Volatility Spillovers from US to SA Markets

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Abstract

I investigate the relationship between the voaltilities of S&P 500 and the JSE Top 40. The purpose of this study is to investigate if this relationship changes in any significant way during the two biggest crisis periods in the last two decades, namely the Global Financial Crisis and Covid-19. I first do a stratification analysis which reveals significant evidence of these two indices sharing periods of high volatility. I then fit multiple multivariate GARCH models to further investigate the volatility relationship and find…

Keywords: Multivariate GARCH, Spillovers

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1. Introduction

2. Data

Three return series are used in the analysis that follows. These are the monthly returns for the S&P 500 and the JSE Top 40, as well as thhe ZAR/USD exchange rate. The exchange rate is represented as the amount of Rands required to buy one US Dollar. Since the series is represented as a growth rate, a postive growth rate represents a depreciation of the Rand, and conversely, an appreciation of the Dollar. The inclusion of an exchange rate serves primarily as a control variable and as such analysis regarding the exchange rate is kept to a minimum in the final analysis. The returns for these 3 series' are visualised below in Figures 2.1 to 2.3.

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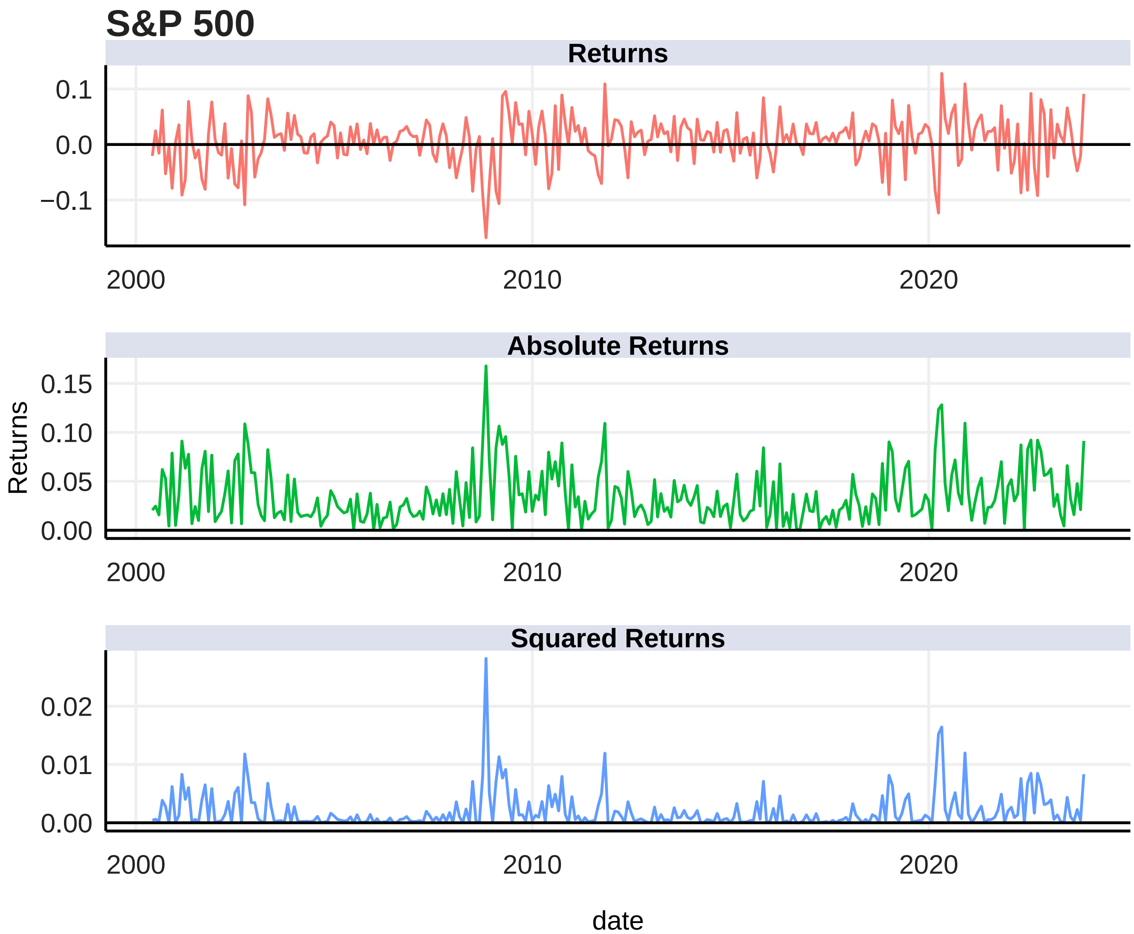

Figure 2.1: S&P 500 Returns

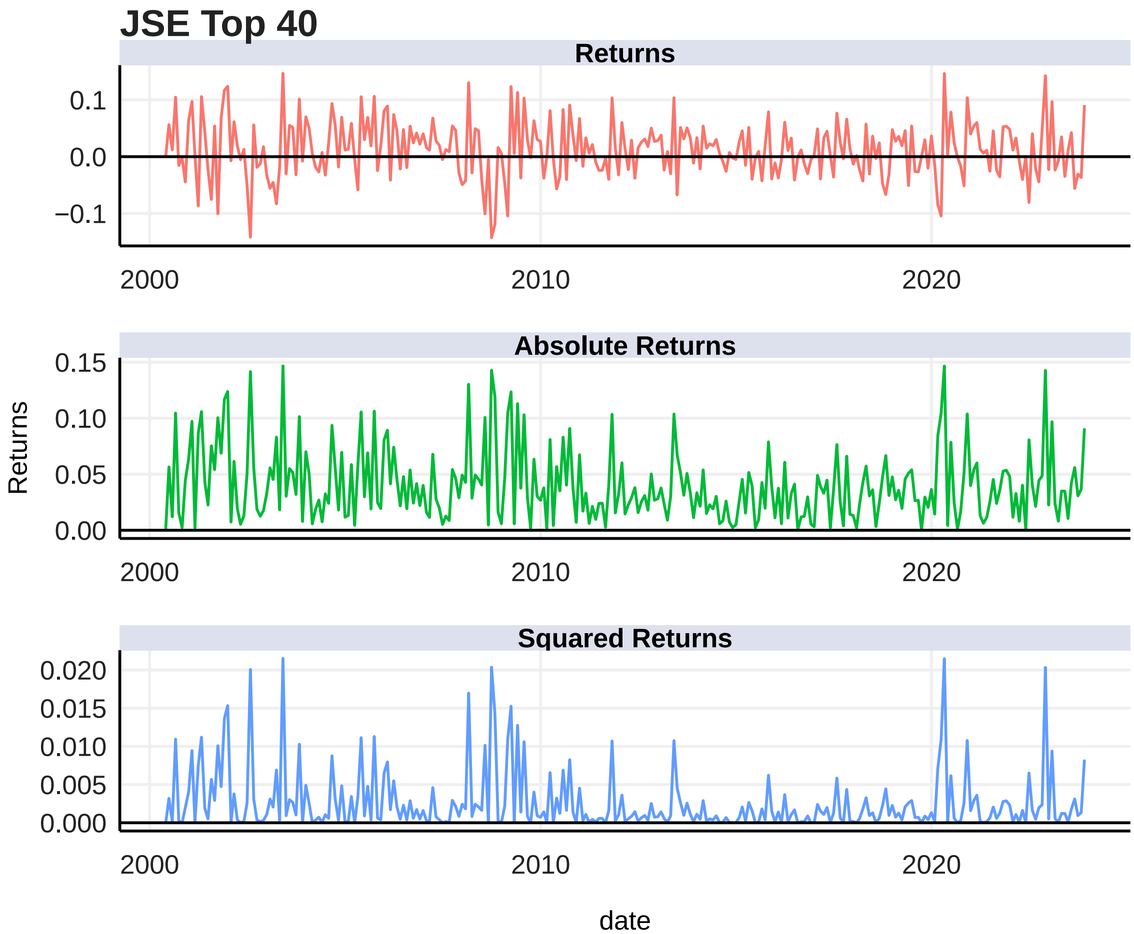

Figure 2.2: JSE Top 40 Returns

Not much information can be revealed through simply observing the returns over time. However, when investigating the squared returns as a measure of volatility, it is clear to see that the JSE Top 40 is substantially more volatile than the S&P 500. This result is reinforced by Table 2.1, where the JSE showcases a standard deviation considerably higher than that of the S&P. Interestingly, the JSE Top 40 showcases higher average monthly returns, however that comes at the cost of the increased volatility as described above. Lastly, as shown in Table 2.1 the S&P experienced the largest draw down, while the JSE experienced the largest uptick.

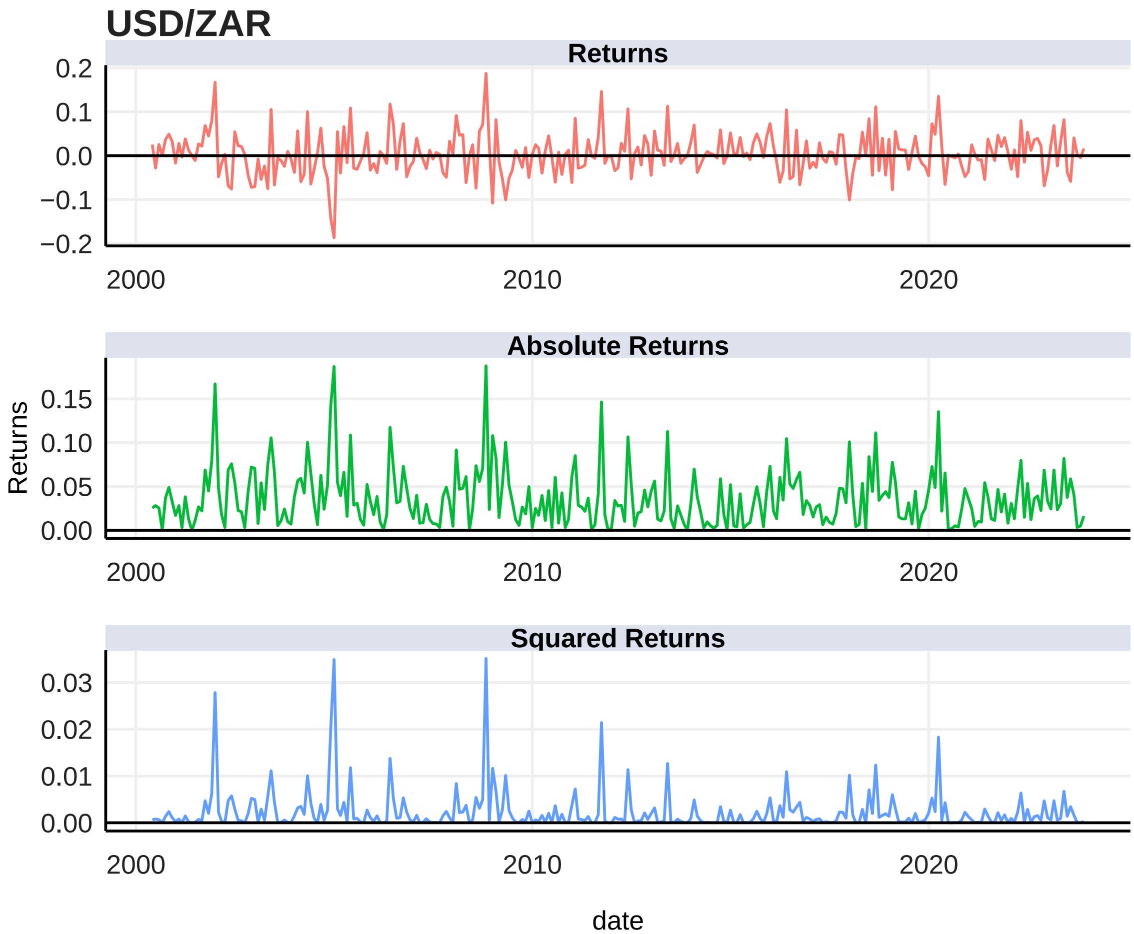

Figure 2.3: ZAR/USD Returns

Table 2.1: Summary Statistics

	S&P 500	JSE Top 40	ZAR/USD
Mean	0.0066	0.0121	0.0040
Median	0.0124	0.0115	0.0018
Std. Dev.	0.0445	0.0502	0.0485
Kurtosis	3.7769	3.3257	4.5258
Skewness	-0.5048	0.0359	0.2933
Minimum	-0.1680	-0.1427	-0.1868
Maximum	0.1282	0.1467	0.1875

Before GARCH models can be fitted, ARCH tests need to be conducted in order to see if controlling for conditional heteroskedasticity is appropriate. I employ two tests. First, a univariate Ljung-Box test is conducted on each series. Practically, to test for ARCH effects a simple AR(1) model is fitted for each series and then Ljung-Box tests are done on the residuals of each AR(1). Next, multivariate Portmanteau tests are conducted to incorporate all variables simultaneously. As outlined by Tsay (2014), 3 tests are tun. The results can found in the tables below

Table 2.2: Ljung-Box Tests

Series	TestStatistic	PValue	LagOrder
SP500	66.5254	0.0000	12
JSE40	65.7493	0.0000	12
ZARUSD	16.0904	0.1378	12

Table 2.3: MV Portmanteau Tests

	Test Statistic	p-value
Q(m) of squared series(LM test)	81.3244	0.0001
Rank-based Test	92.5271	0.0001
Q_k(m) of squared series:	165.2131	0.0001

As can be seen in Table 2.2, for the S&P 500 and the JSE Top 40, the p-values are functionally zero. This means that we can reject the null of no ARCH effects for these series. The same does not hold for ZAR/USD exchange rate, which has a p-value greater than the critical level of 0.05. However when conducting multivariate tests, all tests report p-values that are functionally zero. As such the analysis continues with the assumption that ARCH effects are present within the data. This serves as motivation for the use of GARCH models in this essay.

3. Methodology

I first perform stratification analysis on all three series to determine if periods of high or low volatility are shared across the S&P, the JSE and the ZAR/USD exchange rate. I then fit multiple univariate GARCH models on all three variables to determine an appropriate specification for the multivariate models to come. I then fit three multivariate GARCH models, namely a DCC model, a Go-GARCH model, and a BEKK-GARCH model. Formal definitions and explanations of these models follow below.

3.1. DCC GARCH

Dynamic Conditional Correlation (DCC) models, developed by Engle (2002), are a class of multivariate GARCH models that allow for time varying correlation between variables. This is especially useful to study how specific time periods, like crises, affect the relationship between different stock indices and financial variables. Consider the following GARCH(1,1) model:

$$R_{it} = \mu_i + \epsilon_{it}, \quad \epsilon_{it} = \sigma_{it} z_{it}, \quad z_{it} \sim N(0,1) \tag{3.1}$$

In the equation above, $R_t = (R_{1t}, R_{2t}, ...R_{nt})$, for n assets/variables, is a vector of asset returns at time t. Here μ_i is the mean return, ϵ_{it} is the residual, σ_{it}^2 is the conditional variance and z_{it} is the standard normal innovation. The conditional variance σ_{it}^2 is modeled as:

$$\sigma_{it}^2 = \alpha_0 + \alpha_1 \epsilon_{it-1}^2 + \beta_1 \sigma_{it-1}^2 \tag{3.2}$$

To specify the DCC model, the standardized residuals are defined as $\tilde{\epsilon_t} = (\epsilon_{1t}/\sigma_{1t}, \ldots, \epsilon_{nt}/\sigma_{nt})'$. The correlation matrix of $\tilde{\epsilon_t}$, denoted by Q_t, evolves over time as:

$$Q_t = \bar{Q}(1 - a - b) + a\tilde{\epsilon}_{t-1}\tilde{\epsilon}'_{t-1} + bQ_{t-1} \tag{3.3}$$

where $\bar{Q}$ is the unconditional correlation matrix of $\tilde{\epsilon_t}$. Parameters, a and b are positive and adhere to $a + b < 1$ to ensure stationarity. To obtain the dynamic conditional correlation, the elements of Q_t are standardized.

In order to estimate a DCC GARCH model, two steps are followed. First, to obtain σ_{it} and $\tilde{\epsilon}_t$, a univariate GARCH is fitted for each return series. Then, secondly, the DCC parameters a and b are estimated using a likelihood function derived from the conditional multivariate distribution of $\tilde{\epsilon}_t$.

3.2. GO-GARCH

Generalized Orthogonalized (GO) GARCH models are another class of multivariate GARCH models. Developed by Van der Weide (2002), the model is based on the assumption that asset returns can be decomposed into orthogonal components, thus simplifying the modeling of their covariance structure. Note that GO-GARCH models can become computationally intensive quickly, as the number of variables in the model increases. Once again consider a a set of n asset returns, $R_t = (R_{1t}, R_{2t}, ...R_{nt})$. These returns can be expressed as a linear combination of its orthogonal components:

$$R_t = B_t F_t \tag{3.4}$$

Here B_t is a time-varying $n \times n$ matrix of loadings and F_t are the orthogonal components, $F_t = (F_{1t}, F_{2t}, ...F_{nt})'$, that are assumed to follow a univariate GARCH process:

$$F_{it} = \sigma_{it} z_{it} \quad z_{it} \sim N(0,1) \tag{3.5}$$

where σ_{it}^2 is the conditional variance of F_{it}. In order to estimate the model, the loading matrix B_t is estimated based on the observed correlation of the series', while the volatilities of the orthogonal components, σ_{it}^2, are estimated using standard GARCH procedures.

3.3. BEKK-GARCH

The BEKK-GARCH models, initially developed by Engle & Kroner (1995), are designed specifically to study spillovers between series. The model's ability to capture dynamic covariances and correlations makes it particularly useful for analyzing the interdependencies in financial markets, especially in the context of crises periods. As before consider n assets with returns, $R_t = (R_{1t}, R_{2t}, ...R_{nt})$. For a GARCH(1,1) these returns are given by:

$$R_t = \mu + \epsilon_t, \quad \epsilon_t = H_t^{1/2} z_t, \quad z_t \sim N(0, I) \tag{3.6}$$

In Equation 3.6, as before μ is the vector of mean returns and ϵ_t is the vector of residuals. Now, H_t is the conditional covariance matrix and z_t is a vector of i.i.d. standard normal innovations. The conditional covariance matrix H_t is modeled as:

$$H_t = C + A\epsilon_{t-1}\epsilon_{t-1}' A' + B H_{t-1} B' \tag{3.7}$$

where C, A and B are coefficient matrices. Notably, C is a triangular matrix with positive diagonal elements, ensuring that it is positive definite. This attribute removes the need for additional constraints. Lastly, estimation is done via maximum likelihood.

4. Results

Like stated above, I first employ stratification analysis. I then fit three multivariate GARCH models. The univariate specification these models are based on are selected by fitting different specifications and selecting the best one based on various selection criteria. These results show a gjrGARCH to be the best specification. [1].

4.1. Stratification

Stratification analyses allows for the investigation of a particular assets volatility during a particular period of volatility of another asset. While such analysis does not lend itself to causal interpretation regarding volatility spillovers, it does paint a picture of whether the indices that are being investigated tend to have periods of high or low volatility at the same time. In turn, this could point to a direction of interconnectedness between markets which can then be revealed through more robust analysis. The stratification analyses follows below.

Table 4.1: S&P 500 High Volatility

Index	SD	Full_SD	Period	Ratio
JSE40	0.23	0.17	High_Vol SP500	1.35
ZARUSD	0.19	0.16	High_Vol SP500	1.17

Table 4.2: S&P 500 Low Volatility

Index	SD	Full_SD	Period	Ratio
ZARUSD	0.16	0.16	Low_Vol SP500	1.03
JSE40	0.14	0.17	Low_Vol SP500	0.81

Tables 4.1 and 4.2 showcase the stratification of high and low volatility of the S&P 500. The "SD" column report volatility during that particular period while the "Full_SD" column shows the volatility for the entire sample period. As such a ratio greater than 1 indicates that particular index or series has a higher than usual volatility in a given period. Analysing periods of high volatility of the S&P show both the JSE and the ZAR/USD also have significantly higher volatility in these periods, indicated by the ratio > 1. The JSE also reports lower volatility in periods where the S&P 500 is less volatile, although the same is not true for the exchange rate.

[1] These results are not reported in this document since they serve little purpose other than model construction. For a full table showing the test results see : https://github.com/RuanGeldenhuys/fmx_project_22550801

Table 4.3: JSE Top 40 High Volatility

Index	SD	Full_SD	Period	Ratio
SP500	0.20	0.15	High_Vol JSE40	1.37
ZARUSD	0.19	0.16	High_Vol JSE40	1.17

Table 4.4: JSE Top 40 Low Volatility

Index	SD	Full_SD	Period	Ratio
ZARUSD	0.13	0.16	Low_Vol JSE40	0.82
SP500	0.10	0.15	Low_Vol JSE40	0.70

Tables 4.3 and 4.4 now report the stratification for the JSE Top 40. For high volatility periods, the relationship holds with both the S&P 500 and the exchange rate showcasing higher than average volatility. In low volatility periods, again, both the S&P and the ZAR/USD report lower than usual volatility. Like this stated earlier, this does not allow for causal interpretation, but it does point to the fact that the two indices tend to be in periods of high or low volatility at the same time.

Table 4.5: ZAR/USD High Volatility

Index	SD	Full_SD	Period	Ratio
JSE40	0.18	0.17	High_Vol ZARUSD	1.04
SP500	0.16	0.15	High_Vol ZARUSD	1.07

Table 4.6: ZAR/USD Low Volatility

Index	SD	Full_SD	Period	Ratio
JSE40	0.13	0.17	Low_Vol ZARUSD	0.79
SP500	0.13	0.15	Low_Vol ZARUSD	0.88

Lastly, analysing stratification of the ZAR/USD reveals an interesting result. In periods of high volatility (Table 4.5), both the JSE and S&P also report higher volatility. However this volatility is only slighter higher, with both indices reporting a standard deviation that is only 0.01 larger than the full sample. Conversely, in periods of low volatility (Table 4.6), these indices show significant lower volatility as well. This indicates that a highly volatile Rand does not necessarily mean volatile stock markets, however a low volatile Rand tends to be associated with low volatility in these indices.

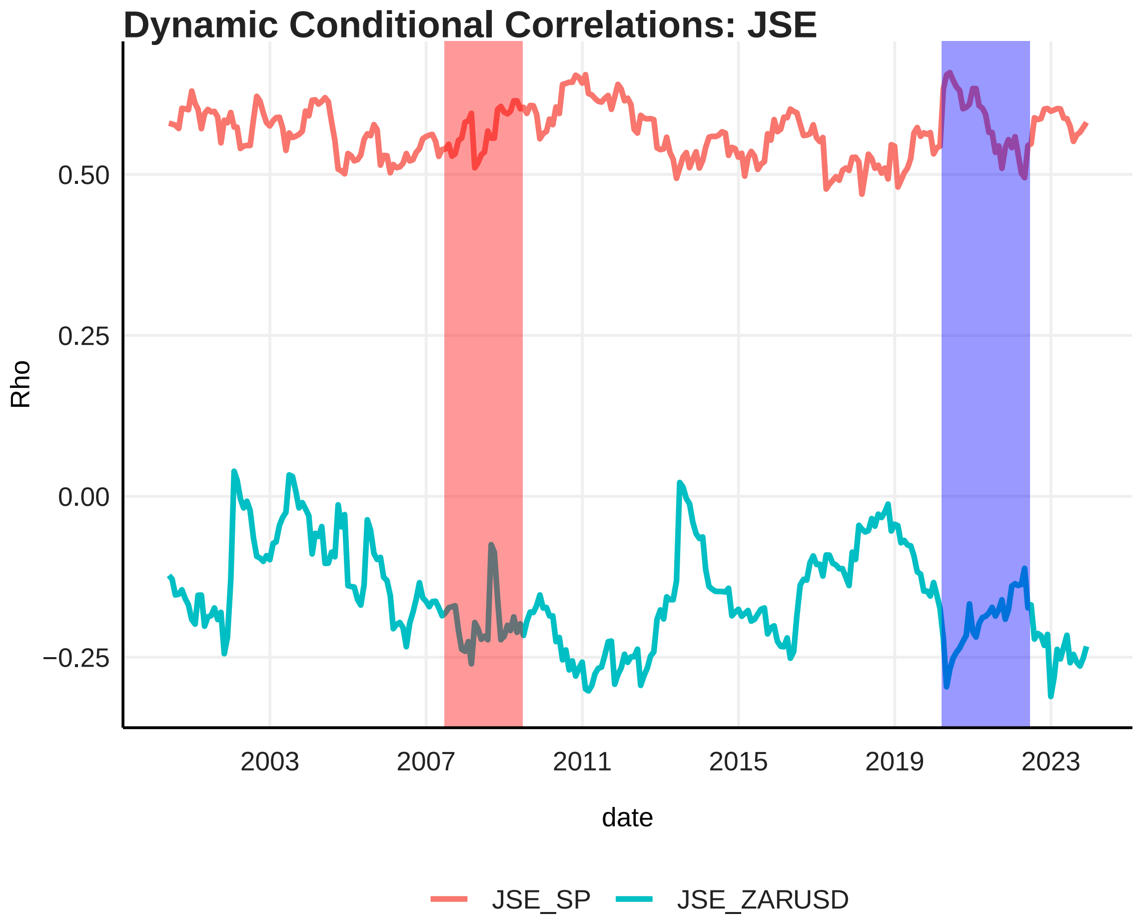

Figure 4.1: DCC GARCH

The dynamic conditional correlation for the JSE Top 40, as reported by the DCC-GARCH model, are shown in Figure 4.1. Intuitively, it shows a noise reduced correlation between the JSE and other variables in the system over time. What is immediately clear is that the JSE shares a significantly higher correlation with the S&P 500, than it does with the ZAR/USD exchange rate. In fact, the correlation with the ZAR/USD is negative for much of the sample period. This result does make sense since "increases" in the exchange rate indicate a depreciation of the Rand.

Analyzing the crisis periods, particularly in the context of the correlation between the JSE Top 40 and the S&P 500, reveals an interesting result. Note, the GFC period is indicated by the red shaded area, while Covid-19 is shown in blue. During the GFC the correlation jumps up initially, then turns sharply downward, before tending upward for the rest of the crisis. The difference in correlation between the start of the crises and the maximum correlation is equal to 0.075, while the difference between the start and end of the crisis is equal to 0.063.

Correlation during Covid-19 behaves differently. Here the model reports a sharp increase in correlation that fades out as the crisis draws to a close. As such the maximum correlation appears early in the crisis period. The difference in correlation between the start of the crisis period and the maximum is equal to 0.115. The difference between the start and end of the crisis is smaller at 0.010.

4.3. Go-GARCH

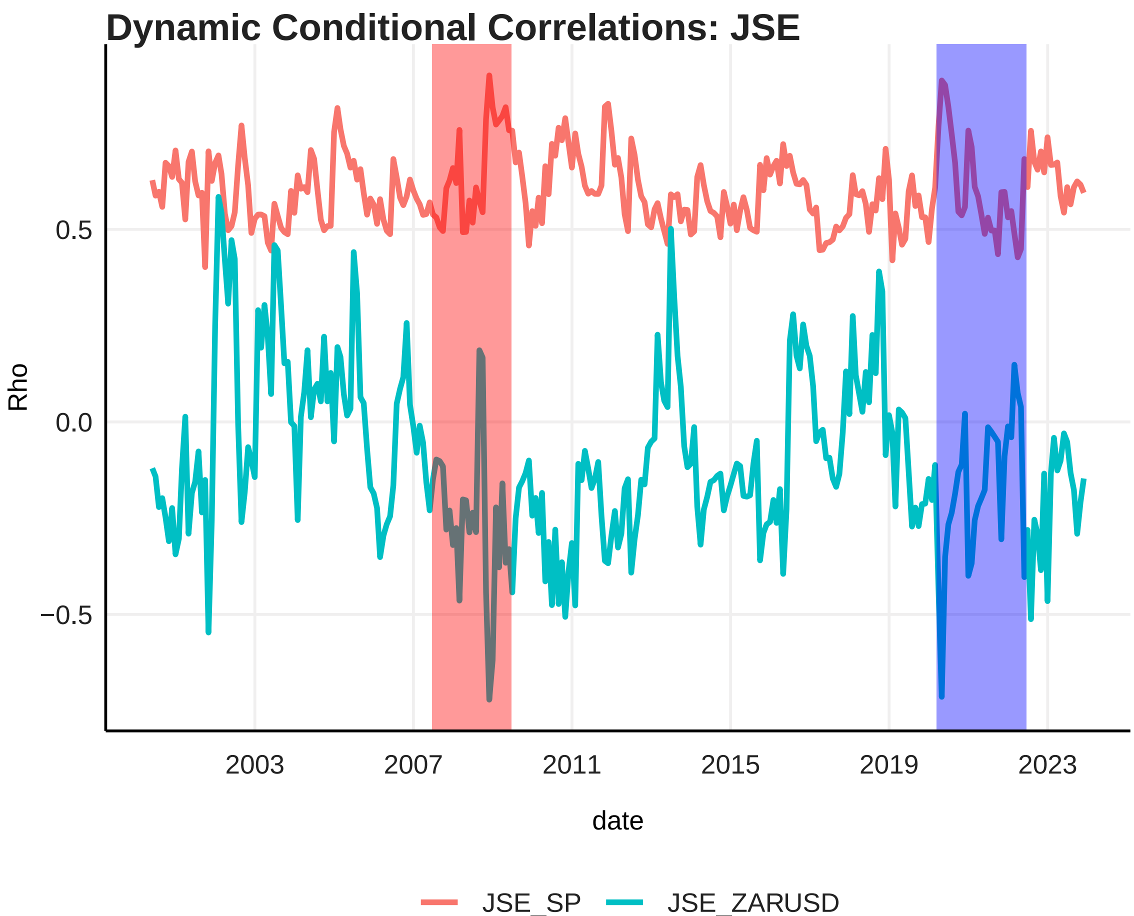

Figure 4.2: GO-GARCH

The dynamic conditional correlations for the JSE Top 40, as estimated by the GO-GARCH model is reported in Figure 4.2. It is immediately apparent that the correlation with the exchange rate is much more volatile than in the DCC model. It is still negative for most of the sample, however experiences large spikes, turning the relationship positive. The correlation with the S&P 500 follows a similar time path to the DCC model, in crises periods. As such the GO-GARCH serves as a robustness check for the results found by the DCC model.

A key difference between the model is the fact that the jump in correlation during crisis periods are more pronounced. The difference in correlation between the start and maximum of the GFC is equal to 0.362, while the difference between the first and last correlation is equal to 0.219. For Covid-19, the difference between the first correlation and maximum correlation is 0.278, and interestingly the difference between the first and last is negative at -0.011.

4.4. BEKK-GARCH

Table 4.7: BEKK-GARCH Constants

	SP	JSE	Rand
SP	0.0381	0.0182	-0.0143
JSE	0.0000	0.0150	-0.0091
Rand	0.0000	0.0000	0.0391

Table 4.8: BEKK-GARCH ARCH estimates

	SP	JSE	Rand
SP	0.0539	-0.3134	-0.3344
JSE	-0.3058	0.1428	0.5942
Rand	0.2485	0.2343	-0.0622

Table 4.9: BEKK-GARCH GARCH estimates

	SP	JSE	Rand
SP	-0.0013	-0.0216	-0.0150
JSE	0.0323	0.0814	-0.0049
Rand	-0.2760	-0.8576	-0.0662

5. Conclusion

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Appendix

Appendix A

Some appendix information here

Appendix B