Synthetic Unruh effect using Python

Rubén Piles Delgado

January 3, 2026

1

If the grading is a 7 or more, I do not need to come on Friday 10th to discuss it in class.

1 Introduction

The Unruh effect is a theoretical prediction from quantum field theory, which suggests that an observer uniformly accelerating through empty space will perceive a thermal bath. In other words, while accelerating, the observer will detect a temperature, relative to an inertial observer, known as the *Unruh temperature*. This temperature can be theoretically derived as follows:

$$T_U = \frac{\hbar a}{2\pi c k_B} \approx 4 \times 10^{-21} \cdot a \tag{1.1}$$

where a is the observer's acceleration.

Interestingly, this temperature is analogous to the *Hawking temperature* associated with the emission of *Hawking radiation* by black holes, which is given by:

$$T_H = \frac{\hbar g}{2\pi c k_B} \tag{1.2}$$

Here, g represents the gravitational acceleration near the event horizon of a black hole. The equivalence between the Unruh temperature and the Hawking temperature arises from the equivalence principle of General Relativity, which states that an acceleration (a) is locally indistinguishable from a gravitational field (g).

Looking at (1.1), it is clear that the temperature increase predicted by the Unruh effect will be extremely small unless the observer experiences very high accelerations. This is one of the main reasons why measuring this effect experimentally has not been possible so far. However, physicists have proposed alternative experimental setups to better understand this phenomenon.

In this project, we consider the method described by Javier Rodríguez-Laguna et al. in their paper *Synthetic Unruh Effect in Cold Atoms.* The proposal utilizes ultracold atoms in a 2D optical lattice as a controllable quantum system to create an *analog gravity* setup.

The *Minkowski vacuum*, which refers to the system as seen in an inertial frame of reference, is simulated by tuning the tunneling amplitudes between all lattice sites to be uniform. This resembles flat space-time. On the other hand, the *Rindler vacuum*, as perceived by an accelerated observer, is simulated by instantly replacing the uniform tunneling with position-dependent tunneling amplitudes that increase linearly with distance from a central line, which corresponds to the event horizon. This creates an effect that simulates acceleration.

Both systems can be described mathematically by their corresponding Hamiltonians, which can be defined in a 2D grid in terms of the ladder operators as:

$$H_M = -\sum_{m,n} t_0 \left[e^{i\frac{\pi}{2}(m-n)} a^\dagger_{m+1,n} + e^{i\frac{\pi}{2}(m-n)} a^\dagger_{m,n+1} \right] a_{m,n} + H.c. \tag{1.3}$$

$$H_R = -\sum_{m,n} t_r \left[\left(m + \frac{1}{2} \right) e^{i\frac{\pi}{2}(m-n)} a^\dagger_{m+1,n} + m e^{i\frac{\pi}{2}(m-n)} a^\dagger_{m,n+1} \right] a_{m,n} + H.c. \tag{1.4}$$

Where t_0 and t_r are the tunneling terms.

2 Project Goals

In this project, the proposal has been implemented and simulated using Python. The Hamiltonians have been written and manipulated using the *Quantum Toolbox in Python (QuTiP)* library to visualize and test the experimental setup. All the code is available at `https://github.com/Ruba18-Code/Unruh_effect`. ***It is recommended to download and run the functions.py and unruh_eff.ipynb files to gain a better understanding of the results and the work behind this project, as a significant portion of the effort was spent on programming.***

3 Hamiltonian magnitudes

After writing both Hamiltonians as QuTiP sparse matrices, the matrix magnitudes have been plotted in order to visualize the event horizon (please zoom in).

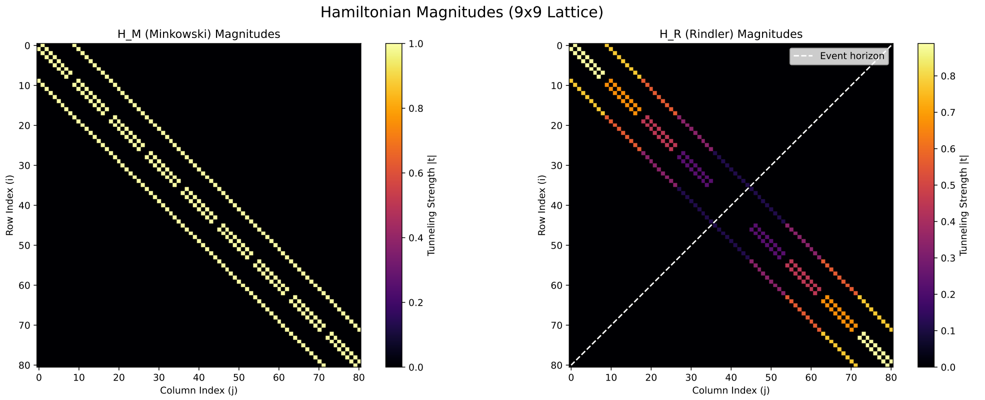

Figure 1.1: Minkowski and Rindler Hamiltonians visualized as matrices

As shown in the figure, the Minkowski Hamiltonian is filled with terms that are either equal to 0 or t_0, which has been set to 1 for simplicity. In contrast, the Rindler Hamiltonian contains tunneling amplitudes that decrease as we approach the center of the matrix, thus creating an *event horizon*, represented by the dashed white line. This event horizon effectively *splits* space into two isolated regions. This is analogous to the event horizon of a black hole, with the upper left region representing the outside and the lower right region representing the inside, for instance.

For the sake of readability, a 9×9 lattice is employed. However, from now on, the lattice size will be increased to 51×51 in order to obtain more meaningful results. This number should be increased as much as the hardware allows. It is important to recall that the grid size must be an odd number, as explained in the paper.

4 Testing the physics of the event horizon

In order to check the behavior of the horizon, a Gaussian wavepacket, initially placed to the left of the grid and moving towards the center, is considered:

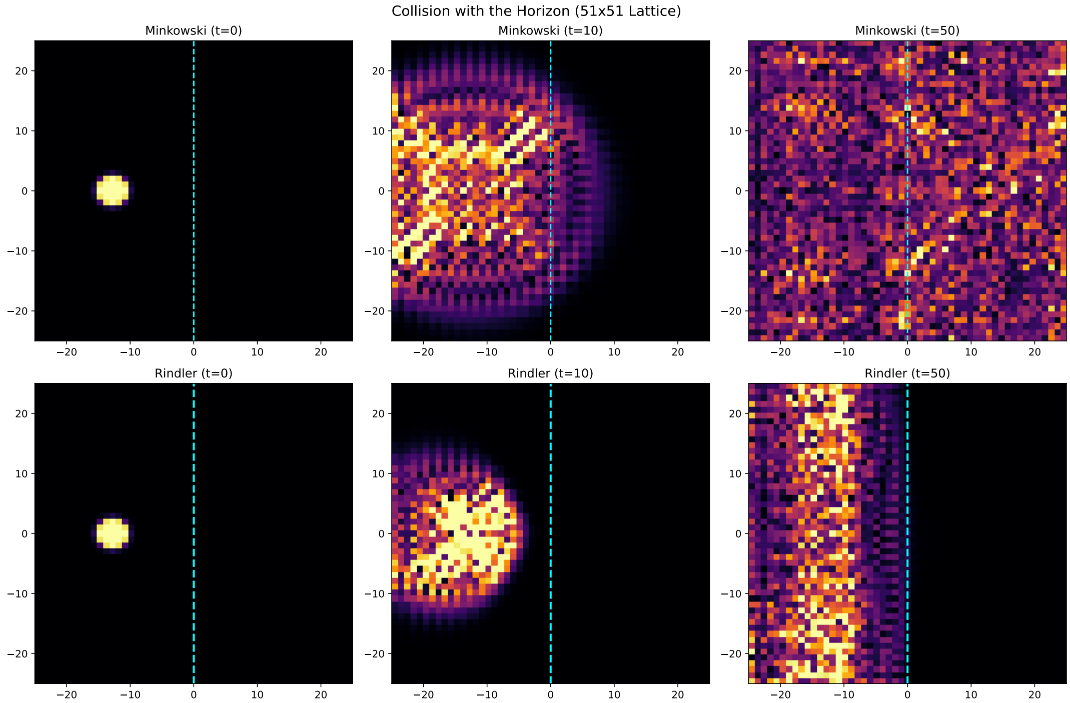

Figure 1.2: Gaussian wavepacket moving towards $x = 0$

From this visualization, it can be deduced that the wave expands freely in Minkowski's flat spacetime, as represented in the first row. However, when considering the behavior of the wave in the Rindler system, it can be seen that the wave stops right at the middle of the lattice. This supports the idea that an event horizon, which splits space into two uncommunicated regions, is indeed created by this experimental setup.

Additionally, an animation has been created to better understand the evolution of the gaussian wavepacket,click here to view the GIF :)

5 Comparing Minkowski and Rindler Vacuum

Now that the behavior of the event horizon has been tested, we can proceed to observe the Unruh effect. For this purpose, we compare the number of particles that an observer perceives in the Minkowski and Rindler vacuums.

To do this, we compute the sum of the probability densities of all the occupied negative-energy states, which correspond to the vacuum state in Quantum Field Theory. Then, we compute the difference between the Minkowski and Rindler results. The outcome is an estimation of the difference in particle density, which is expected to reflect the Unruh effect:

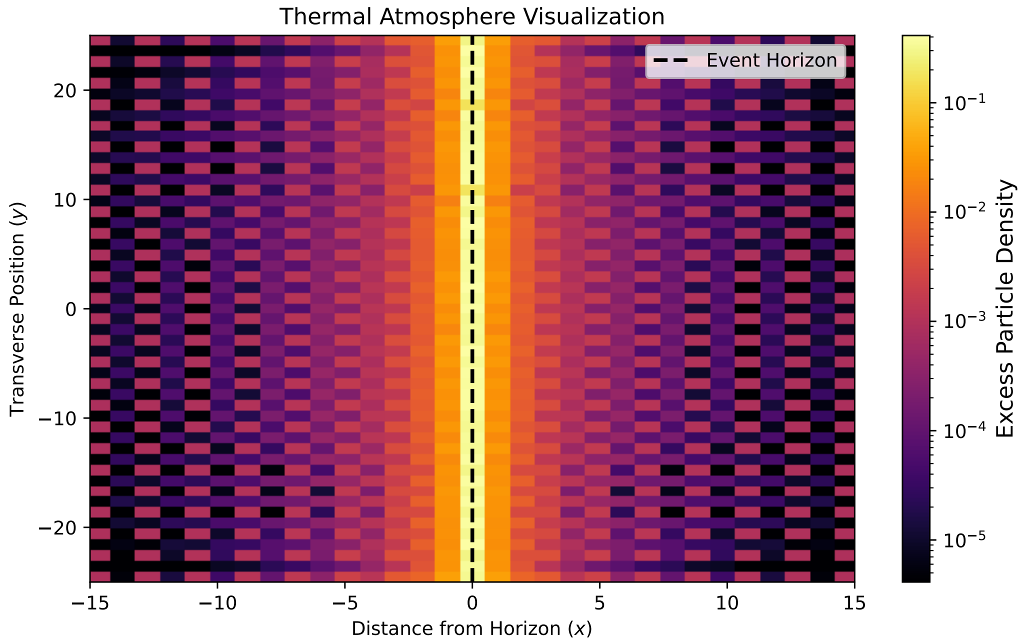

Figure 1.3: Excess particle density in Rindler's vacuum compared to Minkowski's, 2D view

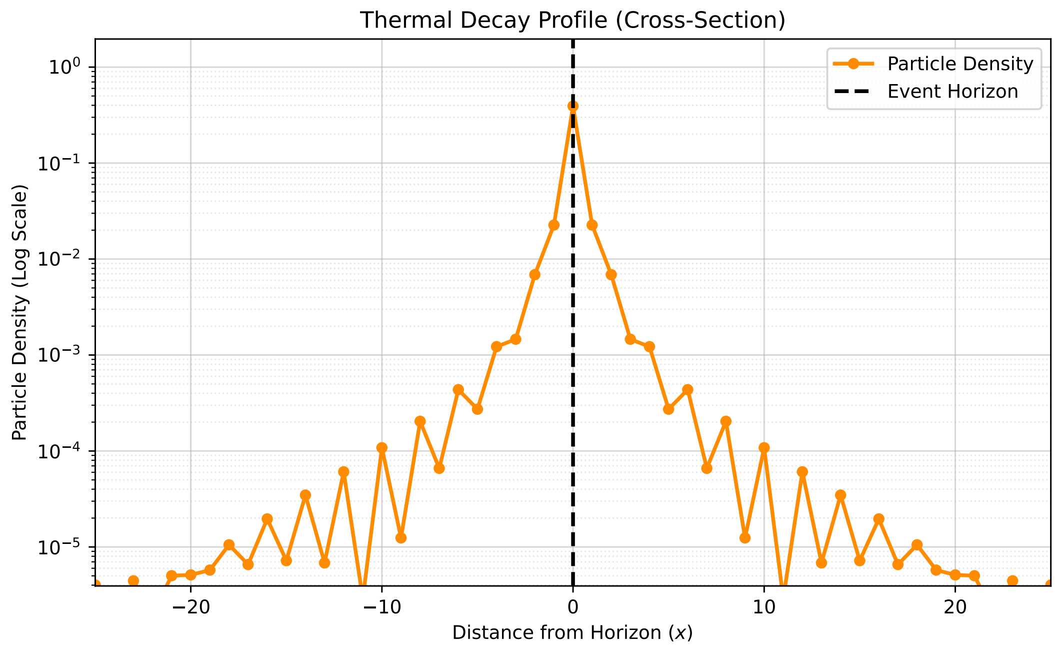

Figure 1.4: Excess particle density in Rindler's vacuum compared to Minkowski's, 1D view (cross section)

As expected, the difference in particle density increases as we approach the event horizon, since *acceleration* is strongest in this region. As we move further away from $x = 0$, this difference tends to zero.

6 Conclusions

In this project, some of the main ideas from the paper *Synthetic Unruh Effect in Cold Atoms* were tested. An event horizon on a 2D optical lattice was created by simulating the Minkowski and Rindler Hamiltonians using QuTiP, and its behavior was verified by observing the interaction with a Gaussian wavepacket. Finally, a qualitative estimation of the Unruh effect was made by comparing the particle densities in the Minkowski and Rindler vacuums.

These results suggest that the approach proposed in the paper could indeed contribute to the challenging task of measuring the Unruh effect.

7 References

1. Javier Rodríguez-Laguna et al., *Synthetic Unruh Effect in Cold Atoms* `https://arxiv.org/abs/1606.09505`

2. Shin Takagi, *Vacuum Noise and Stress Induced by Uniform Acceleration: Hawking-Unruh Effect in Rindler Manifold of Arbitrary Dimension* `https://academic.oup.com/ptps/article/doi/10.1143/PTP.88.1/1938595?login=false`

3. Paul M. Alsing et. al., *Simplified derivation of the Hawking-Unruh temperature for an accelerated observer in vacuum* `https://arxiv.org/abs/quant-ph/0401170`