Lab 1:

1] Given $\frac{dy}{dx} = x+y$, $y(0)=0$, To find y at $x=0.4$, $h=0.2$

Initial values $\Rightarrow x_0=0, y_0=0$

Using formula $\Rightarrow$ $y_1 = y_0 + h f(x_0+y_0)$

$y_1 = 0 + 0.2(0.0) = 0.2(0)$ as $f(x+y)=0$

Iteration 2 $\Rightarrow$ $y_2 = 0 + 0.2(0.2,0)$

$= 0 + 0.2(0.2)$

$= 0.04$

Solution $\rightarrow y_2 = y(0.4) \approx 0.04$

2] Given $\frac{dy}{dx} = -y$, $y(0)=1$, To find y at $x=0.2$, $h=0.1$

Initial values : $x_0=0, y_0=1$

Using formula : $y_1 = y_0 + h f(x_0, y_0)$

Iteration 1 $\Rightarrow$ $y_1 = 1 + 0.1(-1) = 0.9$

Iteration 2 $\Rightarrow$ $y_2 = 0.9 + 0.1(-0.9)$

$= 0.9 - 0.09$

$= 0.81$

Solution $\Rightarrow$ $y(0.2) \approx 0.81$

lab 2:

1] Given $\frac{dy}{dx} = x+y$, $y(0)=0$. To find y at $x=0.4$,

$h = 0.2$

Initial values - $x_0 = 0$, $y_0 = 0$

Using formulas

1st Iteration:-

$$y_1^0 = y_0 + hf(x_0, y_0)$$

$$= 0 + 0.2(0)$$

$$= 0$$

$$y_1^1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^0)]$$

$$= 0 + \frac{0.2}{2}[(0+0.2)]$$

$$= 0.02$$

$$y_1^2 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^1)]$$

$$= 0 + \frac{0.2}{2}[0 + 0.22]$$

$$= 0.022$$

$$y_1^3 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^2)]$$

$$= 0.0222$$

2nd Iteration:

$$y_2^0 = y_1 + hf(x_1, y_1)$$

$$= 0.0222 + 0.2 \times f(0.2, 0.0222)$$

$$= 0.06664$$

$$y_2^1 = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^0)]$$

$= 0.0222 + \frac{0.2}{2}[0.2222 + 0.46664]$

$= 0.091084$

$y_2^2 = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2')]$

$= 0.0222 + \frac{0.2}{2}[0.2222 + 0.491084]$

$= 0.0935284$

$y_2^3 = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^2)]$

$= 0.0222 + \frac{0.2}{2}[0.2222 + 0.4935284]$

$= 0.0937284$

$y_2^4 = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^3)]$

$= 0.0222 + \frac{0.2}{2}[0.2222 + 0.49377284]$

$= 0.09379728 4$

2] Given $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$, To find y at $x = 0.2$, $h = 0.1$

Given $x_0 = 0$, $y_0 = 1$

$x = 0.1, 0.2$

$h = 0.1$

1st iteration $\Rightarrow$

$y_1^0 = y_0 + hf(x_0, y_0)$

$= 1 + 0.1(0^2 + 1)$

$= 1.1$

$y_1' = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^0)]$

$$= 1 + \frac{0.1}{2}\left[(0^2+1) + (0.1^2+1.1)\right]$$

$$= 1.105$$

$$y_1^2 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1)\right]$$

$$= 1.1057$$

$$y_1^3 = 1.10578$$

2nd Iteration:

$$y_2^0 = y_1 + hf(x_1, y_1)$$

$$= 1.1057 + 0.1(0.1^2 + 1.1057).$$

$$= 1.21727$$

$$y_2^1 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^0)\right]$$

$$= 1.224$$

$$y_2^2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^1)\right]$$

$$= 1.2246$$

$$y_2^3 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^2)\right]$$

$$= 1.2247$$

Lab 3]

1] $\frac{dy}{dx} = y - x$, $y(0) = 2$, $x = 0.1, 0.2$, $h = 0.1$

$x_0 = 0$

$y_0 = 2$

① $\rightarrow K_1 = hF(x_0, y_0)$

$= 0.1 \times (2 - 0)$

$= 0.2$

$\rightarrow K_2 = hF(x_0 + h, y_0 + k_1)$

$= 0.1 \times ((2 + 0.2) - (0 + 0.1))$

$= 0.21$

$\rightarrow$ ~~K_3~~ $y(0.1) = y(0) + \frac{1}{2}(K_1 + K_2)$

$= 2 + \frac{1}{2}(0.2 + 0.21)$

$= 2.205$

②

$\rightarrow K_1 = hF(x_1, y_1)$

$= 0.1 (2.205 - 0.1)$

$= 0.2105$

$\rightarrow K_2 = hF(x_1 + h, y_1 + k_1)$

$= 0.1 \times ((2.205 + 0.2105) - (0.1 + 0.1))$

$= 0.22155$

$y(0.2) = y(1) + \frac{1}{2}(k_1 + k_2)$

$= 2.205 + \frac{1}{2}(0.2105 + 0.22155)$

$= 2.421$

2] $\frac{dy}{dx} = 1+y^2$, $y(0)=0$, $h=0.2$

$x_0 = 0$

$y_0 = 0$

① $K_1 = hf(x_0, y_0)$

$= 0.2 \times (1+0)$

$= 0.2$

② $K_2 = hf(x_0+h, y_0+K_1)$

$= 0.2 \times (1 + (0.2)^2)$

$= 0.208$

$y(0.2) = y(0) + \frac{1}{2}(K_1+K_2)$

$= 0 + \frac{1}{2}(0.2 + 0.208)$

$= 0.204$

② $K_1 = hf((x_1, y_1))$

$= 0.2 \times (1 + (0.204)^2)$

$= 0.2083$

$K_2 = hf(x_1+h, y_1+k_1)$

$= 0.2 \times (1 + (0.204+0.2083)^2)$

$= 0.2339$

$y(0.4) = y(0.2) + \frac{1}{2}(k_1+k_2)$

$= 0.204 + \frac{1}{2}(0.2083 + 0.2339)$

$= 0.4251$

Lab 4]

1) Given $\frac{dy}{dx} = y - x$, $y(0) = 2$, $h = 0.1$

To find $y(0.1)$ & $y(0.2)$

Initial → $x_0 = 0$

$y_0 = 2$

$f(x, y) = y - x$

$K_1 = hf(x_0, y_0)$

$= 0.1 * (2)$

$= 0.2$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= 0.1 \times (2.1 - 0.05)$

$= 0.2050$

$K_3 = 0.1\,(2.1025 - 0.05)$

$= 0.20525$

$K_4 = 0.1\,(2.20525 - 0.1)$

$= 0.210525$

& $y_1(0.1) = 2 + \frac{0.1}{6}\,(0.2 + 0.41 + 0.4105 + 0.210525)$

$= \underline{2.2052}$

Now for $y_2(0.2)$

Initial → $x_1 = 0.2$

$y_1 = 2.2052$

$f(x, y) = y - x$

finding →

$K_1 = hf(x_0, y_0)$

$= 0.21052$

$K_2 = hf(x_1 + h/2, y_1 + K_1/2)$

$= 0.21604$

$K_3 = hf(x_1 + h/2, y_1 + K_2/2)$

$= 0.21632$

$K_4 = hf(x_1 + h, y_1 + K_3)$

$= 0.22215$

$y_2 = y_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= 2.41809$

2] $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, To find $y(0)$
$y(0.4)$

$h = 0.2$

Initial ⇒ $x_0 = 0$

$y_0 = 0$

$f(x, y) = 1 + y^2$

$h = 0.2$

$K_1 = hf(x_0, y_0)$

$= 0.2\ (1 + 0)$

$= 0.2$

$K_2 = hf(x_0 + h/2,\ y_0 + K_1/2)$

$= 0.2 \times (1 + (0 + 0.2/2)^2)$

$= 0.202$

$K_3 = hf(x_0 + h/2, y_0 + K_2/2)$

$= 0.2020402$

$K_4 = hf(x_1 + h, y_0 + K_3)$

$= 0.20848$

$\therefore y_1(0.2) = 0 + \frac{1}{6}[0.2 + 2(0.202) + 2(0.2020402) + 0.20848]$

$= 0.2027$

Now, for $y(0.4) \Rightarrow$

$K_1 = hf(x_1, y_1)$

$= 0.2082$

$K_2 = hf(x_1 + h/2, y_1 + K_1/2)$

$= 0.2 \times (0.1 + (0.2027 + 0.2082/2)^2)$

$= 0.2188$

$K_3 = hf(x_1 + h/2, y_1 + K_2/2)$

$= 0.2194$

$K_4 = hf(x_1 + h, y_1 + K_3)$

$= 0.2356$

$y(0.4) = y_1 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= 0.4227$

Lab 5]

Q1] $y' = \frac{x+y}{2}$

x_i	0.5	0.5	1	1.5
y_i	2.0	2.636	3.595	4.968

Find $y(2)$

$y_0' = F(0,2)$

$y_0' = \frac{0+2}{2} = 1$

$y_1' = (0.5, 2.636) = 1.568$

$y_2' = (1, 3.595) = 2.2975$

$y_3' = (1.5, 4.968) = 3.234$

$$y_{4,p} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 4.968 + \frac{0.5}{24}\left[(55 \times 3.234) - (59 \times 2.2975) \times (37 \times 1.568) - (9 \times 1)\right]$$

$y_{4,p} = 6.8708$

$y_4' = F(2.0, 6.8708) = \frac{2+6.8708}{2} = 4.4354$

$$y_{4,c} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' + 5y_2' + y_1'\right]$$

$$= 4.968 + \frac{0.5}{24}\left[(9 \times 4.4354) + (19 \times 3.234) - (5 \times 2.2975) + (1.568)\right]$$

$y_{4,c} = 6.8731$

Q2] $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$

$y(0.2) = 0.2027$

$y(0.4) = 0.4227$

$y(0.6) = 0.6839$

$y(0.8) = ?$

$y_0' = 1 + 0 = 1$

$y_1' = 1 + (0.2027)^2 = 1.041$

$y_2' = 1 + (0.4227)^2 = 1.1787$

$y_3' = 1 + (0.6839)^2 = 1.4677$

$$y_{4,P} = y_3 + \frac{h}{24}\left[55y_3' + 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 0.6839 + \frac{0.2}{24}\left[(55 \times 1.468) - (59 \times 1.1786) + (37 \times 1.041) - 9\right]$$

$y_{4,P} = 1.02323$

$y_4' = f(x_4, y_{4,P}) = (1 + (1.02323)^2) = 2.047$

$$y_{4,C} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$y_{4,C} = 1.02935$

LAB-6

Q1] $y' = x^2 + y^2 - 2$

x_i	-0.1	0	0.1	0.2
y_i	1.09	1.0	0.89	0.7605

$y_0' = ((0.1)^2 + (1.09)^2 - 2) = -0.8019$

$y_1' = ((0)^2 + (1)^2 - 2) = -1$

$y_2' = (0.1)^2 + (0.89)^2 - 2) = -1.1979$

$y_3' = '(0.1)^2 + (0.7605)^2 - 2 = -1.4116$

$$y_{4,p} = y_0 + \frac{4}{3}h\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 1.09 + \frac{4}{3}(0.1)\left[2\times(-1) + 1.979 + (2\times -1.4116)\right]$$

$y_{4,p} = 0.6066$

$y_4' = (0.3)^2 + (0.6066)^2 - 2 = -1.542$

$$y_{4,c} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 0.89 + \frac{0.1}{3}\left[-1.979 + (4\times -1.4116) + (-1.542)\right]$$

$y_{4,c} = 0.6105.$

Q2] $\frac{dy}{dx} = 1 + y^2 \quad y(0) = 0$ at $y(0.8)$

Solution: $y_0 = 0$

$y_1' = 1.041$

$y_2' = 1.1786$

$y_3' = 1.468$

$$y_{4,P} = 0 + \frac{4 \times 0.2}{3} [2(1.041) - 1.1786 + 2(1.468)]$$

$$= 1.02384$$

$$y_{4'} = 1 + (1.02384)^2 = 2.048248346$$

$$y_{4,C} = 0.422 + \frac{0.2}{3} [1.1786 + (4 \times 1.468) + 2.048]$$

$$= 1.029$$