Ruba Khalid Alsulami
2110618

**Optimization and regression**

**Lab 1' & 2'**

**All the equations must be verified by the tools: www.desmos.com/calculator**

**Exercise 1 :**

Find all critical points :
1) what is the max of an area of a rectangle with perimeter equal to 12 ?
2) $f(x)$=1-$x^2$
3) $f(x)$= $x^2$-4$x$+2
4) $f(x,y)=1+x^2+y^2$
5) $f(x,y)=1-(x-2)^2+(y+3)^2$
6) $f(x,y)=(3x-2)^2+(y-4)^2$

**Exercise 2 :**

Find the critical points of the function and test for extrema or saddle points by using algebraic techniques.

1) $f(x,y)=1+x^2+y^2$
2) $f(x,y)=x^4+y^4-16xy$
3) $f(x,y)=15x^3-3xy+15y^3$

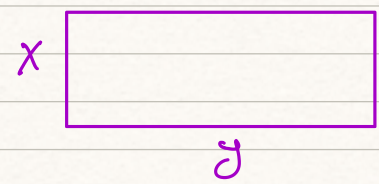
**Exercise 1 :**

Find all critical points :

1) what is the max of an area of a rectangle with perimeter equal to 12 ?

$P = 12$

The Perimeter of the rectangle is:-




$2x + 2y = 12$

$= x + y = 6 \implies y = 6 - x$

Maximize the area of the rectangle it will be:-

Area $= xy$

$A = x(6 - x)$

$A = 6x - x^2$

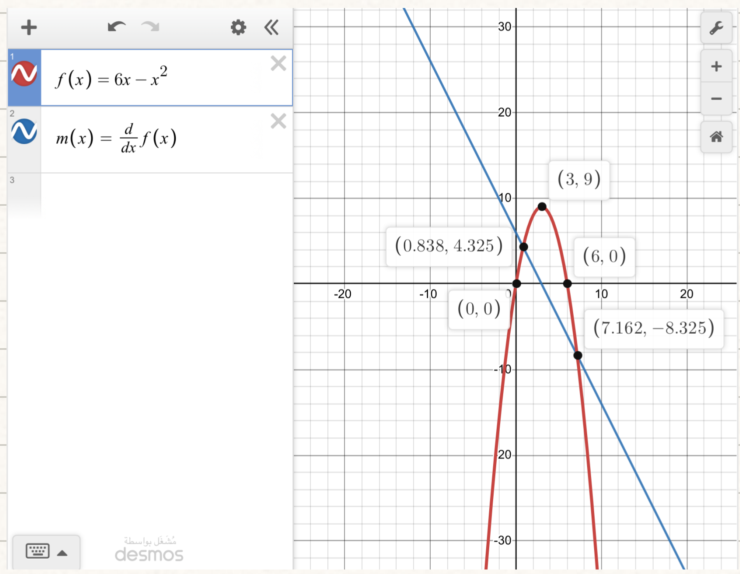
$A'(x) = 6 - 2x$

$A'(x) = 0$

$6 - 2x = 0$

$\frac{-2x}{-2} = \frac{-6}{-2}$

$x = 3$




using $y = 6 - x$

$y = 6 - 3$

$y = 3$

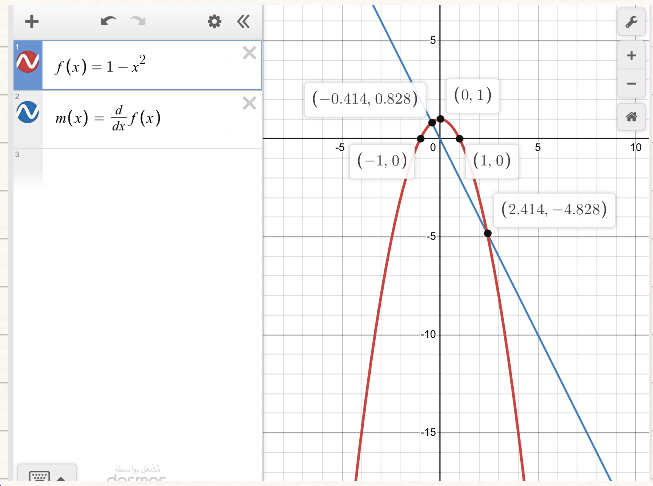
The maximum area of rectangle $= A = xy \rightarrow A = 3(3) = 9$

$\boxed{\text{The critical Point is } = (3, 3)}$

## Exercise 1 :

Find all critical points :

### 2) $f(x)=1-x^2$




$f'(x)=-2x$

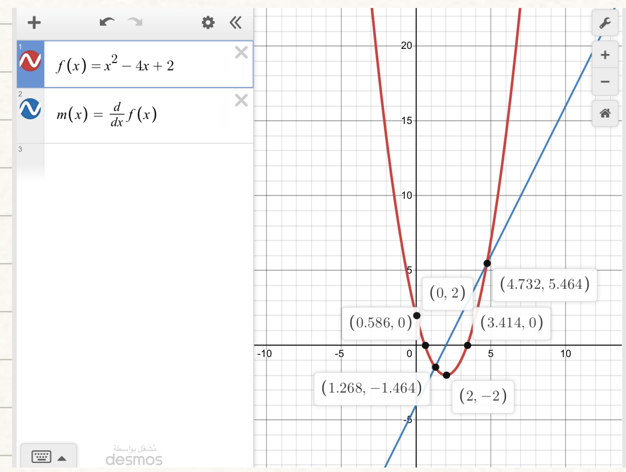
$f''(x)=-2$

$f'(x)=0 \rightarrow -2x=0$ , $\boxed{x=0}$ , This point can be Stationary Point

| $x$ | $f(x)$ | $f''(x)$ |
|---|---|---|
| 0 | 1 | $-2$ $\rightarrow$ local maximum |

The Critical Point is $=(0,1)$

### 3) $f(x)=x^2-4x+2$




$f'(x)=2x-4$

$f''(x)=2$

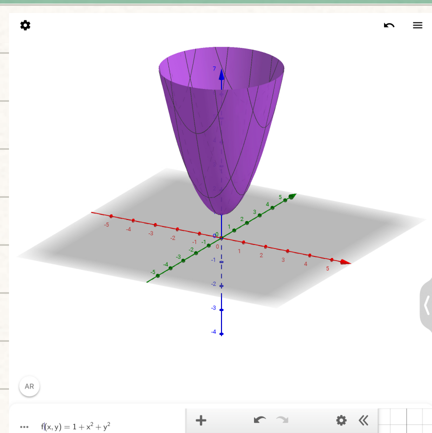
$f'(x)=0 \rightarrow 2x-4=0$ , $\frac{2x}{2}=\frac{4}{2}$ , $\boxed{x=2}$

This Point can be Stationary Point

| $x$ | $f(x)$ | $f''(x)$ |
|---|---|---|
| 2 | $-2$ | 2 $\rightarrow$ local minimum |

The Critical Point is $=(2,-2)$

### 4) $f(x,y)=1+x^2+y^2$




$f_x=2x$ | $f_y=2y$

$f_{xx}=2$ | $f_{yy}=2$

$f_{xy}=0$ | $f_{yx}=0$

$D=f_{xx}\cdot f_{yy}-(f_{xy})^2$

$D=2(2)-(0)^2$

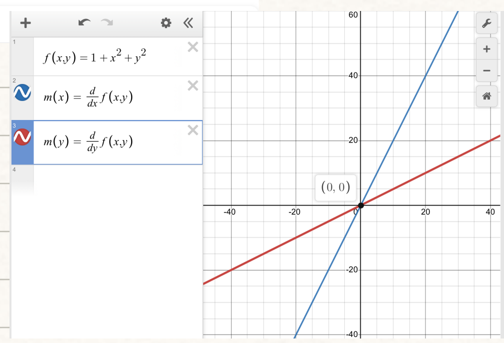
$D=4>0$

$f_{xx}=2>0$

$f_x=0 \longrightarrow 2x=0 \quad x=0$

$f_y=0 \longrightarrow 2y=0 \quad y=0$

The Critical Point is $=(0,0)$




we have $D(x,y)=4$ and in Particular, $D(0,0)=4$

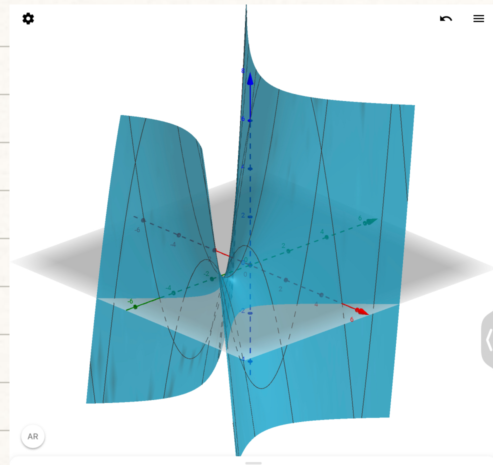
Since $D(0,0)>0$ and $f_{xx}=2>0$ we conclude that $f$ has a relative minimum at Point $(0,0)$. The relative minimum value $f(0,0)=1$, also happens to be the absolute minimum of $f$.

## 5) $f(x,y)=1-(x-2)^2+(y+3)^2$

$f_x = -2(x-2) = -2x+4$ | $f_y = 2(y+3) = 2y+6$

$f_{xx} = -2$ | $f_{yy} = 2y$

$f_{xy} = 0$ | $f_{yx} = 0$



$f_x=0 \longrightarrow -2x+4=0$ , $-2x=-4$ , $x=2$

$f_y=0 \longrightarrow 2y+6=0$ , $2y=-6$ , $y=-3$

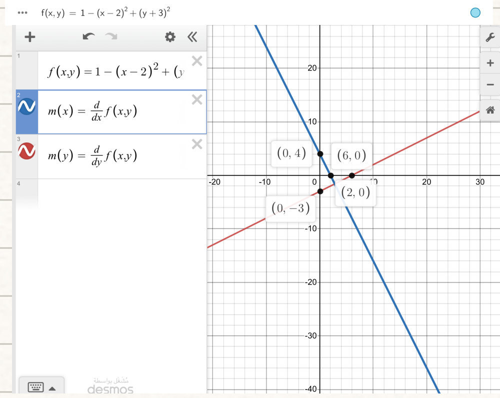
The critical point is $=(2,-3)$

$D=f_{xx}\cdot f_{yy}-(f_{xy})^2$

$D=-2(2)-(0)^2$

$D=-4<0$

$f_{xx}=-2<0$




we have $D(x,y)=-4$ and in particular, $D(0,0)=-4$ since $D(2,-3)>0$ and $f_{xx}=-2<0$ we conclude that f has a saddle point at point $(0,0)$. The relative saddle value $f(0,0)=1$ , also happens to be the absolute saddle of f.

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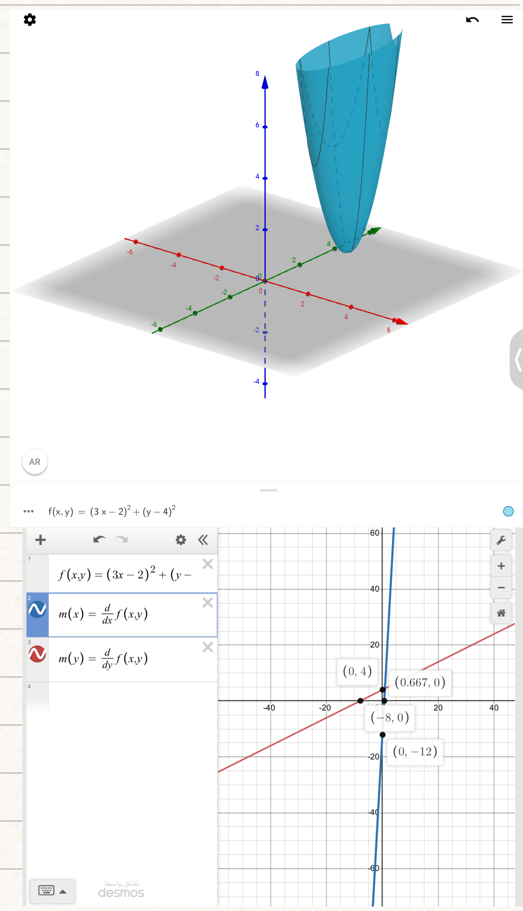



## 6) $f(x,y)=(3x-2)^2+(y-4)^2$

$f_x = 2(3x-2)(3) = 6(3x-2) = 18x-12$ | $f_y = 2(y-4)(1) = 2(y-4) = 2y-8$

$f_{xx} = 18$ | $f_{yy} = 2$

$f_{xy} = 0$ | $f_{yx} = 0$

$f_x=0 \longrightarrow 18x-12=0$ , $18x=12$ , $x=\frac{2}{3}$

$f_y=0 \longrightarrow 2y-8=0$ , $2y=8$ , $y=4$

The critical point is $=(\frac{2}{3},4)$

$D=f_{xx}\cdot f_{yy}-(f_{xy})^2$

$D=18(2)-(0)^2$

$D=36>0$

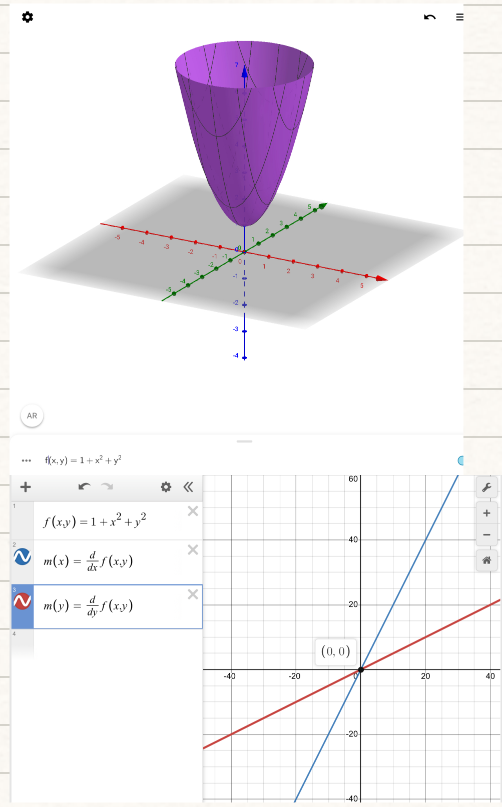
$f_{xx}=18>0$

we have $D(x,y)=36$ and in particular, $D(\frac{2}{3},4)=36$ since $D(\frac{2}{3},4)>0$ and $f_{xx}=2>0$ we conclude that f has a relative minimum at point $(\frac{2}{3},4)$. The relative minimum value $f(\frac{2}{3},4)=0$ , also happens to be the absolute minimum of f.

**Exercise 2 :**

Find the critical points of the function and test for extrema or saddle points by using algebraic techniques.

**1) $f(x,y)=1+x^2+y^2$**




$f_x = 2x$ | $f_y = 2y$

$f_{xx} = 2$ | $f_{yy} = 2$

$f_{xy} = 0$ | $f_{yx} = 0$

$f_x = 0 \longrightarrow 2x = 0 \quad x = 0$

$f_y = 0 \longrightarrow 2y = 0 \quad y = 0$

The critical point is $= (0,0)$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D = 2(2) - (0)^2$

$D = 4 > 0$

$f_{xx} = 2 > 0$

we have $D(x,y) = 4$ and in particular, $D(0,0) = 4$ since $D(0,0) > 0$ and $f_{xx} = 2 > 0$ we conclude that $f$ has a relative minimum at point $(0,0)$. The relative minimum value $f(0,0) = 1$, also happens to be the absolute minimum of $f$.

## 2) $f(x,y)=x^4+y^4-16xy$

$f_x = 4x^3 - 16y$

$f_{xx} = 12x^2$

$f_{xy} = -16$

$f_y = 4y^3 - 16x$

$f_{yy} = 12y^2$

$f_{yx} = -16$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D = 12x^2(12y^2) - (-16)^2$

$D(0,0) = 12(0)^2(12(0)^2) - (-16)^2 = -256 < 0$

The point $(0,0)$ is saddle point

$f_x = 0$

$y = \frac{x^3}{4}$

$4\left(\frac{x^3}{4}\right)^3 - 16x = 0$

$4\frac{x^9}{64} - 16x = 0$

$\frac{x^9}{16} - 16x = 0$

$x^9 - 256x = 0$

$x(x^8 - 256) = 0$

$x = 0 \quad , \quad x^8 - 256 = 0$

$\sqrt[8]{x^8} = \sqrt[8]{256}$

$x = \pm 2 \rightarrow x = 2 \ , x = -2$

$f_y = 0$

$y = \frac{x^3}{4}$

if $x = 2$

$y = \frac{2^3}{4} = 2$

if $x = -2$

$y = \frac{(-2)^3}{4} = -2$

if $x = 0$

$y = \frac{0^3}{4} = 0$

The critical point is $= (0,0), (2,2), (-2,-2)$

$D(2,2) = 12(2)^2(12(2)^2) - (-16)^2 = 2048 > 0$

$f_{xx} = 12(2)^2 = 48 > 0$

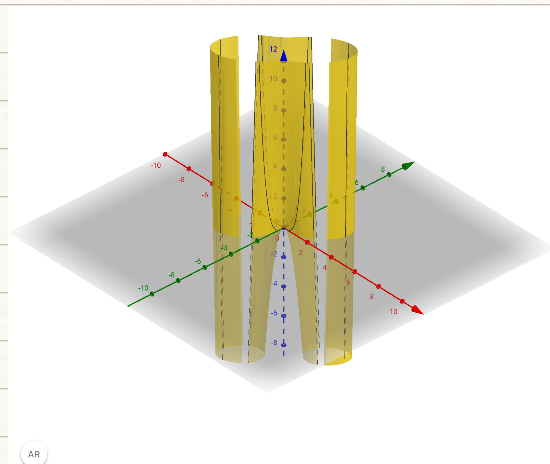
The point $(2,2)$ is relative minimum

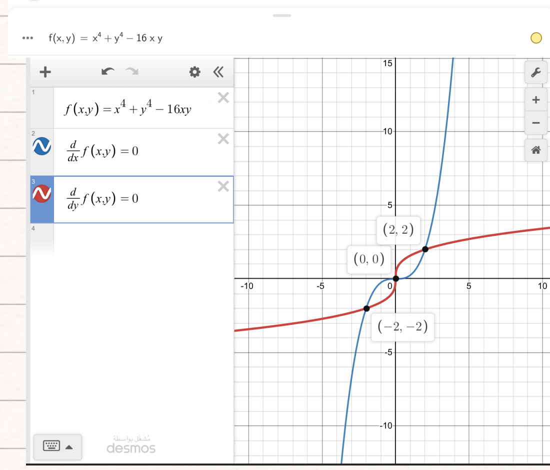
with value $f(2,2) = -32$

$D(-2,-2) = 12(-2)^2(12(-2)^2) - (-16)^2 = 2048 > 0$

$f_{xx} = 12(-2)^2 = 48 > 0$

The point $(-2,-2)$ is relative minimum with value $f(-2,-2) = -32$





3) $f(x,y)=15x^3-3xy+15y^3$

| | |
|---|---|
| $f_x = 45x^2 - 3y$ | $f_y = -3x + 45y^2$ |
| $f_{xx} = 90x$ | $f_{yy} = 90y$ |
| $f_{xy} = -3$ | $f_{yx} = -3$ |

$f_x = 0 \rightarrow (45x^2 - 3y = 0) \div 3$

$15x^2 - y = 0$

$15x^2 = y$

$f_y = 0 \longrightarrow (-3x + 45y^2 = 0) \div 3$

$-x + 15y^2 = 0$

$-x + 15(15x^2)^2 = 0$

$-x + 3375x^4 = 0$

$x(-1 + 3375x^3) = 0$

$x = 0 \qquad \frac{3375x^3}{3375} = \frac{1}{3375}$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{1}{3375}}$

$x = \frac{1}{15} \rightarrow y = 15\left(\frac{1}{15}\right)^2 = \frac{1}{15}$

The critical points when $x=0$ $y=0$ $(0,0)$

when $x = \frac{1}{15}$ $y = \frac{1}{15}$ $\left(\frac{1}{15}, \frac{1}{15}\right)$

$\boxed{\text{The critical Point is } = (0,0), \left(\frac{1}{15}, \frac{1}{15}\right)}$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D = (90x)(90y) - (-3)^2 = 8100xy - 9$

$D(0,0) = (90(0))(90(0)) - (-3)^2 = -9 < 0$

The Point $(0,0)$ is saddle Point

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D = (90x)(90y) - (-3)^2 = 8100xy - 9$

$D\left(\frac{1}{15}, \frac{1}{15}\right) = \left(90\left(\frac{1}{15}\right)\right)\left(90\left(\frac{1}{15}\right)\right) - 9 = 27 > 0$

$f_{xx} = 90\left(\frac{1}{15}\right) = 6 > 0$

The Point $\left(\frac{1}{15}, \frac{1}{15}\right)$ is relative minimum