# EE2703: Applied Programming Lab
# Assignment 8
# The Digital Fourier Transform

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# Contents

# 1 Aim:

The aim of this assignment is to:

1. Learn Digital fourier transform (DFT) and its properties.
2. Use the in-built Python library to generate DFT of various discrete-time signals using the Fast fourier transform (FFT) algorithm.
3. Analyse the continuous time fourier transform (CTFT) of continuous-time signals by plotting the DFT of their sampled DT versions.

# 2 Theory:

## 2.1 Sampling:

Sampling is an important concept in digital signal processing. It is the process of creating a discrete-time signal from a known continuous-time signal by sampling it at regular time intervals. Also, all the digital devices and computers we use can perform arithmetic and algorithms only on discrete data. Hence, sampling is a very important concept in signal processing.

Let's say the CT signal is $x_c(t)$. Now the sampled DT signal will be:

$$x[n] = x_c(t)|_{t=nT_s} = x_c(nT_s)$$

where $T_s$ is known as sampling period. The associated sampling frequency is:

$$\Omega_s = \frac{2\pi}{T_s}$$

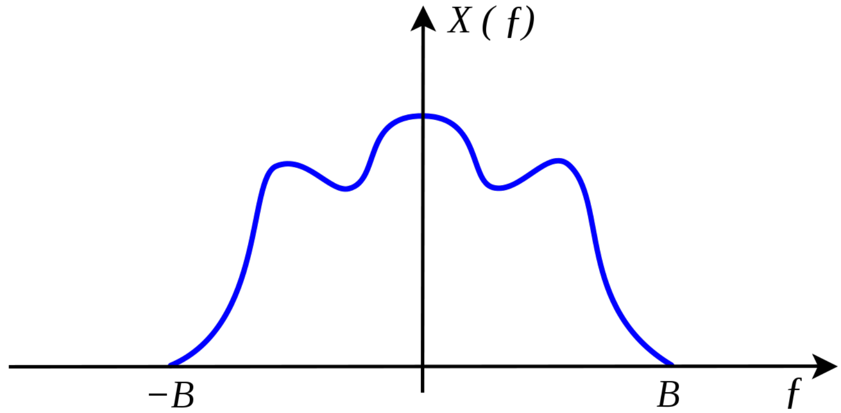
(In this report, $\Omega$ denotes frequency in CT signals and $\omega$ denotes frequency in DT signals)

## 2.2 Nyquist-Shannon sampling theorem:

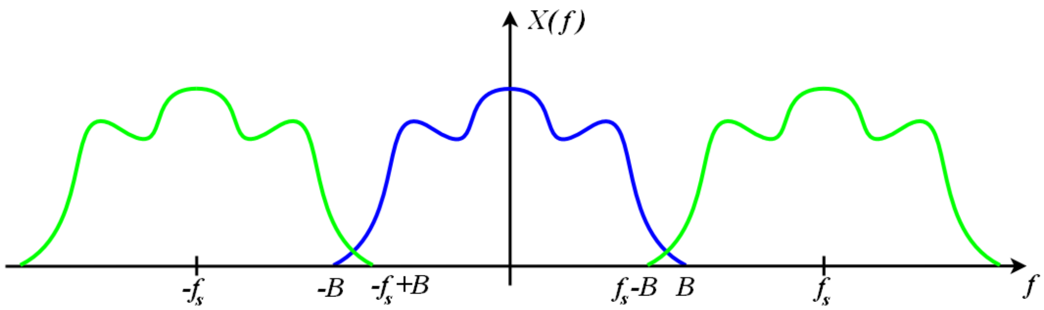
Whenever we sample a CT signal $x_c(t)$ and obtain a DT signal $x[n]$, their spectra: $X_c(\Omega)$ and $X(\omega)$ are related as:

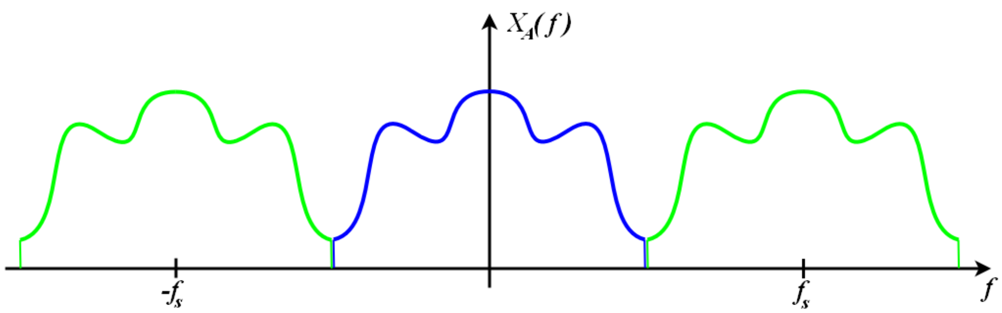
$$X(\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X_c(\frac{\omega}{T_s} - k\frac{2\pi}{T_s})$$

Now, the Nyquist-Shannon sampling theorem states: For a bandlimited signal $x_c(t)$, the sampling frequency $\Omega_s$ should be atleast twice the maximum frequency present in the spectrum of $x_c(t)$ in order to prevent overlapping (or Aliasing). Only if this condition is satisfied, the DTFT of $x[n]$ will preserve the original information about the CT signal. A visual representation is shown below:




The spectrum of the CT signal $x_c(t)$ (Bandlimited)




The time-scaled version of DTFT of signal $x[n]$. Clearly, depending on $\Omega_s$, we may or may not have aliasing

To avoid overlapping, we need to satisfy the condition: $f_s - B > B$ or $f_s > 2B$. Hence, the sampling frequency should be atleast twice the maximum frequency present in the CT signal to avoid overlapping.

## 2.3 Digital fourier transform:

Digital fourier transform (DFT) is an important concept in digital signal processing. As we already saw, we have a continuous variable spectrum pertaining to DT signals known as Dsicrete-time fourier transform (DTFT) which is given by:

$$X(w) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

This is a real variable function and is defined for a continuum of values. DFT, on the other hand, is a discrete variable signal and is defined only for certain integer values. Let's assume we have an N-periodic DT signal, i.e., if we know its first N samples $(x[0], x[1], ...x[N-1])$, we know the full signal. Now, its DFT is defined as:

$$a[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

$$x[n] = \sum_{k=0}^{N-1} \frac{a[k]}{N} e^{j\frac{2\pi}{N}kn}$$

and both k and n belong to the set $0, 1, 2, ...N-1$. Also, the DT signal x[n] calculated by the inverse DFT formula is inherently periodic with period 'N'. Hence, DFT is usually used to analyse periodic DT signals. Their values present in one single period contains all information about that signal and its DFT. Note that both $x[n]$ and its DFT $a[k]$ are periodic with period as 'N' samples.

## 2.4 Connection between CTFT and DFT:

Let's say we have a finite length CT signal and this signal, when periodically extended and sampled, gives us a DT signal. Let us define the following functions:

1. $x(t) =$ Finite length CT signal of length $T_0$
2. $X_{fin}(\Omega) =$ CTFT of $x(t)$
3. $x_c(t) =$ Periodic extension of $x(t)$ with period $T_0$
4. $\Omega_0 =$ Fundamental frequency of $x_c(t) = \frac{2\pi}{T_0}$
5. $a_k = k^{th}$ CTFS coefficient of $x_c(t)$
6. $X_c(\Omega) =$ CTFT of periodic CT signal $x_c(t)$
7. $x[n] =$ Sampled version of $x_c(t)$, i.e., $x_c(nT_s)$

8. $\Omega_s$= Sampling frequency = $\frac{2\pi}{T_s}$
9. $\omega_o$= DFT resolution frequency = $\frac{2\pi}{N}$
10. $X(\omega)$ = DTFT of signal $x[n]$
11. $a[k]$ = $k^{th}$ DFT coefficient of signal $x[n]$

Let's assume the first N samples that define $x[n]$ are given by sampling the $T_o$ period signal $x(t)$. Hence, $T_s = \frac{T_0}{N}$ We have the following **important** equations which relate the spectra of CT signal and the sampled DT signal:

$$X_{fin}(k\Omega_o) = T_o a_k \tag{1}$$

$$X_c(\Omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\Omega - k\Omega_o) \tag{2}$$

$$X(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_c(\frac{\omega}{T_s} - k\frac{2\pi}{T_s}) \tag{3}$$

$$X(\omega) = \sum_{k=0}^{N-1} \frac{2\pi a[k]}{N} \delta(\omega - k\omega_o) \tag{4}$$

(The last equation relates the DTFT and DFT of periodic DT signal $x[n]$ in the first period [-$\pi$,$\pi$).) Now, let's assume that the CTFT of the periodic CT signal $x_c(t)$ is bandlimited (or) atleast has a decreasing trend in the fourier series coefficients such that we can assume that the spectrum is approximately bandlimited after a particular value of $k$. Now, we can choose a sampling frequency $\Omega_s$ such that the Nyquist criteria is met, i.e., $\Omega_s > 2\Omega_m$ where $\Omega_m$ is the maximum frequency component present in $X_c(\Omega)$.

Any CT frequency $\Omega$ we have will become $\Omega T_s$ in the DT domain. Hence, a freqeuncy of $\frac{\Omega_s}{2}$ in CT domain becomes $\frac{\Omega_s T_s}{2} = \pi$ in the DT domain. So, a range of $[-\Omega_s,\Omega_s)$ in CT domain becomes [-$\pi$,$\pi$) in the DT domain. Also, since the Nyquist criteria is assumed to be satisfied, we can safely say:

$$X(\omega) = \frac{1}{T_s} X_c(\frac{\omega}{T_s}) \tag{5}$$

for $\omega \in [-\pi, \pi)$. We are able to get this equation only because Nyquist criteria is satisfied. Else, the shifted spectrum terms will also contribute to $X(\omega)$.With this assumption, let's see how $X(\omega)$ will be related to the DFT coefficients. From equation (4), (5),we can say:

$$X_c(\frac{\omega}{T_s}) = \sum_{k=0}^{N-1} \frac{2\pi.T_s.a[k]}{N} \delta(\omega - k\omega_o) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\frac{\omega}{T_s} - k\frac{2\pi}{T_o}) = \sum_{k=-\infty}^{\infty} 2\pi.T_s.a_k \delta(\omega - k\frac{2\pi}{N}) \tag{6}$$

Since, we have assumed that the CTFS coefficients are limited within [-$\frac{\Omega_s}{2}$,$\frac{\Omega_s}{2}$) in the CT domain, they will be restricted in [-$\pi$,$\pi$) on the DT domain (or) the w-axis. So, our eqn. (6) reduces to:

$$\sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \frac{2\pi.T_s.a[k]}{N}\delta(\omega - k\omega_o) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} 2\pi.T_s.a_k\delta(\omega - k\frac{2\pi}{N})$$

Clearly, we have:

$$a_k = \frac{a[k]}{N} \tag{7}$$

Now, we also have:

$$X_{fin}(k\Omega_o) = T_o a_k \tag{8}$$

From eqns. (7), (8),we can say:

$$X_{fin}(k\Omega_o) = T_o\frac{a[k]}{N} = T_s a[k] \tag{9}$$

The final takeaway point is that if we can safely assume the signal is bandlimited and the sampling frequency satisfies the Nyquist criteria, then the CTFT of the finite length CT signal $x(t)$ can be computed at integer values of frequency $\Omega_o$ by multiplying the N-point DFT of the sampled version by $T_s$.

- CT frequency interval : $[\frac{-\pi.N}{T_o},\frac{\pi.N}{T_o})$
- CT frequency least count : $\Omega_o = \frac{2\pi}{T_0}$

Hence, to get the CTFT for a higher range, we can either increase $N$ or decrease $T_o$ or do both. However, the resolution can be improved only by increasing $T_o$. Hence, when we aim to increase the resolution, the range automatically gets shortened. To maintain the same range, we need to proportionately increase $N$.

# 3 Assignment:

## 3.1 Spectrum of $sin(5t)$:

Sin(5t) is a very simple continuous time periodic function. Let's analyse some of its parameters and properties:

- Has a single CT freqeuncy of $\Omega = 5$ rad/s.
- Spectrum is bandlimited, i.e., only two impulses are present at $\Omega = \pm 5$ rad/s.
- Since it is periodic, its CTFT will have impulses. So, its better to plot its CTFS coefficients.

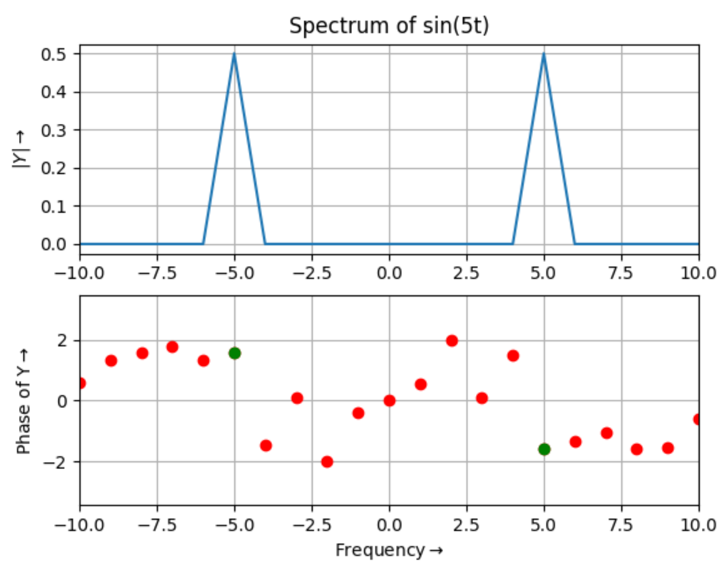
We can write sin(5t) as:

$$sin(5t) = \frac{e^{j5t} - e^{-j5t}}{2j} = -0.5je^{j5t} + 0.5je^{-j5t}$$

Clearly, the CTFS coefficients are -0.5j and 0.5j. To get this plot using the DFT of the sampled version, we need to satisfy the Nyquist criteria. Also, for the Fast fourier transform (FFT) algorithm to work faster, we need to choose an $N$ which is a power of 2. Hence, let's choose $N = 128$. Also, let the finite length signal whose periodic representation is $sin(5t)$ have a length of $2\pi$, i.e., it is $sin(5t)$ for $t \in [0, 2\pi)$ and zero outside. Hence, $T_o = 2\pi$. Thus, $T_s = \frac{2\pi}{128}$ and $\Omega_s = 128$. Clearly, $128 > 2 * 5 = 10$. Hence, Nyquist criteria is also satisfied. Now, we have:

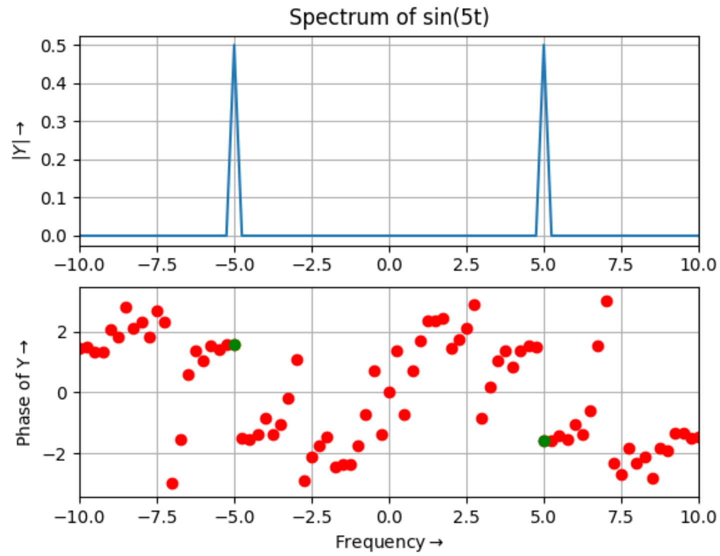
$$a_k = \frac{a[k]}{N}$$

So in our code, after finding the DFT coefficients, we should divide the vector by $N$ to get the CTFS coefficients. After doing all that and plotting the magnitude and phase response plots, we have the following plot:




There are peaks of length 0.5 at $\Omega_o = \pm 5$ as expected

The phase is $\frac{\pi}{2}$ at $\Omega_o = -5$ and $\frac{-\pi}{2}$ at $\Omega_o = 5$ as expected. Hence, the plot is correct. To get a more precise plot, we need to increase $T_o$ as resolution will increase but at the same time, we should increase $N$ to maintain the same frequency range for which the CTFT (or CTFS) is calculated. The plot given below is for $N = 1024$ and $T_o = 8\pi$:




A much more accurate plot with better resolution

## 3.2 Spectrum of amplitude modulated signal:

AM modulation is a common technique used in communication of signals. A signal which is to be transmitted, will be modulated over a carrier frequency so that the modulated signal can be transmitted using antennas of small length. The modulated signal can be again converted back to the original signal using signal processing techniques on the reciever end. Let us consider the following example:

$$x(t) = (1 + 0.1cos(t))cos(10t)$$

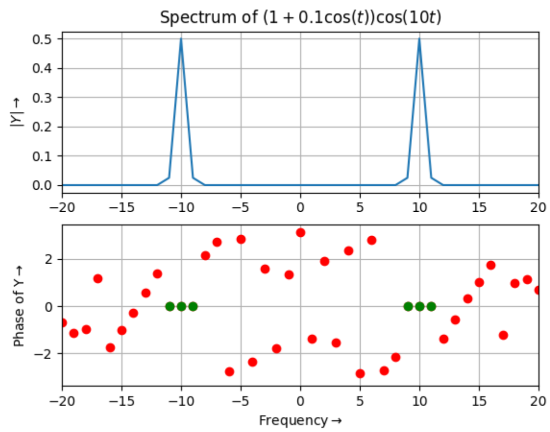
The properties of this signal are:

- Has three CT frequencies of $\Omega = 9, 10, 11$ rad/s.
- Spectrum is bandlimited, i.e., only two impulses are present at $\Omega = \pm 9, \pm 10, \pm 11$ rad/s.
- Since it is periodic, its CTFT will have impulses. So, its better to plot its CTFS coefficients.

We can write this signal as:

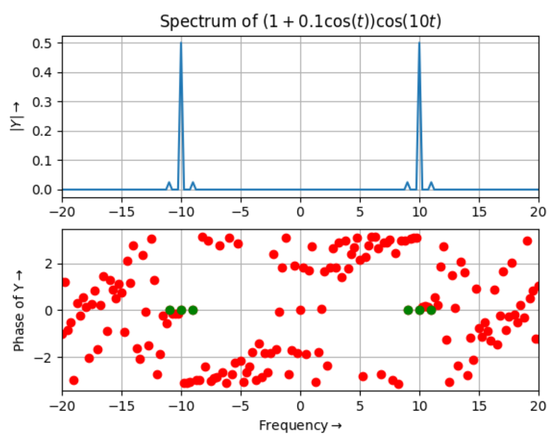
$$x(t) = 0.025e^{j9t} + 0.025e^{-j9t} + 0.5e^{j10t} + 0.5e^{-j10t} + 0.025e^{j11t} + 0.025e^{-j11t}$$

In this question, let's assume we choose $T_o = 2\pi$ and $N = 128$. Now, the least count is $\Omega_0 = \frac{2\pi}{T_0} = 1\text{rad/s}$. And the CT frequencies we have also differ by 1 rad/s. Hence, when we plot, we won't get distinct impulses. We will get an interpolated plot which is wrong. The plot is given below:




We get interpolated plot which is not correct

To resolve this problem, we need to set the least count less than 1. Our previous least count was $\Omega_0 = \frac{2\pi}{T_0} = 1\text{rad/s}$. Now, let's say $T_o = 8\pi$ and $N = 1024$. Now, the new least count is $\Omega_0 = \frac{2\pi}{T_0} = 0.25$ rad/s. Clearly, now we won't have any wrong interpolation and the graph will be much better and visually appealing. The plot will be:




The plot contains magnitudes which are exactly same as the mathematical results we got and the phases are also zero at those frequencies

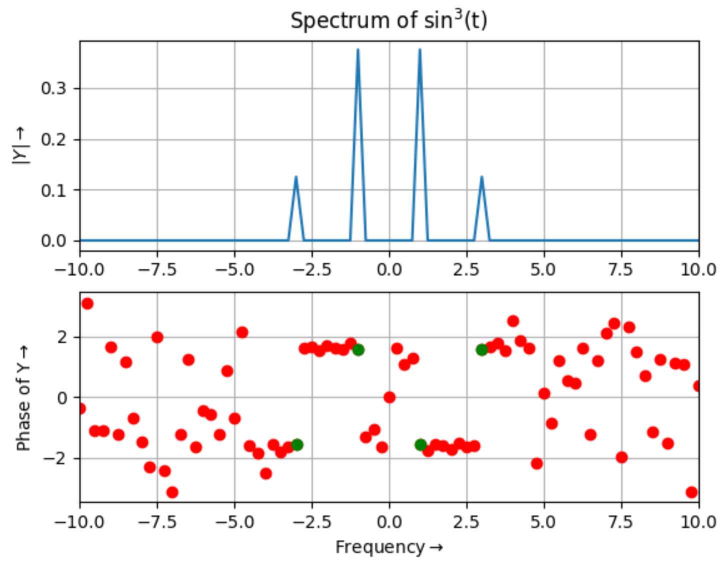
### 3.3 Spectrum of $sin^3(t), cos^3(t)$ :

Both signals the $sin^3(t)$ and $cos^3(t)$ are periodic with bandlimited spectrum. We can write them as:

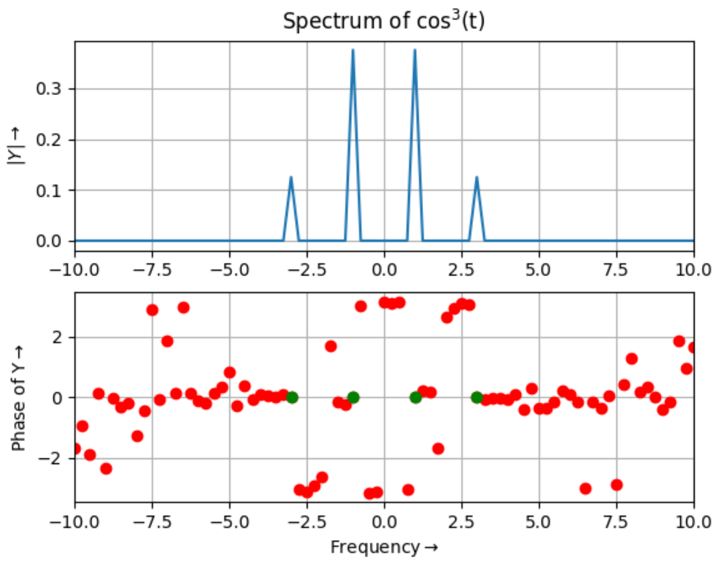
$$sin^3(t) = \frac{3}{4}sin(t) - \frac{1}{4}sin(3t) = -0.375je^{jt} + 0.375je^{-jt} + 0.125je^{j3t} - 0.125je^{-j3t}$$

$$cos^3(t) = \frac{3}{4}cos(t) + \frac{1}{4}cos(3t) = 0.375e^{jt} + 0.375e^{-jt} + 0.125e^{j3t} + 0.125e^{-j3t}$$

Hence, the spectrum of both the signals will contain distinct impulses at frequencies 1 and 3 rad/s. the plots are given below:




As expected we have peaks of lengths 0.375 and 0.125. Also, we have the phases as -$\pi/2$ and $\pi/2$ at the correct frequencies




Similar to previous graph, we have peaks of lengths 0.375 and 0.125. However, now the phases are purely zero as expected

(Both these plots are generated for $N = 1024$ and $T_o = 8\pi$. Hence, Nyquist criteria is also met since $\Omega_s = \frac{2\pi}{T_s} = 256 > 2 * 3$)

### 3.4 Spectrum of frequency modulated signal:

**Note:** Only in this section, both $\Omega$ and $\omega$ denote CT frequencies.

Sometimes, signals are also freqeuncy modulated before transmission. One such example is:

$$x(t) = cos(20t + 5cos(t))$$

We can write this function as a sum of complex exponentials based on the **Laurent's expansion of Bessel's function**:

$$exp(\frac{\beta}{2}(z - \frac{1}{z})) = \sum_{k=-\infty}^{\infty} J_k(\beta)z^k$$

Substituting $z = jexp(jw_m t)$, we have:

$$exp(j\beta cos(w_m t)) = \sum_{k=-\infty}^{\infty} J_k(\beta)j^k exp(jkw_m t)$$

Now, we have:

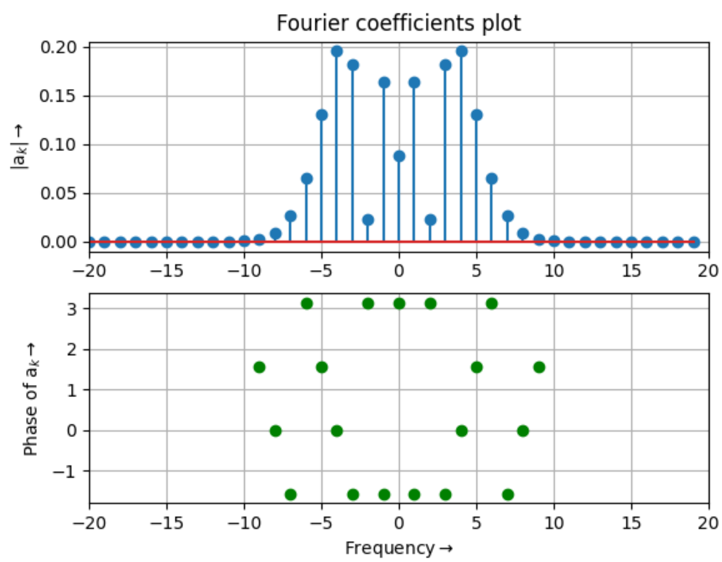
$$cos(w_ct+\beta cos(w_mt)) = Re\{exp(jw_ct+j\beta cos(w_mt))\} = Re\{e^{jw_ct}\sum_{k=-\infty}^{\infty} J_k(\beta)e^{j(w_mt+\frac{\pi}{2})k}\}$$

$$cos(w_ct+\beta cos(w_mt)) = \sum_{k=-\infty}^{\infty} J_k(\beta)cos(w_ct+kw_mt+k\frac{\pi}{2})$$

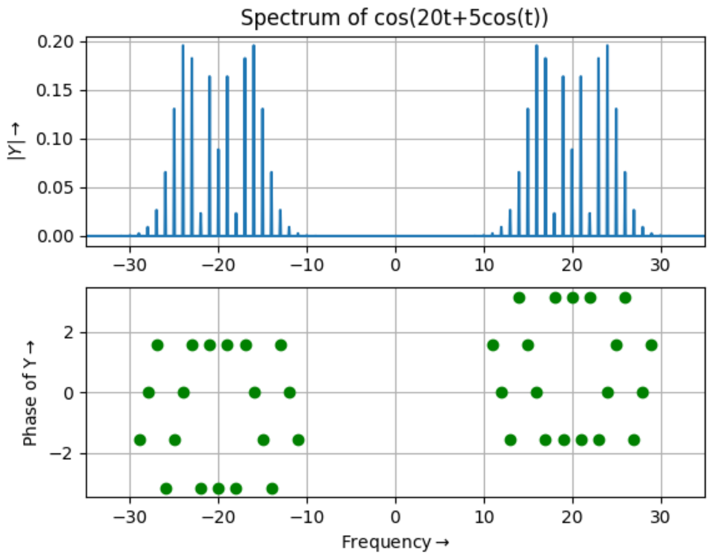
Writing this as a sum of sinusoids, we have:

$$cos(w_ct+\beta cos(w_mt)) = \sum_{k=-\infty}^{\infty}\frac{J_k(\beta)}{2}e^{(w_ct+kw_mt+k\frac{\pi}{2})}+\sum_{k=-\infty}^{\infty}\frac{J_k(\beta)}{2}e^{-(w_ct+kw_mt+k\frac{\pi}{2})}$$

Hence, when we plot the CTFT, we will have impulses of length $J_k(\beta)/2$ centered around $w_c$ and $-w_c$. Also, the phase of the impulses will depend on the $k$ value. The fourier coefficients, i.e., $\frac{J_k(\beta)}{2}j^k$ are plotted for $\beta = 5$ and $w_m = 1$ as given in the question:




Now, we have an idea about the fourier coefficients of the FM signal. We see that the signal has a bandlimited spectrum with the maximum frequency at $w = \pm 10$ rad/s. So, in order to prevent aliasing, we just need to choose a sufficient $N$. Let's assume $T_o = 32\pi$ and $N = 4096$. Hence, maximum frequency is $\frac{\Omega_s}{2} = 4096\frac{\pi}{32\pi} = 128 > (20+10) = 30$. Hence, we can safely assume that the Nyquist criteria is satisfied. (We consider 20+10 and not just 10 since now, the impulses will be around $w_c = 20$. Hence, we need to check whether $w_c + 10$ is less than $\frac{\Omega_s}{2}$).




We have peaks at frequency $\Omega = 20$ and a few frequenices surrounding it. Also, the magnitude response is even and the phase response is odd since the CT signal is real

The plot obtained from DFT exactly matches with the Bessel's function plot generated manually. Hence, the DFT gave an accurate plot of the CTFS of the FM signal.

**Note:** The theory about CTFS of FM modulated signal and the relation with Bessel's functions was referred from this link: https://www.dsprelated.com/freebooks/mdft/Sinusoidal_Frequency_Modulation_FM.html

### 3.5 Spectrum of the Gaussian signal:

This Gaussian function is a special signal. Unlike the previous signals, this is a non-periodic sugnal and also the spectrum is not bandlimited. This is because Gaussian is a **self-function,** i.e., it is it's won fourier transform.

Let the signal be of the form:

$$x(t) = exp(\frac{-t^2}{2\sigma^2})$$

Then the continuous time fourier transform of this signal will be:

$$X(\mathit{\Omega}) = \sqrt{2\pi\sigma^2}exp(\frac{-\mathit{\Omega}^2\sigma^2}{2})$$

In the example given in the assignment, $\sigma = 1$. Hence, the expected plot should be:

$$X(\mathit{\Omega}) = \sqrt{2\pi}exp(\frac{-\mathit{\Omega}^2}{2})$$

Now, before procceding forward, we must first analyse some important properties of the signal and make some valid approximations based on that. The following properties are important:

- It is not a bandlimited signal (Satisfying Nyquist criteria exactly is impossible)
- It is a non-periodic signal
- It is a steeply decreasing positive even function and almost looks like a finite length signal

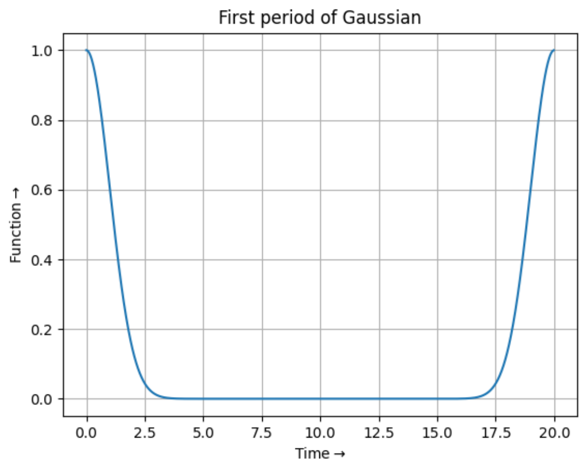
Since, the gaussian is a non-periodic signal, it's CTFT won't necessarily have impulses (Infact, it doesn't have any as we have already derived its CTFT). So, in order to apply the DFT concepts on it, we must first make a periodic CT signal out of it and sample it. Let's first define a new finite length signal like this:

$$x_{finite}(t) = exp(\frac{-t^2}{2})\forall t \in [-\frac{T_o}{2}, \frac{T_o}{2})$$

and the signal is zero outside this interval. This new function is not the same gaussian function but a finite piece (or window) of it is taken. Now, let's find its CTFT:

$$X(\Omega) = \int_{-\infty}^{\infty} exp(\frac{-t^2}{2})exp(-j\Omega t)dt \approx \int_{-T_o/2}^{T_o/2} exp(\frac{-t^2}{2})exp(-j\Omega t)dt$$

Hence, the CTFT of the new finite length function is approximately equal to the CTFT of the Gaussian function. **This is the first assumption.** Now, this signal is periodically repeated and a new periodic signal $x_c(t)$ is generated and sampled. The first period plot of this signal for $T_o = 20$ would look like this:




This is the first period of the periodic repetition of the finite length signal ($T_o = 20$)

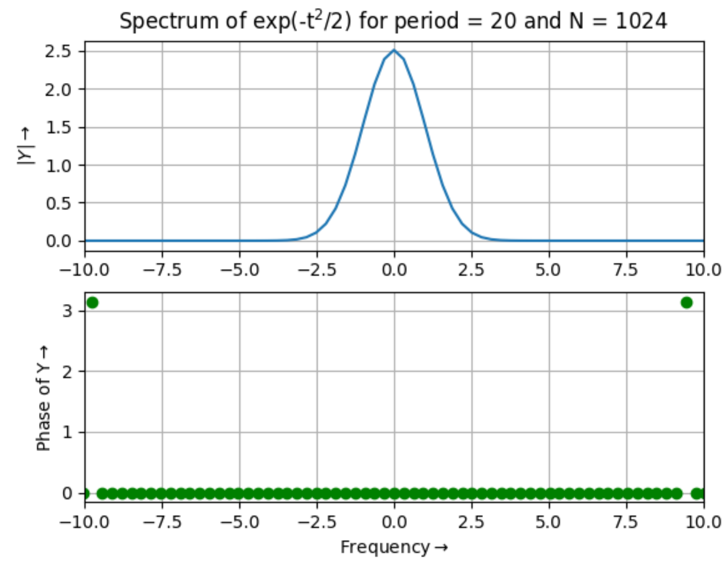
This new periodic function has a period of $T_o$ and hence a fundamental frequency of $\Omega_0 = \frac{2\pi}{T_0}$. Hence, the signal will have CTFS coefficients at frequencies which are multiples of $\Omega_o$. Now, we already proved in section (2.4) that:

$$X(k\Omega_o) = T_o a_k$$

and we know $X(\Omega)$ decreases as $\Omega$ increases. Hence, we can safely say the CTFS coefficients are nearly zero at high values of $k$. Now, the only final step is to choose a sampling frequency such that we can safely assume the CTFT of this periodic signal is bandlimited within that range. Let's choose $N = 1024$ and $T_o = 20$. Hence, $\Omega_o = \frac{2\pi}{T_o} = \frac{\pi}{10}$ and $\Omega_s = N\frac{2\pi}{T_o} = 1024\frac{\pi}{10}$. Clearly, $\Omega_s$ is sufficiently high to assume that the Nyquist criteria is satisfied. **This is the second assumption.** With these two assumptions in mind, we can use all the results we got in section (2.4). The main result which we want is the following equation:

$$X(k\Omega_o) = T_s a[k]$$

Hence, whatever DFT coefficients vector we are getting, we need to multiply it by $T_s$. The resulting coefficients are nothing but the CTFT values of the windowed Gaussian signal at integral values of $\Omega_o$. The plot given below is for $T_o = 20$ and $N = 1024$.




The curve looks very much similar to the CTFT formula we derived

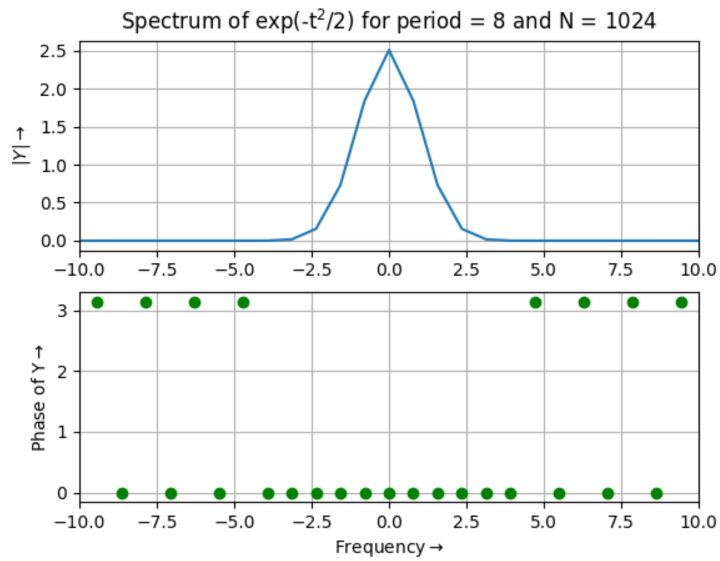
### 3.6 Effect of $T_o$ and N on the CTFT of the gaussian:

We already derived that the least count of CT frequency is $\Omega_o = \frac{2\pi}{T_o}$ and the frequency range is $[-\frac{\Omega_s}{2}, \frac{\Omega_s}{2}) = [-N\frac{2\pi}{T_o}, N\frac{2\pi}{T_o})$.

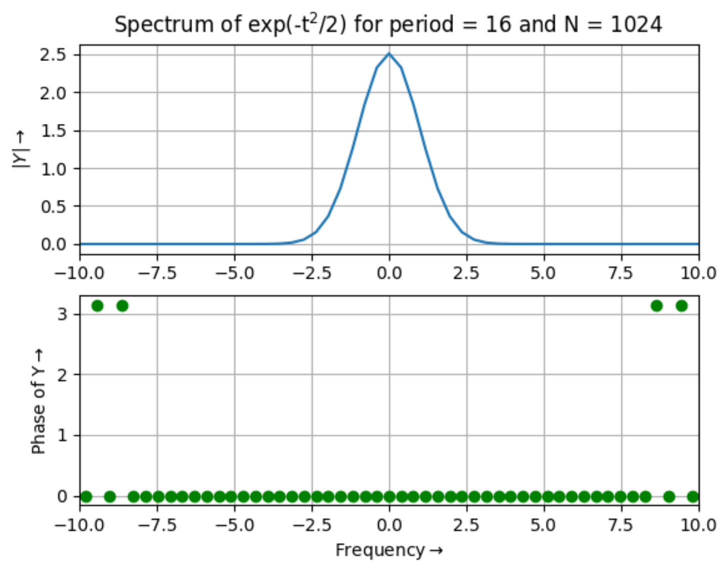
With these facts in mind, let's understand the important points to improve the spectrum plot of the Gaussian function:

1. Increasing $T_o$ has two benefits:
   (a) The window of the finite length signal increases and hence, the approximation is much more accurate.
   (b) The resolution improves and hence, the plot will be much more smoother.
2. Increasing N also has two benefits:
   (a) The sampling frequency $\Omega_s = N\frac{2\pi}{T_o}$ increases and hence, the assumption that the Nyquist criteria is satisfied is much more credible.
   (b) We can get the CTFT for a broader range of frequency as the range is determined by $\Omega_s$.

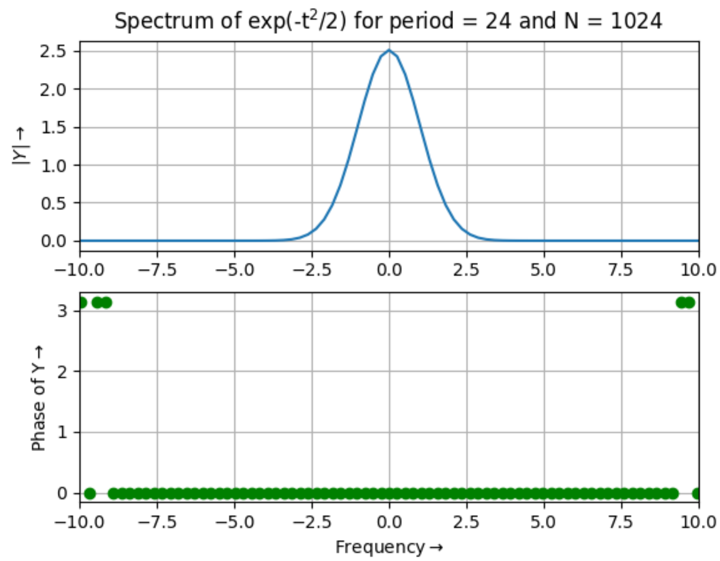
Hence, we expect the graphs to become smoother with increase in period. Let's see if that's the case:

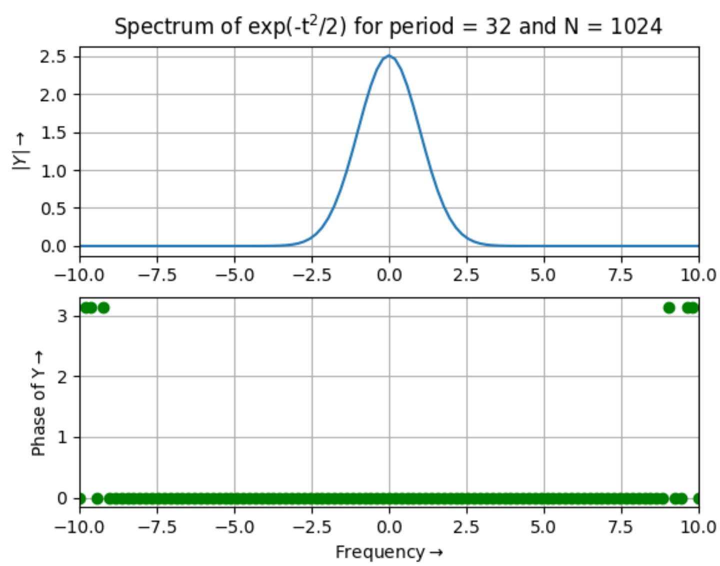


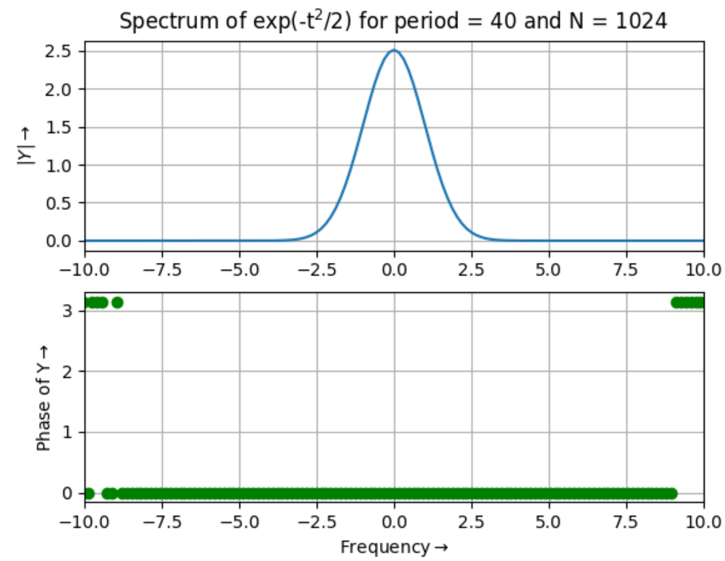
Spectrum of exp(-t²/2) for period = 8 and N = 1024
|Y| →
Phase of Y →
Frequency →

Spectrum of exp(-t²/2) for period = 16 and N = 1024
|Y| →
Phase of Y →
Frequency →

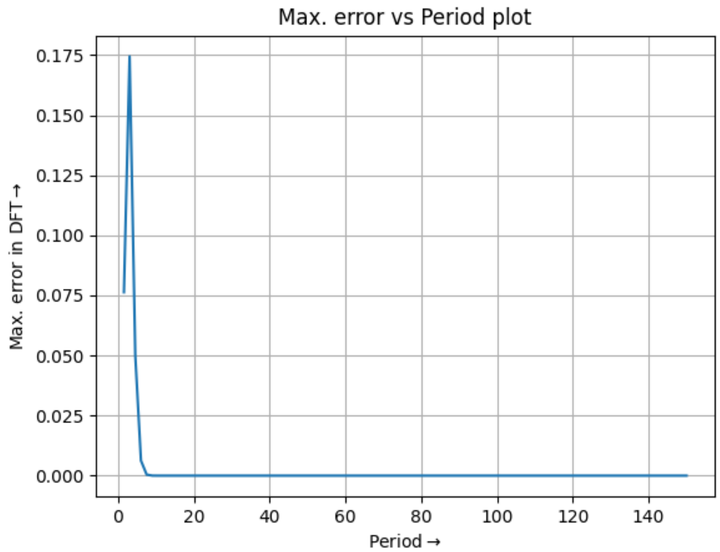

Spectrum of exp(-t²/2) for period = 24 and N = 1024
|Y| →
Phase of Y →
Frequency →


Spectrum of exp(-t²/2) for period = 32 and N = 1024
|Y| →
Phase of Y →
Frequency →




Clearly, the graph becomes smoother with increasing $T_o$ value. Also, note that the maximum frequency range for which the CTFT is calculated is also decreasing because $\Omega_s$ inversely depends on $T_o$ for a given value of $N$. However, in all the plots, the limit of the x-axis is set to [-10,10). And the lowest frequency range in these set of plots is $\frac{\Omega_s}{2} = 512\frac{2\pi}{40} > 10$. Hence, we aren't able to see the decreasing trend in the range of frequencies.

**Error plot:** As already said, whatever plots we are getting is an approximation and not the exact spectrum of the gaussian. That's why we are even getting phase values of $\pi$ at high frequencies (Could be due to computer precision and some other non-idealities). However, the graph we get is fairly accurate in the significant range of frequencies, i.e., where the spectrum magnitude is more than 0.001. Now, let's see how the maximum error between the computed CTFT values and actual CTFT values change with the period. We expect a decreasing trend with increasing period as we have already seen higher period means better approximation. Let's see if the plot obeys this fact:




In this example, the period $T_o$ starts from 1.5 and ranges till 150 in steps of 1.5. As expected, the error is decreasing with increasing $T_o$

# 4 Conclusions:

1. The spectrum plot of $sin(5t)$ is exact and the CTFS coefficients are correctly computed.

2. The spectrum plot of amplitude modulated signal is also exact. Only the resolution had to be improved there to get an accurate plot.

3. The spectra plots of signals $sin^3(t)$ and $cos^3(t)$ are also exact.

4. The spectrum plot of frequency modulated signal is very much accurate when the chosen sampling frequency is sufficiently large and it exactly matched the Bessel's function plot generated manually.

5. In all the above cases, the Nyquist criteria is satisfied and hence, the plots are exact.

6. In the gaussian example, the CTFT spectrum plot obtained is very accurate when the window (or period) chosen is high because of:

   (a) High resolution

   (b) Better approximation of the windowed signal with respect to the actual gaussian signal

7. To improve the resolution, the only method is to increase $T_o$. However, we can control the maximum frequency range (For which the CTFT is computed) by both $N$ as well as $T_o$.

8. The maximum error between computed CTFT and actual CTFT decreases with increasing $T_o$ as expected.

9. The window length $T_o(ideal)$ must be chosen so that the periodic repetition of the windowed gaussian almost mimics a sinusoid. In that case, the frequency components in the periodic signal would be less and hence, the Nyquist criteria will also be satisfied much more accurately.

10. However, we have a trade-off here. Any increase in the period $T_o(ideal)$ will improve the approximation that the CTFT of the windowed signal is same as the CTFT of the original gaussian. However, now the periodic signal will have more frequency components and hence, the Nyquist criteria may not be obeyed fully. To resolve this issue, we must increase the value of $N$.