Damped Gyromagnetic Switching

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***Abstract*—The gyromagnetic switching behavior of a single-domain grain or particle with uniaxial magnetocrystalline and shape anisotropy is solved analytically by the gyromagnetic torque equation using physically plausible Gilbert damping. The sole restriction is that the anisotropy easy axis and the applied field be collinear. The analytical solution for the magnetization switching time is then used to calculate precisely various phenomena which occur in pulsed-current stripline testing of magnetic recording storage media. Also, the analytic relationship between the polar and azimuthal angles of the magnetization during damped precession is derived.**

***Index Terms*—Damping, gyromagnetic effect, hard disks, magnetic tapes, magnetization reversal, single domain particles.**

I. Introduction

STARTING with studies of magnetic core and film memories, and later in magnetic recording studies, first for high bandwidth video recording and then more recently for high data-rate digital recording, the very high-speed switching behavior of a single-domain magnetic particle or grain has been the subject of many investigations. Theoretical studies of the problem include contributions by Kikuchi at Armour Research in 1956 [1], Gillette and Oshima at Stanford Research in 1958 [2], Mallinson at Ampex in 1971 [3], Doyle *et al.* at the University of Alabama [4], and Hannay and Chantrell at the University of N. Wales [5] throughout the 1990's. Experimental studies include the work of Thornley and Williams at IBM in 1974 [6] and, in the 1990's, Doyle *et al.* at the University of Alabama [4] and Rizzo *et al.* at NIST, Boulder [7].

Kikuchi solved the gyromagnetic reversal problem for an isotropic single-domain sphere using the Gilbert equation [8]. In this somewhat artificial case, the magnetization was held down with a down applied field that was then switched instantaneously up. Switching, thus, could occur in arbitrarily small applied fields.

Gillette and Oshima gave the gyromagnetic equations in component form but did not provide either analytical or numerical solutions.

In the 1971 work, which appears to have been overlooked by subsequent investigators, Mallinson published the closed-form solution for the reversal of magnetization in a single-domain sphere of anisotropic material using the Gilbert equation. This case remains of great interest because it corresponds very closely to the physical situation prevailing in modern thin-film disks that have isolated, highly anisotropic, but equi-axed grains. Now, switching cannot occur until the applied field exceeds the anisotropy field.

Manuscript received November 15, 1999; revised February 24, 2000.
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Publisher Item Identifier S 0018-9464(00)05841-6.

In recent theoretical work, Doyle *et al.* have followed the magnetization reversal using strictly numerical methods [9]. Among their many fascinating findings is the fact that when the Gilbert damping coefficient α is small ($\alpha \ll 1$), irreversible switching can occur inside the well-known Stoner–Wohlfarth astroid because the precessing magnetization can cross the equatorial plane stability limit of uniaxial anisotropy even when the applied field is lower than is the anisotropy field. Another interesting discovery of this group is that switching times of only a few picoseconds are attainable with the Gilbert equation and $\alpha < 0.5$, again, for the same reason [10].

The experimental work in very high-speed switching was initiated by Thornley and Williams in 1974, and all subsequent investigators have followed the same basic method. A coaxial cable, charged to a high voltage (≈20 000 V) is discharged, using a fast switch, into a strip line. The resulting high current pulse generates a short magnetic field pulse that switches the recording medium sample placed in the strip-line. By observing (VSM, magneto-optical, Hall probe, etc.) the remanent magnetization or magnetic moment of the sample, it can be determined if and to what extent the sample switched magnetization irreversibly. Note that the measurement of the magnetization is not made in a nanosecond time scale. On the contrary, the remanence may be determined as long as several minutes after the initial strip-line pulse. The switching time measured (usually a few nanoseconds) is the duration of the strip-line current pulse required for the precessing magnetization to first cross the equatorial plane stability limit and, thus, to reverse its remanence.

As the anisotropy and coercivity of recording media have increased (γ-Fe_2O_3: $H_c = 300$ Oe, MP: $H_c = 1500$ Oe, thin-film disks 1987: $H_c = 600$ Oe and thin-film disks 1999: $H_c = 3000$ Oe), it has become increasingly difficult to generate sufficiently high pulse fields to switch the media. This appears to be caused, principally, by the difficulties that develop with the high charging voltages required. In order to alleviate this problem, investigators have taken to applying a dc bias field in the same direction as the strip-line pulse field. In some recent studies, the bias field magnitude has been almost 50% of that of the pulse field [7].

The precise effect of this bias field has been an outstanding unsolved problem. Merely noting that the bias field alone does not change the media remanence appreciably is clearly insufficient, because applying a static stability criterion to a dynamic precessional process is not appropriate. It has, nevertheless, been observed that, when the bias field magnitudes are increased, the apparent switching times are decreased. The effect of the bias field is solved exactly in this paper.

It has been shown recently that for small particles, which are close to the superparamagnetic limit, and, therefore, display high magnetic viscosity, thermal energy effects play a significant role even in high-speed (<10 ns) switching. Here, thermal phenomena are discussed only in terms of the statistically most probable deviation from the uniaxial easy axis of the magnetization of a single domain particle or grain.

The case of the sphere with uniaxial magnetocrystalline anisotropy is treated anew, and the minor typographical errors existing in the 1971 solution are corrected. Then, the case of an elongated (ellipsoidal) particle with shape anisotropy is analyzed, showing that the mathematical form of the solution is identical to that of the uniaxial magnetocrystalline anisotropy sphere. This makes it obvious that, provided the easy axes are parallel, the uniaxial magnetocrystalline anisotropy and the uniaxial shape anisotropy may simply be added.

Next, the general uniaxial anisotropy switching time results are discussed for the usual strip-line experiment, and the precise effect of the addition of a dc bias field parallel to the strip-line pulse field is analyzed. Then, it is shown that the trajectory or path of the switching is determined solely by the damping constant, independent of the applied field.

In the discussion section of the paper, there are three main topics: the relationship of the present results to those of Kikuchi, the effect of thermal energy on the gyromagnetic switching times, and the slow field-free switching behavior that can occur after the applied field is terminated.

II. Damped Gyromagnetic Precession

Magnetization is, by definition, the volume average of the vector sum of the electron spin magnetic moments. Spinning electrons have angular momentum, and it follows, from the law of conservation of angular momentum, that whenever the magnetization changes direction without dissipation, it does so by precession, in a manner analogous to that of a mechanical gyroscope. Dissipation is, in this case, the conversion of magnetic energy into heat. Mathematically, the gyromagnetic equation is

$$\overline{\dot{M}} = -\gamma(\overline{M} \times \overline{H}) \tag{1}$$

where

- $\overline{\dot{M}}$ is the time rate of change of $\overline{M}$;
- $\overline{M}$ is the magnetization;
- $\overline{H}$ is the applied field;
- γ is the gyromagnetic ratio (17.6 10^6 rad/Oe).

In order for the magnetization to switch irreversibly, dissipation or damping is required. In this paper, only the Gilbert form of the damped gyromagnetic equation is used

$$\overline{\dot{M}} = -\gamma(\overline{M} \times \overline{H}) + \frac{\alpha}{M}\left(\overline{M} \times \overline{\dot{M}}\right) \overline{H}_f. \tag{2}$$

The damping constant α is termed "phenomenological" because it is not derived from a heat or energy transfer mechanism or model.

This equation gives the correct direction of precession if the gyromagnetic ratio γ and the damping constant α are both taken to be positive quantities. Unfortunately, as can be seen by reviewing the references, a certain degree of confusion exists concerning what are the correct algebraic signs to be used in this equation. For example, all equations given previously by Mallinson [3], [11] contain incorrect algebraic signs, and in 1996 alone, both $\gamma < 0$ and $\gamma > 0$ are given by He and Doyle [4], [10].

The vector diagram of the Gilbert equation may be visualized easily on the surface of a sphere of radius equal to the magnetization; see Figs. 1 and 2. In Fig. 2, point A represents the tip of the constant magnitude magnetization vector. $\gamma(\overline{M} \times \overline{H})$ is a vector acting in the negative azimuthal direction. $(\alpha/M)(\overline{M} \times \overline{\dot{M}}) = \alpha\overline{\dot{M}}$ is the damping vector, which is always normal to $\overline{\dot{M}}$. For low values of α, $\overline{\dot{M}}$ is almost entirely precessional. At an intermediate value of the damping constant $\alpha = 1$, $\overline{\dot{M}}$ reaches a maximum giving the fastest possible switching. With higher damping constants, $\overline{\dot{M}}$ decreases, approaching zero for very large values of α.

It is has been found, in recent purely numerical investigations using the Gilbert equation, that the results of strip-line pulse field experiments can be fit best by using $\alpha \approx 1$ [4].

III. The Spherical Grain with Uniaxial Anisotropy

Consider a single-domain, spherical grain made of a magnetic material that has uniaxial magnetocrystalline anisotropy. There is, of course, no shape anisotropy, and because the demagnetizing field $\overline{H}_d = -4\pi/3\overline{M}$, is always colinear with $\overline{M}$, there is no torque, $\overline{M} \times \overline{H}_d$, due to the demagnetizing field. On the other hand, if the magnetocrystalline anisotropy energy density is

$$E_k = K \sin^2\theta \tag{3}$$

where K is the anisotropy constant and θ is the polar angle between the easy axis and $\overline{M}$, there is an anisotropy torque

$$\frac{-dE_k}{d\theta} = -K\sin 2\theta. \tag{4}$$

Consider the special case in which the applied field is coaxial with the easy axis, with the coordinate system, as is shown in Fig. 1. This shows the polar angle θ and the azimuthal angle φ of $\overline{M}$, the initial easy axis direction and the coaxial applied magnetic field direction.

The vector diagram of the Gilbert equation for this problem is shown in Fig. 3. Note that the magnetocrystalline anisotropy torque $-dE_k/d\theta$ opposes the magnetostatic $(\overline{M} \times \overline{H})$ torque in the azimuthal direction. An alternate, but equivalent, way of thinking about the effect of the uniaxial anisotropy is to imagine that it causes a fictitious field, $H_f = (2K/M)\cos\theta$, acting in the easy axis direction. $2K/M$ is usually called the magnetocrystalline anisotropy field H_k. The anisotropy torque $\overline{M} \times \overline{H}_a$ is then again $-MH_k\sin\theta = -K\sin 2\theta$.

Fig. 4 shows how $\overline{\dot{M}}$ is related to the rates of change of the polar angle $\dot{\theta}$ and the azimuthal angle $\dot{\varphi}$. Equating the azimuthal components in Figs. 3 and 4 yields

$$M\sin\theta(-\dot{\varphi}) = \gamma(MH\sin\theta - K\sin 2\theta) - \alpha\dot{M}\sin\beta \tag{5}$$

or

$$M\sin\theta(-\dot{\varphi}) = \gamma(MH\sin\theta - K\sin 2\theta) - \alpha M\dot{\theta} \tag{6}$$

because $\dot{M}\sin\beta = M\dot{\theta}$, as can be seen in Fig. 4.

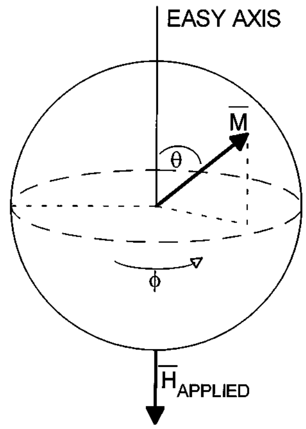

Fig. 1. The coordinate system showing the polar θ and azimuthal φ angles of the magnetization vector M and both the uniaxial anisotropy easy axis and the applied field direction.

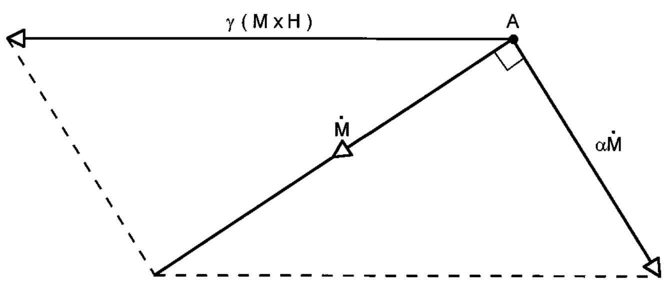

Fig. 2. The Gilbert equation shown as applied torque $\gamma(M \times H)$, damping torque $\alpha\dot{M}$, and their resultant $\dot{M}$ on the surface of a sphere of radius equal to the magnetization. Point A represents the tip of the magnetization vector.

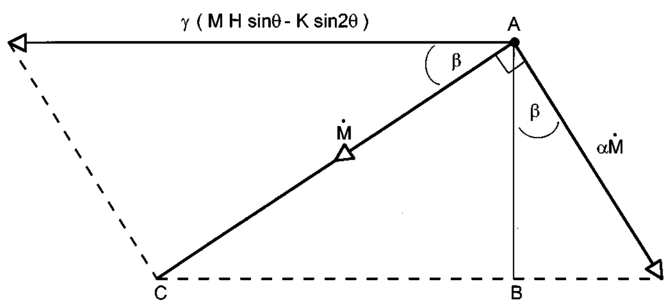

Fig. 3. The Gilbert equation shown as vectors when the uniaxial magnetocrystalline anisotropy easy axis and applied field are coaxial as in Fig. 1.

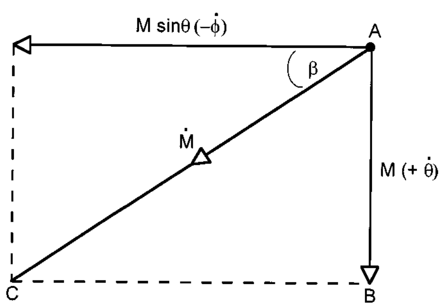

Fig. 4. The Gilbert equation resultant $\dot{M}$ and its relationship to the rate of change of the polar angle θ and azimuthal angle φ.

Equating the polar components gives

$$M\dot{\theta} = \alpha\dot{M}\cos\beta = \alpha M\sin\theta(-\dot{\varphi}) \tag{7}$$

because $\dot{M}\cos\beta = M\sin\theta(-\dot{\varphi})$, from Fig. 4.

Equations (6) and (7) are a pair of simultaneous equations for $\dot{\theta}$ and $\dot{\phi}$. Fascinating though the details of the azimuthal precession may be, in order to determine switching times, it suffices to find $\dot{\theta}$ only. From (6) and (7)

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{\gamma\alpha}{\alpha^2+1}\left(H\sin\theta - H_k\sin\theta\cos\theta\right) \tag{8}$$

which may be inverted and integrated giving

$$\tau = \frac{\alpha^2+1}{\gamma\alpha}\int_{\theta_1}^{\theta_2}\frac{d\theta}{\sin\theta(H-H_k\cos\theta)} \tag{9}$$

or

$$\tau = \frac{\alpha^2+1}{\gamma\alpha}\,\frac{1}{H^2-H_k^2}\left[H\ln\left(\frac{\tan\theta_2/2}{\tan\theta_1/2}\right)\right. \\ \left. + H_k\ln\left(\frac{H-H_k\cos\theta_1}{H-H_k\cos\theta_2}\right) + H_k\ln\left(\frac{\sin\theta_2}{\sin\theta_1}\right)\right]. \tag{10}$$

When this expression for the switching time first appeared, it unfortunately contained two typographical errors [3]. In the second logarithmic term, the terms $(H - H_k\cos\theta)$ were printed as $(M + H_k\cos\theta)$.

If $\theta_1 = 0$ or $\theta_2 = \pi$ the switching time, τ becomes infinite. This shows merely that the starting and finishing magnetostatic torques are precisely zero when the applied field is exactly colinear with the magnetization. This fact will be avoided, in all that follows in this paper, by assuming that the initial angle $\theta_1 = 10^\circ$ and the final angle $\theta_2 < 170^\circ$. The justification for this choice of angles is treated in the discussion section below.

Wherever $\theta_2 = \pi - \theta_1$, the third logarithmic term vanishes and if the applied field is much larger than is the anisotropy field $(H \gg H_k)$, the switching time reduces to

$$\tau = \frac{\alpha^2+1}{\gamma\alpha}\,\frac{2}{H}\ln\left(\text{cotan}\,\frac{\theta_1}{2}\right). \tag{11}$$

IV. The Ellipsoidal Particle with Shape Anisotropy

Consider a single-domain, prolate ellipsoidal particle, with circular cross section, made of an isotropic magnetic material. Let the particle be long in the z-axis, with demagnetizing factor N_{11} and the orthogonal demagnetizing factor $N_\perp$.

The shape anisotropy energy density is

$$E_s = \tfrac{1}{2}M^2(N_{11}\cos^2\theta + N_\perp\sin^2\theta) \tag{12}$$

and this gives rise to a shape anisotropy torque

$$\frac{-dE_s}{d\theta} = \frac{-1}{2}M^2(N_\perp - N_{11})\sin 2\theta. \tag{13}$$

It follows, because both the uniaxial magnetocrystallline and shape anisotropy torques vary as $-\sin 2\theta$, that the analysis given in the previous section for the magnetocrystalline

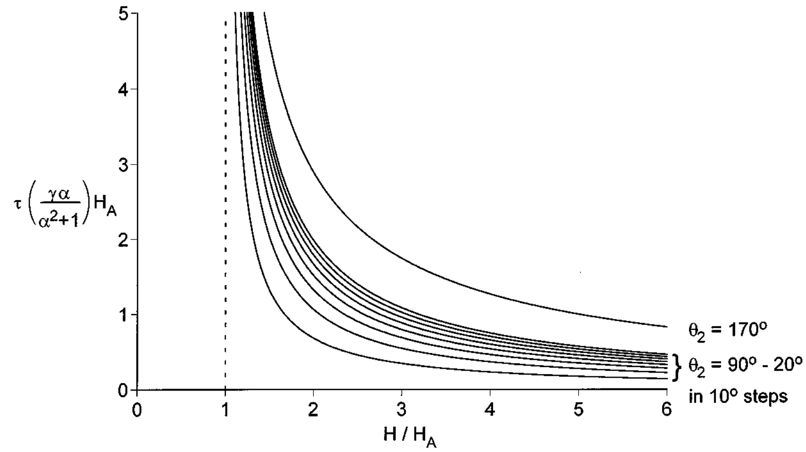

Fig. 5. The normalized switching time [actual time multiplied by the factor $(\gamma\alpha/\alpha^2+1)H_A$] versus the normalized applied field (actual field divided by the total anisotropy field) for switching starting at $\theta_1 = 10°$ and ending at 20°–90° in 10° steps and at 170°.

anisotropy switching time also holds for shape anisotropy upon making the substitution $H_k = M(N_\perp - N_{11})$.

In the more general case, in which uniaxial magnetocrystalline and shape anisotropies coexist and are coaxial, the switching time solution (10) holds valid with $(H_k + M(N_\perp - N_{11}))$ substituted for H_k.

Henceforth, in this paper, the distinction between magnetocrystalline and shape anisotropy will be dropped. In the interests of simplicity, the term $(H_k + M(N_\perp - N_{11}))$ will just be called the total anisotropy field H_A.

V. Strip-Line Pulse Field Experiments

Consider the classic strip-line experiment, in which a short duration pulse field is applied in the opposite direction to the magnetization and no dc bias field is used.

Fig. 5 shows the solution of (10) for the switching time τ. On the ordinate is the normalized switching time $\tau(\gamma\alpha/\alpha^2 + 1)(H_A)$. If the fastest possible switching with $\alpha = 1$ is assumed, the first bracketed factor is 8.8 10^6 radian/s/Oe. The second bracketed factor H_A is the magnitude of the total anisotropy field in Oersteds. Thus, for example, if the total anisotropy field is 1000 Oe, the actual switching time τ has been multiplied by 8.8 10^9 rad/s. Accordingly, a normalized switching time of one unit on the ordinate corresponds to an actual switching time of $(8.8\ 10^9)^{-1}$ s or 0.114 ns. If the total anisotropy field were 2000 Oe, one unit would equal 0.057 ns, and so on.

On the abscissa is the normalized applied field. This is the applied field divided by the total anisotropy field H_A, which is the sum of the magnetocrystalline and shape anisotropy fields.

The starting or initial angle θ_1 for all nine curves is 10°. The final angle θ_2 for the nine curves is 20°–90° in 10° steps and 170°.

Many observations can be made about these results. First, switching does not occur below the limit $H = H_A$ shown dotted. As will be discussed below, however, when the initial angle $\theta_1 = 10°$, the noswitching limit is actually $H = 0.985 H_A$, about 1% lower than is the total anisotropy field.

Second, it is seen that when H is only slightly larger than H_A, the switching starts relatively slowly at low values of θ_2 and then accelerates as θ_2 increases. This is easily understood because at low values of θ, the magnetostatic torque, $MH\sin\theta$, is low.

Third, the time to switch to 90° is always greater than is the time to then proceed to 170°. This is because, of course, when $\theta_2 < 90°$, the precessing magnetization has to "climb up the total anisotropy mountain." After reaching the equatorial hard plane at $\theta_2 = 90°$, the magnetization then precesses more rapidly "down the other side of the mountain." More formally, the total anisotropy torque $(1/2)MH_A\sin 2\theta_2$ changes sign at $\theta_2 = 90°$. From 10° to 90°, the total anisotropy torque opposes the magnetostatic torque but from 90° to 170°, the total anisotropy torque adds.

Fourth, as was mentioned above, it should be realized that $\theta_2 = 90°$ corresponds to the critical pulsewidth measured in the strip-line apparatus. Once the magnetization has crossed the equatorial hard plane, no further applied field is required to complete the switching to 170°. In Fig. 5, however, the applied field is held on for all angles, θ_2.

VI. Strip-Line Experiments with DC Bias Fields

As was discussed above, when media of very high anisotropy or coercivity are being tested, the strip-line pulse field alone is often not of sufficient magnitude, and it is, therefore, augmented with a dc bias field. The precise effect of the dc bias field can be assessed using Fig. 5.

The energy density of the system when magnetized at angle θ is

$$E = MH\cos\theta + K\sin^2\theta + \tfrac{1}{2}M^2(N_\perp\sin^2\theta + M_{11}\cos^2\theta) \tag{14}$$

so that

$$\frac{dE}{d\theta} = -MH\sin\theta + 2K\sin\theta\cos\theta + M^2(N_\perp\sin\theta\cos\theta - N_{11}\cos\theta\sin\theta). \tag{15}$$

It follows that in order to keep the energy density decreasing as θ increases

$$H \geq \left(\frac{2K}{M} + M(N_\perp - N_{11})\right)\cos\theta \tag{16}$$

or

$$H \geq H_A \cos\theta. \tag{17}$$

As θ increases, the effective anisotropy barrier between the easy axis and the equatorial hard plane decreases proportional to $\cos\theta$. This fact has already been used above, in the section on the spherical grain with uniaxial anisotropy, where a fictitious field $2K/M \cos\theta$ was introduced. As was mentioned above, when $\theta_1 = 10°$, the noswitching limit field is $H_A \cos 10° = 0.985\, H_A$.

Suppose now that the sum of the strip-line pulse and the dc bias field rotates $\overline{M}$ to polar angle θ_2. If the bias field is slightly more than is $H_A \cos\theta_2$, the pulse field may then be turned off and the magnetization will complete its switching, albeit slowly, in the dc bias field alone.

For example, if the bias field is 50% of H_A, the pulse field may be removed as soon as θ_2 exceeds 60°, because $\cos 60° = 0.5$.

Using Fig. 5, therefore, the precise effect of the bias field can be measured. It is, specifically, the normalized switching time difference between $\theta_2 = 90°$ and $\theta_2 = \cos^{-1} H_{\text{BIAS}}/H_A$. When there is no dc bias, switching continues with no pulse field after $\theta_2 = 90°$; with a dc bias field, switching continues after $\theta_2 = \cos^{-1} H_{\text{BIAS}}/H_A < 90°$.

The switching time difference can be large when the applied field (pulse plus bias) is relatively small. From Fig. 5, if the applied field is $1.5H_A$ with $H_{\text{BIAS}} = 0.5H_A$ (and pulse field = H_A), the $\theta_2 = 90°$ switching time is about 3.4 and the $\theta_2 = 60°$ time is about 2.8, giving a difference of 0.6. This is a -18% difference.

It is clear that, for a fixed dc bias field, the percentage error increases as the pulse field decreases, just as for a fixed pulse field, the percentage increases as the dc bias field increases.

An analysis of the precise effect of the dc bias field has not been published previously.

VII. The Behavior of the Azimuthal Angle Φ

As has been remarked above, it is not necessary to know the azimuthal angle φ of the precession to deduce the switching times. Nevertheless, it is surprisingly instructive to analyze the behavior of this angle.

From (7)

$$-\dot{\varphi} = \frac{d\varphi}{dt} = \frac{\dot{\theta}}{\alpha\sin\theta} = \frac{d\theta}{dt}\,\frac{1}{\alpha\sin\theta} \tag{18}$$

or

$$\frac{d\varphi}{d\theta} = \frac{d\varphi}{dt}\,\frac{dt}{d\theta} = \frac{-1}{\alpha\sin\theta} \tag{19}$$

and therefore,

$$\varphi = -\int_{\theta_1}^{\theta_2} \frac{d\theta}{\alpha\sin\theta}$$

or

$$\varphi = \frac{-1}{\alpha}\ln\left(\frac{\tan\theta_2/2}{\tan\theta_1/2}\right). \tag{20}$$

In Fig. 6, the product $\alpha(-\varphi)$ is plotted versus θ_2, assuming that φ = 0 at the starting angle $\theta_1 = 10°$. It will be seen that this figure shows odd symmetry about both the vertical line $\theta_2 = 90°$ and the horizontal line $-\alpha\varphi = -\ln(\tan\theta_1/2) = -\ln(\tan 5°) = 2.44$ radians = 140°. In the terms used in navigation, when $\alpha = 1, \overline{\dot{M}}$ makes good its equatorial crossing at longitude 140° W.

The particularly instructive point about this analysis is that the applied field $\overline{H}$ is not a factor in determining the azimuthal angle φ. An explicit relationship exists between the azimuthal angle φ and the polar angle θ that, for a fixed damping constant α, holds no matter whether the switching be rapid with high applied field $\overline{H}$ or slow with low applied field. The path traced by the precessing magnetization vector is fixed by the damping factor alone. With high and low applied fields, the same path is simply traced faster and slower, respectively.

It is easy to see why this fixed path precession occurs. In Fig. 3, which shows the vectors for the Gilbert equation, the angle β is, by geometry, such that $\cos\beta = [\alpha^2+1]^{-1/2}$, independent of the magnitudes of the the applied and anisotropy fields. The angle β is, at all points, a constant determined only by the switching constant α. Again, in navigational terms, when $\alpha = 1, \overline{\dot{M}}$ makes good a constant track of 225° true.

In the classic strip-line switching experiment, it is perhaps surprising to realize that the switching trajectory of the magnetization is exactly the same no matter whether the strip-line field is held on (from $10°$ to $170°$) or it is turned off just past the equator (say, at $100°$), and then the magnetization is allowed to "coast down" the anisotropy hill (from $100°$ to $170°$) in zero applied field. In the first case, the entire switching path is traversed quickly. In the second case, the first part (up to $100°$) goes quickly with the second part ($100°$–$170°$) proceeding slower, but following precisely the same path as in the first case.

The closed-form expression for φ, given in (20) shows, as expected, that in the limits where the damping coefficient is very small or very large, there is a great deal of precession or almost no precession, respectively, because the product $\alpha\varphi$ must remain constant.

Finally, (18) may be combined with (8) to yield the azimuthal precession rate

$$\frac{-d\varphi}{dt} = \left(\frac{\gamma}{\alpha^2+1}\right)(H - H_A\cos\theta). \tag{21}$$

This shows that the absolute azimuthal precession rate $\dot{\varphi}$, which is constant in the isotropic case $(H_A = 0)$, increases monotonically throughout the switching in the anisotropic case.

VIII. Discussion

By putting the total anisotropy field H_A, equal to zero, (10) is, of course, the solution to Kikuchi's problem of the isotropic sphere.

Kikuchi's solution [his (8)] is of the form

$$\tau = \frac{\alpha^2+1}{\gamma\alpha}\,\frac{1}{2H}\ln\left(\frac{(M+M_2)(M-M_1)}{(M-M_2)(M+M_1)}\right). \tag{22}$$

In this expression, M_1 and M_2 have been substituted for Kikuchi's original M_{si} and M_{sf} (i and f indicating initial and final). Also, the argument of the logarithm has been inverted, because Kikuchi analyzed switching from the down to the up direction and not vice-versa, as in the present work. It can be shown that this result is in agreement with (10) [12].

In this study, it has been assumed that the magnetization angle $\theta_1 = 10°$, to avoid the long switching times associated with the low magnetostatic torque at small angles. The choice of $\theta_1 =$

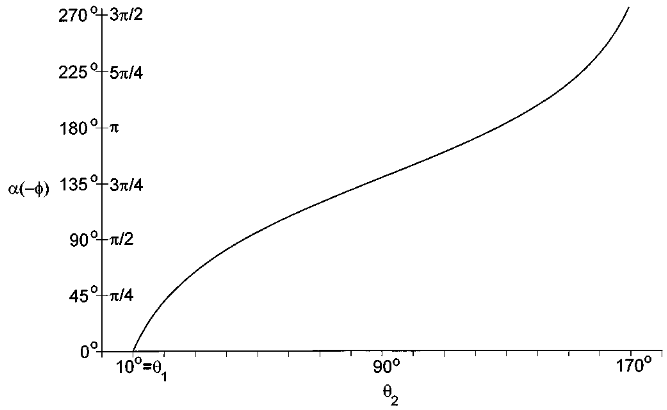

Fig. 6. The explicit relationship between the product $-\alpha\varphi$ and the polar angle θ_2. The azimuthal angle has been set, arbitrarily, to zero at the starting angle $\theta_1 = 10^\circ$.

10° was made based on a simple estimate of the thermal effects expected in the near-superparamagnetic particle or grains that are found in current high areal density recording media.

Consider a particle that has a total anisotropy energy barrier (between the easy axis and hard plane) equal to 50 kT so that $(1/2)MH_AV = 50$ kT. When the magnetization is rotated an angle θ from the easy axis, the increase in energy is $(1/2)MH_AV\sin^2\theta$. For a system in thermal equilibrium, the equipartition theorem of thermodynamics asserts that each degree of freedom has, most probably, 1/2-kT thermal energy. With two degrees of freedom, the most probable thermal energy of the magnetization is kT, corresponding to an angle θ given by $\sin^2\theta = 1/50$ or $\theta = 8^\circ$. It is thus seen that for a 50-kT particle or grain, the statistically most probable, thermally excited, initial angle is approximately 10°. Of course, exactly the same argument applies to the final angle $(\pi - \theta_2)$.

Next, it is pointed out that the switching time expression (10) also holds when there is no applied field $\overline{H}$. Thus, the field-free, "coasting downhill," switching time after the hard plane, from angle θ_1 to θ_2, where θ_1 and $\theta_2 > 90^\circ$, is

$$\tau = \frac{\alpha^2+1}{\gamma\alpha}\,\frac{1}{H_A}\left[\ln\left(\frac{\sin\theta_1}{\sin\theta_2}\right) - \ln\left(\frac{\cos\theta_1}{\cos\theta_2}\right)\right] = \frac{\alpha^2+1}{\gamma\alpha}\,\frac{1}{H_A}\ln\left(\frac{\tan\theta_1}{\tan\theta_2}\right). \quad (23)$$

If $\theta_1 = 90^\circ$, the switching time becomes infinite because the total anisotropy torque is zero in the equatorial hard plane and the magnetization is stuck in unstable equilibrium. This difficulty may be avoided by setting $\theta_1 = 100^\circ$, and with $\theta_2 = 170^\circ$, (23) reduces to

$$\tau = \frac{\alpha^2+1}{\gamma\alpha}\,\frac{1}{H_A}\,(3.4708). \quad (25)$$

For a sample with total anisotropy field $H_A = 1000$ Oe, the "coasting downhill" switching time is 0.394 ns. As may be expected, this field-free switching time is relatively slow; in Fig. 5, the 90°– 170° switching times, with applied field on, are always less than 1 ns.

It is stressed, once more, that the analysis in this paper holds true rigorously only in the very special case in which the uniaxial anisotropy easy axis is colinear with the applied field. However, it will be realized that in cases in which exact colinearity does not exist, the closed-form solution of this paper must be the primary or leading term describing the switching behavior. When a distribution of easy axes exists, which is both small and colinear with the applied field, it may be expected further that considerable cancellation of the secondary or higher order terms will occur, leaving the primary terms describing the switching behavior better than might otherwise have been expected.

IX. Conclusions

The gyromagnetic switching problem of a single-domain grain or particle with uniaxial magnetocrystalline anisotropy or uniaxial shape anisotropy has been solved analytically and completely, under the sole restriction that the uniaxial anisotropy easy axis is colinear with the applied field. Several specific conditions of relevance to strip-line pulse testing have been elucidated. They are the effects of the strip-line pulse field alone, the strip-line pulse field augmented with a dc bias field, and the behavior when the strip-line pulse field is terminated before the completion of switching. The fact that an explicit relationship exists between the polar angle θ and the azimuthal angle φ, which is determined as solely by the damping constant α means that the path or trajectory of the precessing magnetization is fixed, regardless of the magnitude of the applied field $\overline{H}$.

Acknowledgment

The author is deeply indebted to Dr. H. Shute, University of Plymouth, U.K., for producing all of the figures.

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