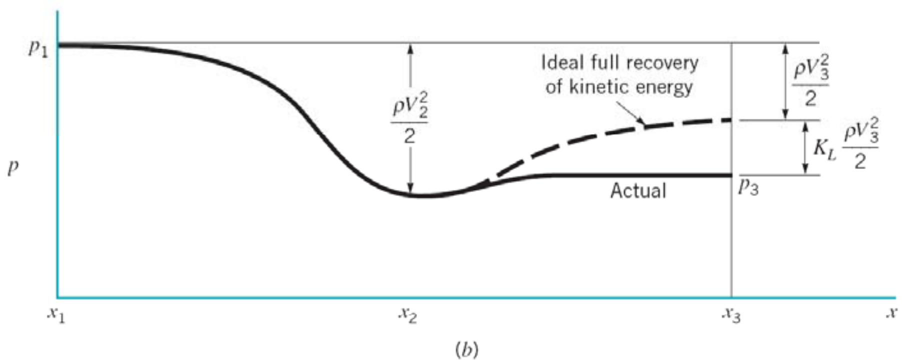
- A vena contract region may result because the fluid cannot turn a sharp right-angle corner. The flow is said to separate from the sharp corner.
- The maximum velocity velocity at section (2) is greater than that in the pipe section (3), and the pressure there is lower.
- If this high speed fluid could slow down efficiently, the kinetic energy could be converted into pressure.

Minor Losses Coefficient Entrance flow 3/3

❖ Such is not the case. Although the fluid may be accelerated very efficiently, it is very difficult to slow down (decelerate) the fluid efficently.

❖ (2)→(3) <u>The extra kinetic energy of the fluid is partially lost because of viscous dissipation, so that the pressure does not return to the ideal value.</u>





Flow pattern and pressure distribution for a sharp-edged entrance

Minor Losses Coefficient Exit flow

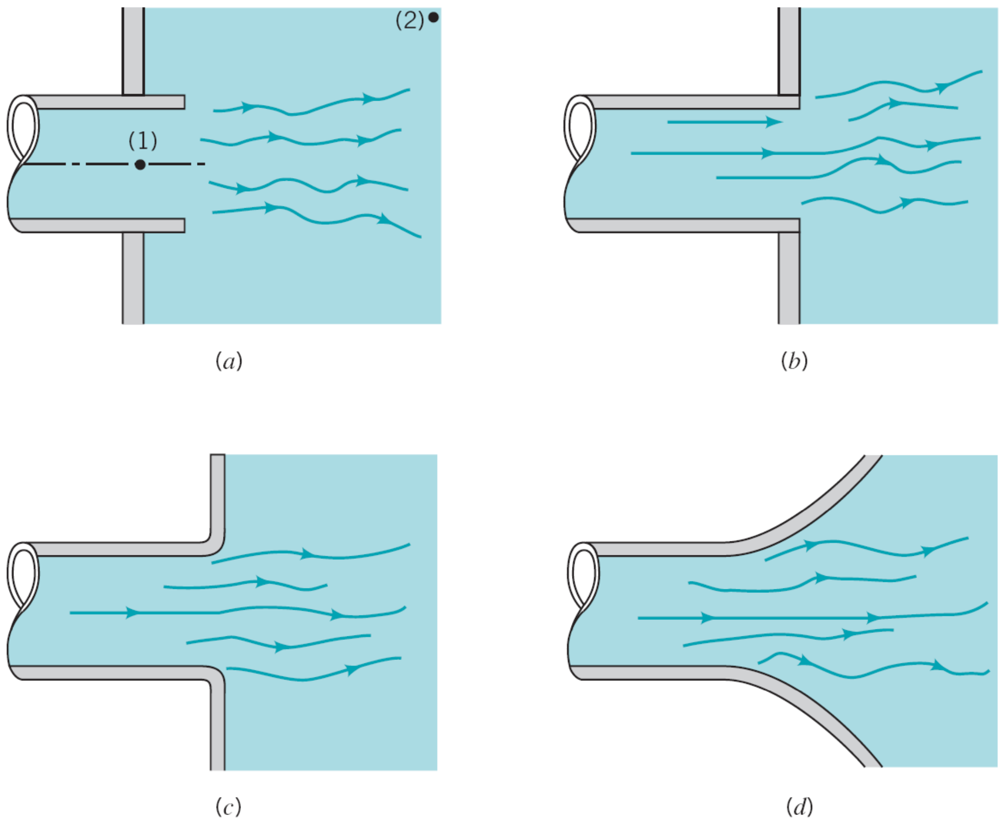
- Exit flow condition and loss coefficient

  (a) Reentrant, $K_L = 1.0$

  (b) sharp-edged, $K_L = 1.0$

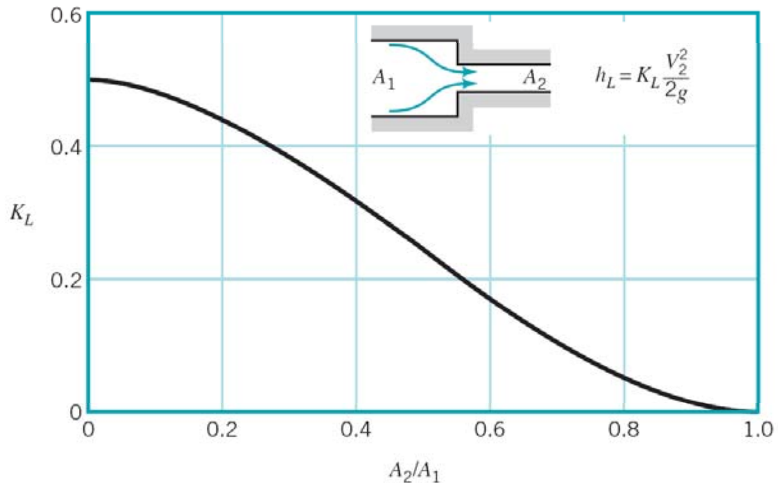
  (c) slightly rounded, $K_L = 1.0$

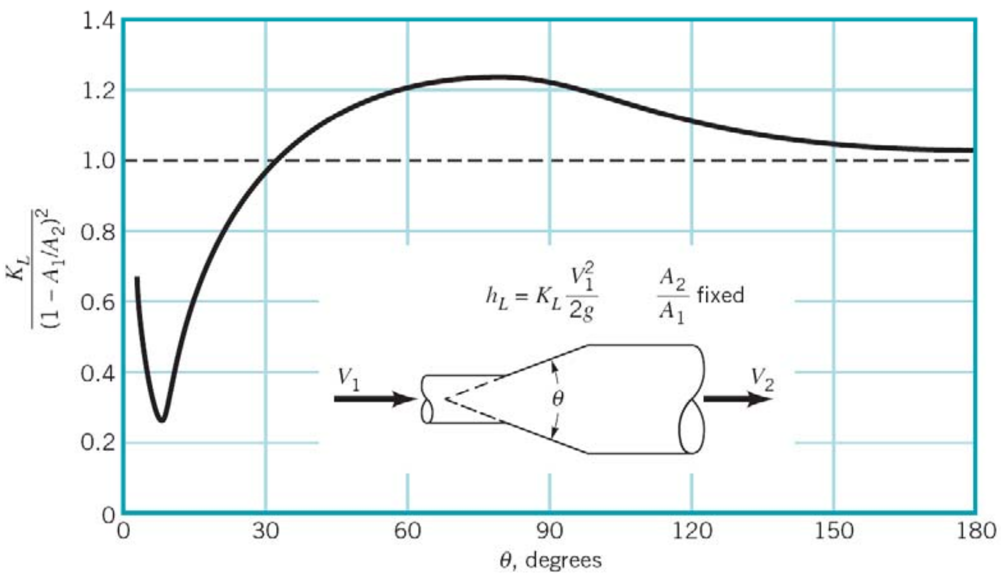
  (d) well-rounded, $K_L = 1.0$

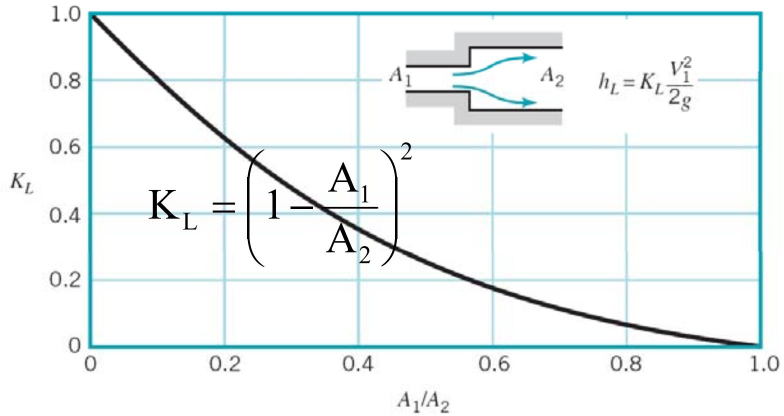


Minor Losses Coefficient varied diameter

- ❖ Loss coefficient for sudden contraction, expansion,typical conical diffuser.



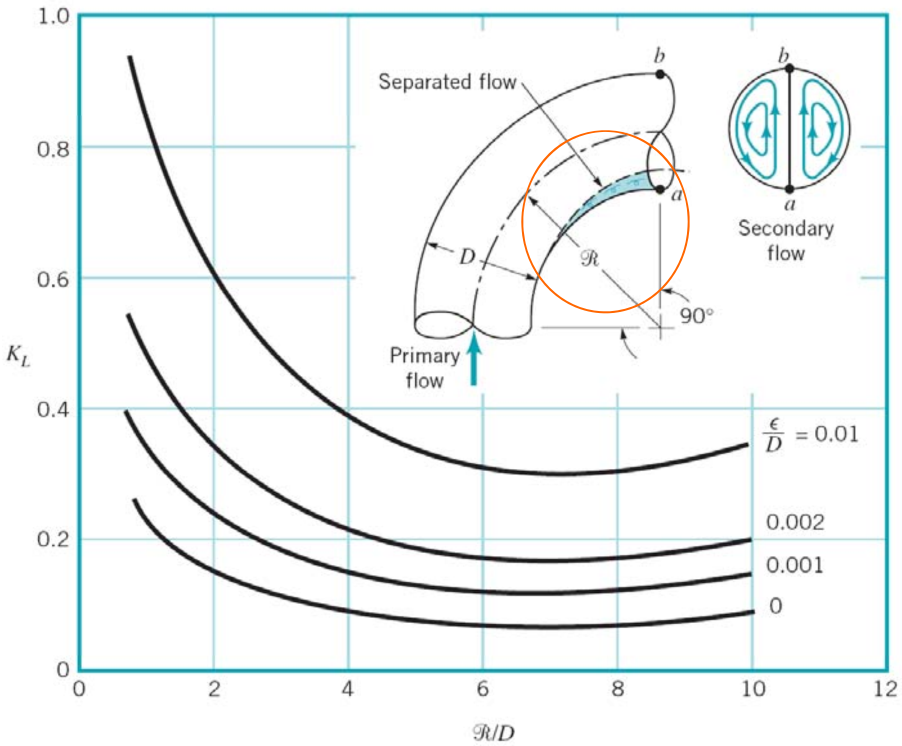


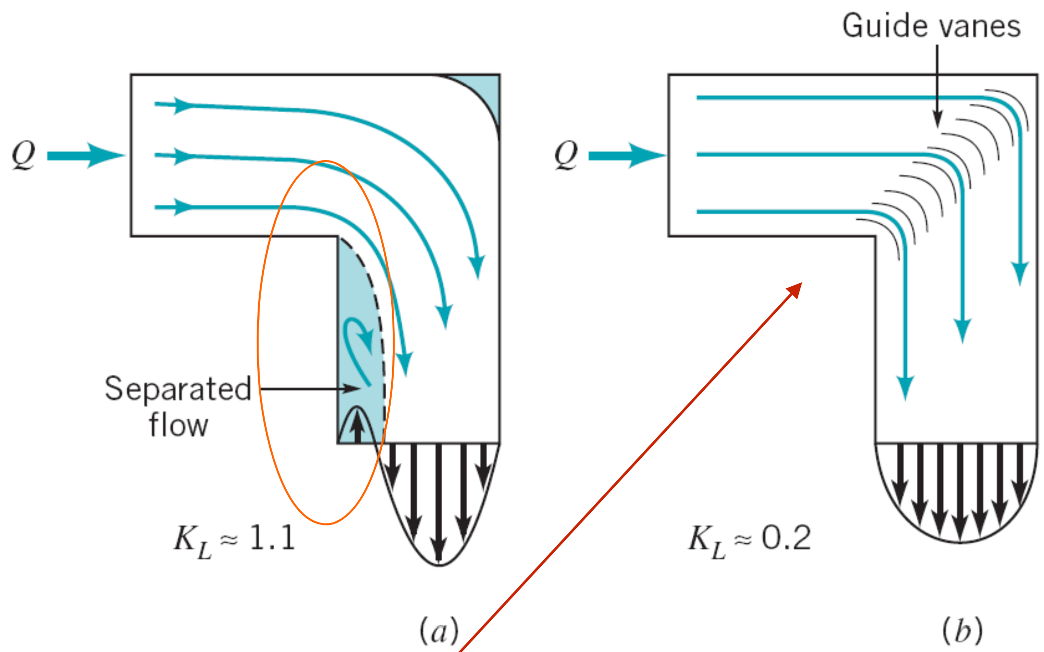


$$K_L = \left(1 - \frac{A_1}{A_2}\right)^2$$

Minor Losses Coefficient Bend

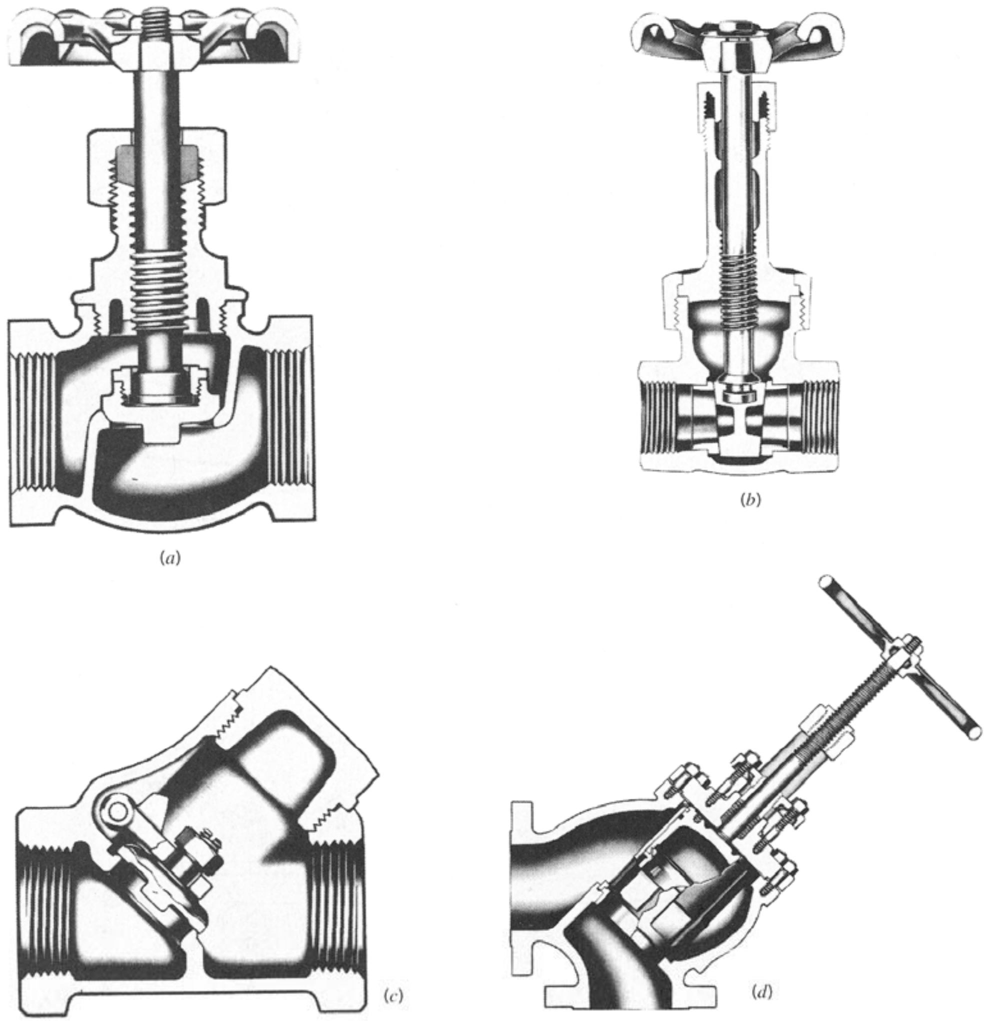
❖ Character of the flow in bend and the associated loss coefficient.





Carefully designed guide vanes help direct the flow with less unwanted swirl and disturbances.

Internal Structure of Valves



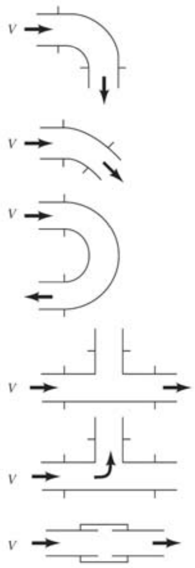
(*a*) globe valve
(*b*) gate valve
(*c*) swing check valve
(*d*) stop check valve

Loss Coefficients for Pipe Components

TABLE 8.2

Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$ (Data from Refs. 5, 10, 27)

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210

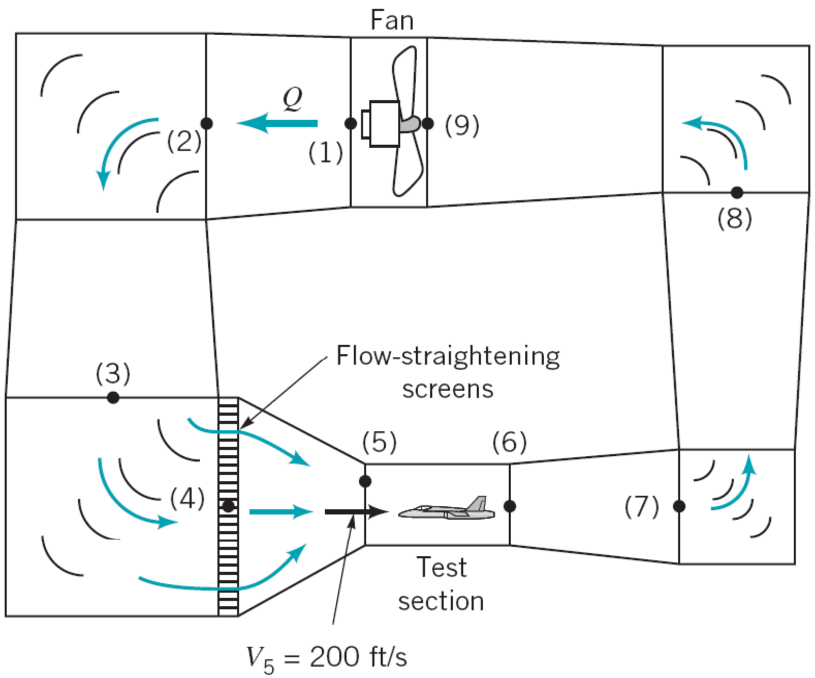


*See Fig. 8.32 for typical valve geometry.

Example 8.6 Minor Loss 1/2

- Air at standard conditions is to flow through the test section [between sections (5) and (6)] of the closed-circuit wind tunnel shown if Figure E8.6 with a velocity of 200 ft/s. The flow is driven by a fan that essentially increase the static pressure by the amount p_1-p_9 that is needed to overcome the head losses experienced by the fluid as it flows around the circuit. Estimate the value of p_1-p_9 and the horsepower supplied to the fluid by the fan.

Example 8.6 Minor Loss 2/2



Location	Area (ft^2)	Velocity (ft/s)
1	22.0	36.4
2	28.0	28.6
3	35.0	22.9
4	35.0	22.9
5	4.0	200.0
6	4.0	200.0
7	10.0	80.0
8	18.0	44.4
9	22.0	36.4

Example 8.6 Solution[1/3]

The maximum velocity within the wind tunnel occurs in the test section (smallest area). Thus, the maximum Mach number of the flow is $Ma_5 = V_5/c_5$

$$V_5 = 200 \text{ft/s} \qquad c_5 = (KRT_5)^{1/2} = 1117 \text{ft/s}$$

The energy equation between points (1) and (9)

$$\frac{p_1}{\gamma} + \frac{V_1^{\ 2}}{2g} + z_1 = \frac{p_9}{\gamma} + \frac{V_9^{\ 2}}{2g} + z_9 + h_{L1-9}$$

$$\Rightarrow \quad h_{L1-9} = \frac{p_1}{\gamma} - \frac{p_9}{\gamma}$$

The total head loss from (1) to (9).

Example 8.6 Solution 2/3

The energy across the fan, from (9) to (1)

$$\frac{p_9}{\gamma}+\frac{V_9{}^2}{2g}+z_9+h_p=\frac{p_1}{\gamma}+\frac{V_1{}^2}{2g}+z_1$$

H_p is the actual head rise supplied by the pump (fan) to the air.

$$h_p=\frac{p_1}{\gamma}-\frac{p_9}{\gamma}=h_{L1-9}$$

The actual power supplied to the air (horsepower, P_a) is obtained from the fan head by

$$P_a=\gamma Q h_p=\gamma A_5 V_5 h_p=\gamma A_5 V_5 h_{L1-9}$$

Example 8.6 Solution[3/3]

The total head loss

$$h_{L1-9} = h_{L_{corner\,7}} + h_{L_{corner\,8}} + h_{L_{corner\,2}} + h_{L_{corner\,3}} + h_{L_{dif}} + h_{L_{noz}} + h_{L_{scr}}$$

$$h_{L_{corner}} = K_L \frac{V^2}{2g} = 0.2\frac{V^2}{2g} \qquad h_{L_{dif}} = K_{L_{dif}} \frac{V^2}{2g} = 0.6\frac{V^2}{2g}$$

$$K_{L_{noz}} = 0.2 \qquad K_{L_{scr}} = 4.0$$

$$p_1 - p_9 = \gamma h_{L1-9} = (0.765\text{lb}/\text{ft}^2)(560\text{ft}) = \ldots = 0.298\text{psi}$$

$$P_a = \ldots = 34300\text{ft}\cdot\text{lb}/\text{s} = 62.3\text{hp}$$

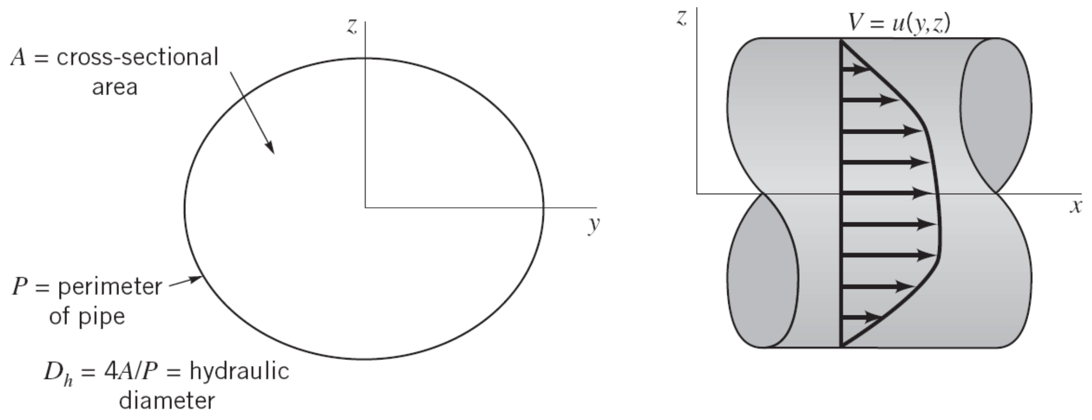
Noncircular Ducts 1/4

❖ The empirical correlations for pipe flow may be used for computations involving noncircular ducts, provided their cross sections are not too exaggerated.

❖ The correlation for turbulent pipe flow are extended for use with noncircular geometries by introducing the hydraulic diameter, defined as

$$D_h \equiv \frac{4A}{P}$$

where A is cross-sectional area, and P is wetted perimeter.



(a) (b)

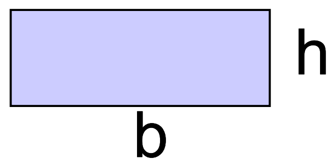
Noncircular Ducts 2/4

- For a circular duct

$$D_h \equiv \frac{4A}{P} = \frac{4\pi R^2}{2\pi R} = D$$

- For a rectangular duct of width b and height h

$$D_h \equiv \frac{4A}{P} = \frac{4bh}{2(b+h)} = \frac{2h}{1+ar} \qquad ar = h/b$$



The hydraulic diameter concept can be applied in the approximate range $\frac{1}{4}<ar<4$. So the correlations for pipe flow give acceptably accurate results for rectangular ducts.

Noncircular Ducts 3/4

- The friction factor can be written as $f=C/Re_h$, where the constant C depends on the particular shape of the duct, and Re_h is the Reynolds number based on the hydraulic diameter. Note: C = 64 for a circular tube.
- The hydraulic diameter is also used in the definition of the friction factor, $h_L = f(\ell / D_h)(V^2 / 2g)$, and the relative roughness ε/D_h.

Noncircular Ducts 4/4

- For <u>Laminar</u> flow, the value of $C=f\cdot Re_h$ have been obtained from theory and/or experiment for various shapes.
- For <u>turbulent</u> flow in ducts of noncircular cross section, calculations are carried out by using the Moody chart data for round pipes with the diameter replaced by the hydraulic diameter and the Reynolds number based on the hydraulic diameter.

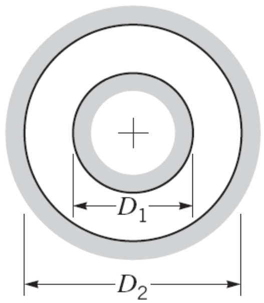
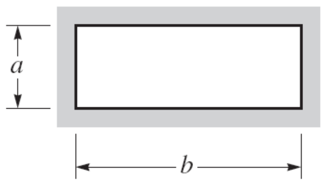
The <u>Moody chart, developed for round pipes, can also be used for noncircular ducts</u>.

Friction Factor for Laminar Flow in Noncircular Ducts

Note: C = 64 for a circular tube.

TABLE 8.3

Friction Factors for Laminar Flow in Noncircular Ducts (Data from Ref. 18)

Shape	Parameter	$C = f\,\mathrm{Re}_h$
I. Concentric Annulus $D_h = D_2 - D_1$ 	D_1/D_2	
	0.0001	71.8
	0.01	80.1
	0.1	89.4
	0.6	95.6
	1.00	96.0
II. Rectangle $D_h = \dfrac{2ab}{a+b}$ 	a/b	
	0	96.0
	0.05	89.9
	0.10	84.7
	0.25	72.9
	0.50	62.2
	0.75	57.9
	1.00	56.9

Example 8.7 Noncircular Duct

- Air at temperature of 120°F and standard pressure flows from a furnace through an 8-in.-diameter pipe with an average velocity of 10ft/s. It then passes through a transition section and into a square duct whose side is of length a. The pipe and duct surfaces are smooth ($\varepsilon=0$). Determine the duct size, a, if the head loss per foot is to be the same for the pipe and the duct.

Example 8.7 Solution[1/3]

The head loss per foot for the pipe

$$\frac{h_L}{\ell} = \frac{f}{D}\frac{V^2}{2g}$$

For given pressure and temperature $\nu = 1.89\times10^{-4}\,\text{ft}^2/\text{s}$

$$\text{Re} = \frac{VD}{\nu} = 35300$$

For the square duct $$\frac{h_L}{\ell} = \frac{f}{D_h}\frac{V_s^2}{2g} = 0.0512$$

$$D_h = \frac{4A}{P} = a \qquad V_s = \frac{Q}{A} = \frac{3.49}{a^2}$$

Example 8.7 Solution[2/3]

$$\frac{h_L}{\ell}=\frac{f}{D_h}\frac{V_s^2}{2g}=0.0512=\frac{f}{a}\frac{(3.49/a^2)^2}{2(32.2)}\Rightarrow a=1.30f^{1/5} \quad (1)$$

The Reynolds number based on the hydraulic diameter

$$Re_h=\frac{V_sD_h}{\nu}=\frac{(3.49/a^2)a}{1.89\times10^{-4}}=\frac{1.89\times10^{-4}}{a} \quad (2)$$

Have three unknown (a, f, and Re_h) and three equation – Eqs. 1, 2, and either in graphical form the Moody chart or the Colebrook equation

Find a

Example 8.7 Solution $^{3/3}$

Use the Moody chart

Assume the friction factor for the duct is the same as for the pipe.

That is, assume f=0.022.

From Eq. 1 we obtain a=0.606 ft.

From Eq. 2 we have $Re_h=3.05\times10^4$

From Moody chart we find f=0.023, which does not quite agree the assumed value of f.

Try again, using the latest calculated value of f=0.023 as our guess.

…… The final result is f=0.023, $Re_h=3.05\times10^4$, and a=0.611ft.

Pipe Flow Examples 1/2

❖ The energy equation, relating the conditions at any two points 1 and 2 for a single-path pipe system, for <u>steady and incompressible</u> flow with <u>zero shaft work</u> is given by

$$\left(\frac{p_1}{\rho g}+\frac{\alpha_1 V_1^2}{2g}+z_1\right)-\left(\frac{p_2}{\rho g}+\frac{\alpha_2 V_2^2}{2g}+z_2\right)=h_L=\sum h_{L_{major}}+\sum h_{L_{minor}}$$

By judicious choice of points 1 and 2, we can analyze not only the entire pipe system, but also just a certain section of it that we may be interested in.

Major loss $h_{L_{major}} \equiv f\dfrac{\ell}{D}\dfrac{V^2}{2g}$ **Minor loss** $h_{L_{minor}} = K_L\dfrac{V^2}{2g}$

Pipe Flow Examples 2/2

- Single pipe whose length may be interrupted by various components.
- Multiple pipes in different configuration
  - Parallel
  - Series
  - Network

Single-Path Systems 1/2

- Pipe flow problems can be categorized by what parameters are given and what is to be calculated.

TABLE 8.4
Pipe Flow Types

Variable	Type I	Type II	Type III
a. Fluid			
Density	Given	Given	Given
Viscosity	Given	Given	Given
b. Pipe			
Diameter	Given	Given	Determine
Length	Given	Given	Given
Roughness	Given	Given	Given
c. Flow			
Flowrate or Average Velocity	Given	Determine	Given
d. Pressure			
Pressure Drop or Head Loss	Determine	Given	Given

Single-Path Systems $^{2/2}$

- Type 1: Given pipe (L and D), and flow rate, and Q, find pressure drop Δp
- Type 1: Given Δp, D, and Q, find L.
- Type 2: Given Δp, L, and D, find Q.
- Type 3: Given Δp, L, and Q, find D.

Given L , D, and Q, find Δp

- The energy equation

$$\left(\frac{p_1}{\rho g}+\frac{\alpha_1 V_1^2}{2g}+z_1\right)-\left(\frac{p_2}{\rho g}+\frac{\alpha_2 V_2^2}{2g}+z_2\right)=h_L=\sum h_{L\text{major}}+\sum h_{L\text{min or}}$$

- The flow rate leads to the Reynolds number and hence the friction factor for the flow.
- Tabulated data can be used for minor loss coefficients and equivalent lengths.
- The energy equation can then be used to directly to obtain the pressure drop.

Given Δp, D, and Q, find L

- The energy equation

$$\left(\frac{p_1}{\rho g}+\frac{\alpha_1 V_1^{\;2}}{2g}+z_1\right)-\left(\frac{p_2}{\rho g}+\frac{\alpha_2 V_2^{\;2}}{2g}+z_2\right)=h_L=\sum h_{L\mathrm{major}}+\sum h_{L\min\mathrm{or}}$$

- The flow rate leads to the Reynolds number and hence the friction factor for the flow.
- Tabulated data can be used for minor loss coefficients and equivalent lengths.
- The energy equation can then be rearranged and solved directly for the pipe length.

Given Δp, L, and D, find Q

- These types of problems required either manual iteration or use of a computer application.
- The unknown flow rate or velocity is needed before the Reynolds number and hence the friction factor can be found.
- First, we make a guess for f^* and solve the energy equation for V in terms of known quantities and the guessed friction factor f^*.
- Then we can compute a Reynolds number and hence obtain a new value for f.

Repeat the iteration process

$f^* \rightarrow V \rightarrow Re \rightarrow f$ until convergence ($f^* = f$)

Given Δp, L, and Q, find D

- These types of problems required either manual iteration or use of a computer application.
- The unknown diameter is needed before the Reynolds number and relative roughness, and hence the friction factor can be found.
- First, we make a guess for f^* and solve the energy equation for D in terms of known quantities and the guessed friction factor f^*.
- Then we can compute a Reynolds number and hence obtain a new value for f.

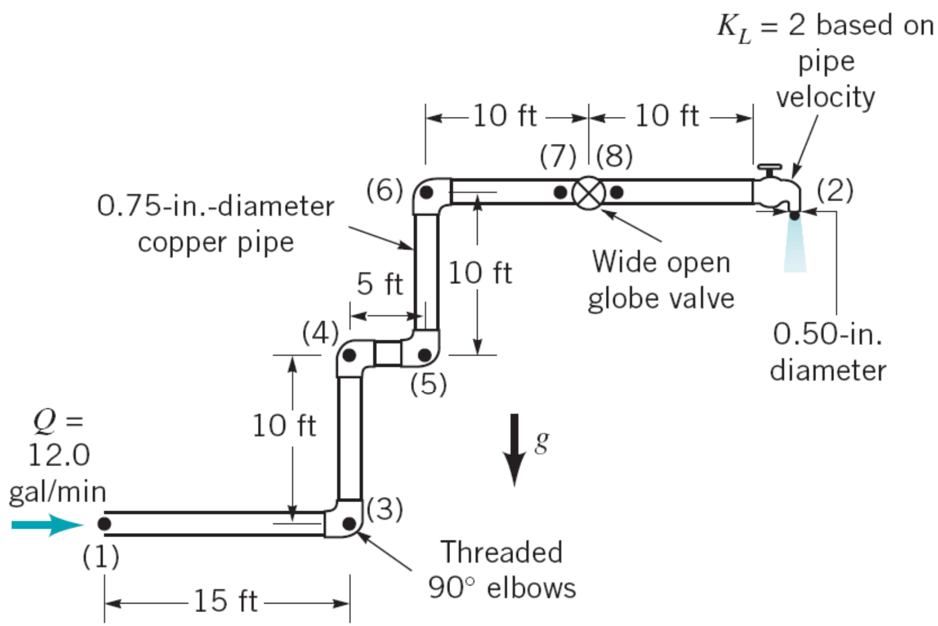
Repeat the iteration process

$f^* \rightarrow D \rightarrow (Re \text{ and } \varepsilon/D) \rightarrow f$ until convergence ($f^*=f$)

Example 8.8 Type I Determine Pressure Drop

- Water at 60°F flows from the basement to the second floor through the 0.75-in. (0.0625-fy)-diameter copper pipe (a drawn tubing) at a rate of Q = 12.0 gal/min = 0.0267 ft^3/s and exits through a faucet of diameter 0.50 in. as shown in Figure E8.8.

Determine the pressure at point (1) if: (a) all losses are neglected, (b) the only losses included are major losses, or (c) all losses are included.



Example 8.8 Solution[1/4]

$$V_1 = \frac{Q}{A_1} = \ldots = 8.70 \mathrm{ft/s} \qquad \rho = 1.94 \mathrm{slug/ft^3}$$

$$\mu = 2.34 \times 10^{-5} \mathrm{lb \cdot s/ft^2} \qquad \mathrm{Re} = \rho V D/\mu = 45000$$

The flow is turbulent

Nearly uniform velocity profile

$$\alpha_1 = \alpha_2 \cong 1$$

The energy equation

$$\frac{p_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_L$$

$$p_1 = \gamma z_2 + \tfrac{1}{2}\rho(V_2^2 - V_1^2) + \gamma h_L$$

$$z_1 = 0, z_2 = 20\ ft, p_2 = 0(free\ \ jet)$$

$$V_2 = Q/A_2 = \ldots = 19.6\ ft/s$$

$$V_1 = Q/A_1 = 8.70\ ft/s$$

Head loss is different for each of the three cases.

Example 8.8 Solution 2/4

(a) If all losses are neglected (h_L=0)

$$p_1 = \gamma z_2 + \tfrac{1}{2}\rho(V_2{}^2 - V_1{}^2) = \ldots = 1547 lb/ft^2 = 10.7 psi$$

(b) If the only losses included are the major losses, the head loss is

$$h_L = f\frac{\ell}{D}\frac{V_1^2}{2g}$$

Moody chart

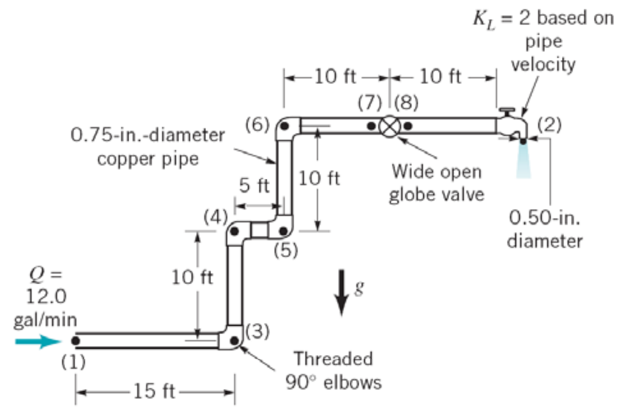
$$\varepsilon = 0.000005,\ \ \varepsilon / D = 8\times 10^{-5},\ \ Re = 45000 \Rightarrow f=0.0215$$

$$p_1 = \gamma z_2 + \tfrac{1}{2}\rho(V_2{}^2 - V_1{}^2) + \rho f\frac{\ell(=60ft)}{D}\frac{V_1{}^2}{2} = \ldots = 3062 lb/ft^2 = 21.3 psi$$

Example 8.8 Solution[3/4]

(c) If major and minor losses are included

$$p_1 = \gamma z_2 + \tfrac{1}{2}\rho(V_2^{\,2} - V_1^{\,2}) + f\gamma\frac{\ell}{D}\frac{V_1^2}{2g} + \sum \rho K_L \frac{V^2}{2}$$



$$p_1 = 21.3\text{psi} + \sum \rho K_L \frac{V^2}{2}$$

$$= 21.3\text{psi} + (1.94\text{slugs}/\text{ft}^3)\frac{(8.70\text{ft}/\text{s})^2}{2}[\underbrace{10}_{\text{Valve (pp. 72)}} + \underbrace{4(1.5)}_{\text{Elbow}} + \underbrace{2}_{\text{Faucet}}]$$

$$p_1 = 21.3\text{psi} + 9.17\text{psi} = 30.5\text{psi}$$

Example 8.9 Type I, Determine Head Loss

- Crude oil at 140°F with γ=53.7 lb/ft^3 and μ= 8×10-5 lb·s/ft^2 (about four times the viscosity of water) is pumped across Alaska through the Alaska pipeline, a 799-mile-along, 4-ft-diameter steel pipe, at a maximum rate of Q = 2.4 million barrel/day = 117ft^3/s, or V=Q/A=9.31 ft/s. Determine the horsepower needed for the pumps that drive this large system.

Example 8.9 Solution 1/2

The energy equation between points (1) and (2)

$$\frac{p_1}{\gamma}+\frac{\alpha_1 V_1^{\ 2}}{2g}+z_1+h_P=\frac{p_2}{\gamma}+\frac{\alpha_2 V_2^{\ 2}}{2g}+z_2+h_L$$

h_P is the head provided to the oil by the pump.

Assume that $z_1=z_2$, $p_1=p_2=V_1=V_2=0$ (<- large, open tank)

Minor losses are negligible because of the large length-to-diameter ratio of the relatively straight, uninterrupted pipe.

$$h_L=h_P=f\frac{\ell}{D}\frac{V^2}{2g}=\ldots=17700\text{ft}$$

Turbulent (α=1)

Table 8.1

f=0.0124 from Moody chart ε/D=(0.00015ft)/(4ft), Re=7.76 x 10^5

For steel pipe

Example 8.9 Solution$^{2/2}$

The actual power supplied to the fluid.

$$P_a = \gamma Q h_P = \ldots \left(\frac{1hp}{550\,ft \cdot lb / s} \right) = 202000hp$$

-> It requires many pump stations to be set up.

Example 8.10 Type II, Determine Flowrate

- According to an appliance manufacturer, the 4-in-diameter galvanized iron vent on a clothes dryer is not to contain more than 20 ft of pipe and four 90° elbows. Under these conditions determine the air flowrate if the pressure within the dryer is 0.20 inches of water. Assume a temperature of 100°F and standard pressure. K_L = 0.5 for an entrance and 1.5 for elbows.

Example 8.10 Solution[1/2]

Application of the energy equation between the inside of the dryer, point (1), and the exit of the vent pipe, point (2) gives

$$\frac{p_1}{\gamma}+\frac{\alpha_1 V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{\alpha_2 V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}+\sum K_L\frac{V^2}{2g}$$

Assume that $z_1=z_2$, $p_2=0$, $V_1=0$ $\quad \frac{p_1}{\gamma}+\cancel{\frac{\alpha_1 V_1^2}{2g}}+\cancel{z_1}=\cancel{\frac{p_2}{\gamma}}+\frac{\alpha_2 V_2^2}{2g}+\cancel{z_2}+f\frac{\ell}{D}\frac{V^2}{2g}+\sum K_L\frac{V^2}{2g}$

$$\frac{p_1}{\gamma_{H_2O}}=0.2\text{in}\Rightarrow p_1=(0.2\text{in.})\left(\frac{1\text{ft}}{12\text{in.}}\right)(62.4\text{lb}/\text{ft}^3)=1.04\text{lb}/\text{ft}^2$$

With $\gamma=0.0709\text{lb/ft}^3$, $V_2=V$, and $\nu=1.79\times10^{-4}\ \text{ft}^2/\text{s}$.

$$\frac{1.04(\text{lb}/\text{ft}^2)}{0.0709(\text{lb}/\text{ft}^3)}=\left[1+f\frac{20\text{ft}}{4/12\text{ft}}+0.5+4\times1.5\right]\frac{V^2}{2(32.2)(\text{ft}/\text{s}^2)} \quad \textbf{(1)}$$

Assume turbulent flow (α=1)

$$945=(7.5+60f)V^2$$

f is dependent on Re, which is dependent on V, and unknown.

Example 8.10 Solution[2/2]

$$\mathrm{Re} = \frac{VD}{\nu} = \ldots = 1860V \qquad \textbf{(2)}$$

For galvanized iron

We have three relationships (Eq. 1, 2, and the ε/D=0.0015 curve of the Moody chart) from which we can solve for the three unknowns f, Re, and V.

This is done easily by iterative scheme as follows.

Assume f=0.022→V=10.4ft/s (Eq. 1)→Re=19,300 (Eq.2)→f=0.029

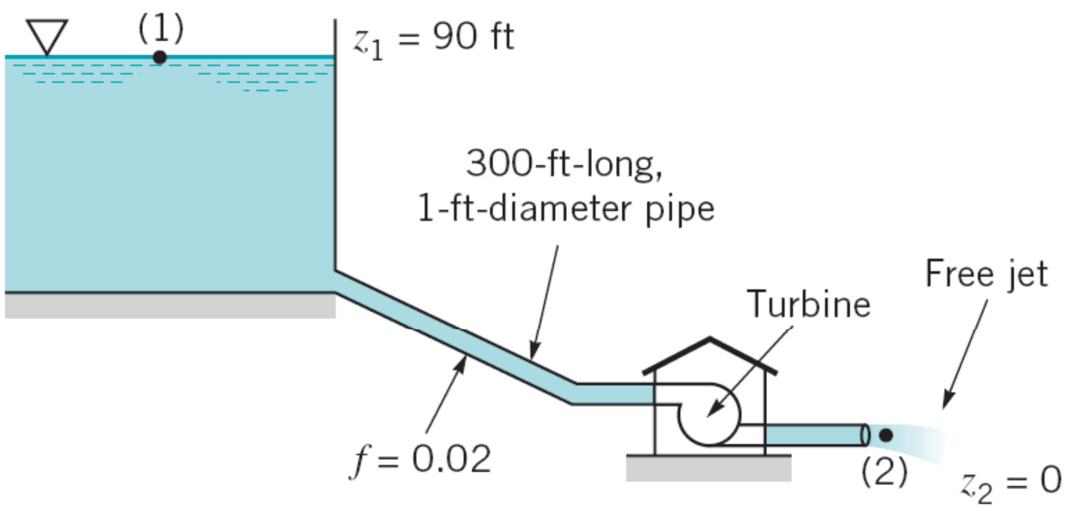
Assume f=0.029 →V10.1ft/s→Re=18,800 →f=0.029 (Final value)

Turbulent

$$Q = AV = \ldots = 0.881\,\mathrm{ft}^3/\mathrm{s}$$

Example 8.11 Type II, Determine Flowrate

- The turbine shown in Figure E8.11 extracts 50 hp from the water flowing through it. The 1-ft-diameter, 300-ft-long pipe is assumed to have a friction factor of 0.02. Minor losses are negligible. Determine the flowrate through the pipe and turbine.



Example 8.11 Solution[1/2]

The energy equation can be applied between the surface of the lake and the outlet of the pipe as

$$\frac{p_1}{\gamma}+\frac{\alpha_1 V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{\alpha_2 V_2^2}{2g}+z_2+h_L+h_T \qquad \frac{\cancel{p_1}}{\gamma}+\frac{\cancel{\alpha_1 V_1^2}}{2g}+z_1=\frac{\cancel{p_2}}{\gamma}+\frac{\alpha_2 V_2^2}{2g}+\cancel{z_2}+h_L+h_T$$

Where $p_1=V_1=p_2=z_2=0$, $z_1=90\text{ft}$, and $V_2=V$, the fluid velocity in the pipe

$$h_L=f\frac{\ell}{D}\frac{V^2}{2g}=0.0932V^2\text{ft} \qquad h_T=\frac{P_a}{\gamma Q}=\ldots=\frac{561}{V}\text{ft}$$

Assume turbulent flow ($\alpha=1$)

$$90\text{ft}=\frac{V^2}{2\times 32.2\left(\text{ft}/\text{s}^2\right)}+0.0932V^2\text{ft}+\frac{561}{V}\text{ft} \;\; -> 0.107V^3-90V+561=0$$

There are two real, positive roots: V=6.58 ft/s or V=24.9 ft/s. The third root is negative (V=-31.4ft/s) and has no physical meaning for this flow.

Example 8.11 Solution$^{2/2}$

Two acceptable flowrates are

$$Q=\frac{\pi}{4}D^2V=\ldots=5.17\,ft^3/s$$ → At 60^oF, 1.21×10^{-5} (ft^2/s) $Re \approx 10^5$

$$Q=\frac{\pi}{4}D^2V=\ldots=19.6\,ft^3/s$$

Example 8.12 Type III Without Minor Losses, Determine Diameter

- Air at standard temperature and pressure flows through a horizontal, galvanized iron pipe (ε=0.0005 ft) at a rate of 2.0ft^3/s. Determine the minimum pipe diameter if the pressure drop is to be no more than 0.50 psi per 100 ft of pipe.

Example 8.12 Solution$^{1/2}$

Assume the flow to be incompressible with ρ=0.00238 slugs/ft^3 and μ=3.74×10^{-7} lb · s/ft^2.

If the pipe were too long, the pressure drop from one end to the other, p_1-p_2, would not be small relative to the pressure at the beginning, and compressible flow considerations would be required.

$$\frac{p_1}{\gamma}+\frac{\cancel{\alpha_1}V_1^2}{2g}+\cancel{z_1}=\frac{p_2}{\gamma}+\frac{\cancel{\alpha_2}V_2^2}{2g}+\cancel{z_2}+f\frac{\ell}{D}\frac{V^2}{2g}+\cancel{\sum K_L}\frac{V^2}{2g}$$

With z_1=z_2, V_1=V_2 , The energy equation becomes $p_1=p_2+f\frac{\ell}{D}\frac{\rho V^2}{g}$

$$p_1-p_2=(0.5)(144)\mathrm{lb/ft^2}=f\frac{(100\mathrm{ft})}{D}(0.00238\mathrm{slugs/ft^3})\frac{V^2}{2g}$$

$$V=\frac{Q}{A}=\frac{2.55}{D^2}$$

$$D=0.404f^{1/5} \quad (1)$$

Example 8.12 Solution 2/2

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \ldots = \frac{1.62 \times 10^4}{D} \qquad (2)$$

$$\frac{\varepsilon}{D} = \frac{0.0005}{D} \qquad (3)$$

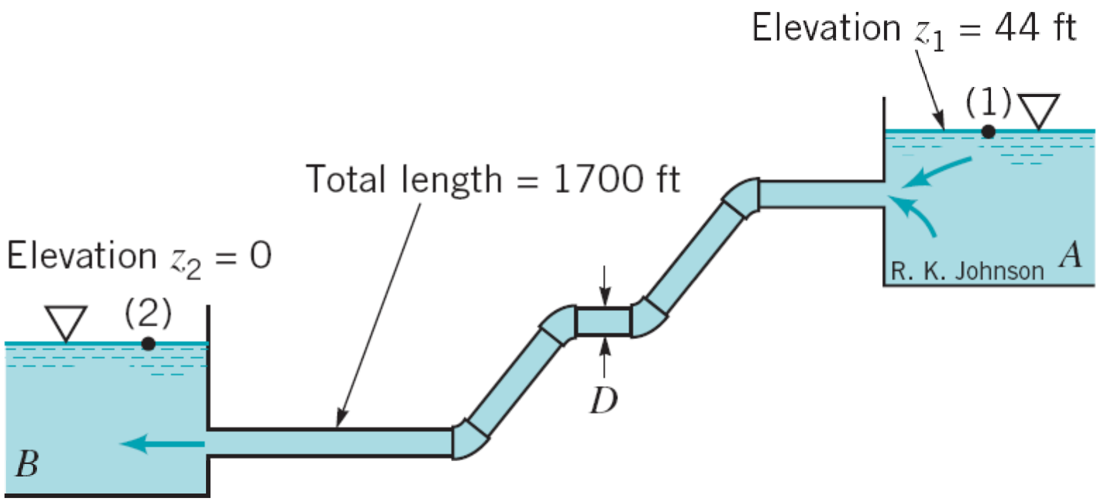
We have four equations (Eq. 1, 2, 3, and either the Moody chart or the Colebrook equation) and four unknowns (f, D, ε/D, and Re) from which the solution can be obtained by trial-and-error methods.

Repeat the iteration process

$$f^* \underset{(1)}{\rightarrow} D \underset{(2)}{\rightarrow} \mathrm{Re} \text{ and } \varepsilon/D \underset{(3)}{\rightarrow} f \quad \textbf{until convergence}$$

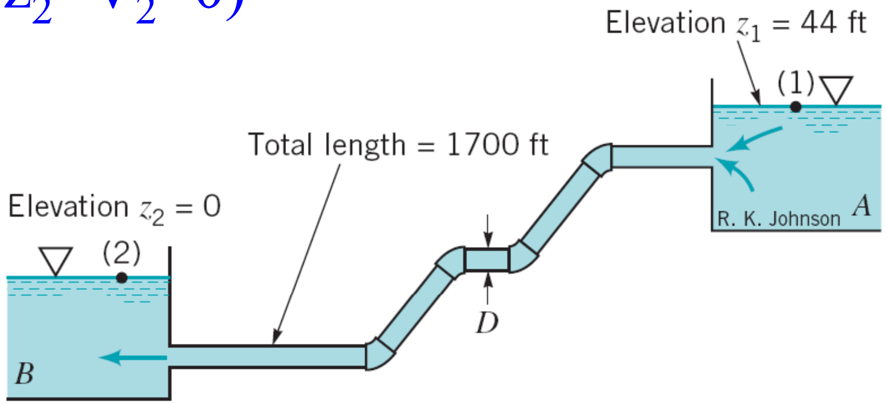
Example 8.13 Type III With Minor Losses, Determine Diameter

- Water at 60°F (ν=1.21×10^{-5} ft^2/s) is to flow from reservoir A to reservoir B through a pipe of length 1700 ft and roughness 0.0005 ft at a rate of Q= 26 ft^3/s as shown in Figure E8.13. The system contains a sharp-edged entrance and four flanged 45° elbow. Determine the pipe diameter needed.



Example 8.13 Solution[1/2]

The energy equation can be applied between two points on the surfaces of the reservoirs ($p_1=V_1=p_2=z_2=V_2=0$)



$$\frac{\cancel{p_1}}{\gamma}+\frac{\alpha_1 \cancel{V_1^2}}{2g}+z_1=\frac{\cancel{p_2}}{\gamma}+\frac{\alpha_2 \cancel{V_2^2}}{2g}+\cancel{z_2}+h_L$$

$$z_1=\frac{V^2}{2g}\left(f\frac{\ell}{D}+\sum K_L\right)$$

$$V=\frac{Q}{A}=\frac{33.1}{D^2}$$

$K_{Lent}=0.5$, $K_{Lelbow}=0.2$, and $K_{Lexit}=1$

$$44\text{ft}=\frac{V^2}{2(32.2\text{ft}/s^2)}\left(f\frac{1700}{D}+[4(0.2)+0.5+1]\right)$$

$$f=0.00152D^5-0.00135D \qquad (1)$$

Example 8.13 Solution 2/2

$$\mathrm{Re} = \frac{VD}{\nu} = \ldots = \frac{2.74 \times 10^6}{D} \qquad (2)$$

$$\frac{\varepsilon}{D} = \frac{0.0005}{D} \qquad (3)$$

We have four equations (Eq. 1, 2, 3, and either the Moody chart or the Colebrook equation) and four unknowns (f, D, ε/D, and Re) from which the solution can be obtained by trial-and-error methods.

Repeat the iteration process

$$\mathbf{D \underset{(1)}{\rightarrow} f^{*} \underset{(2)}{\rightarrow} Re \text{ and } \varepsilon/D \underset{(3)}{\rightarrow} f \text{ until convergence}}$$