EE2703: Applied Programming Lab
End semester examination
Radiation from a loop antenna

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Contents

1 Problem statement:

We have a circular current conducting loop located on the x-y plane with its center being the origin. Also, the current through it varies sinusoidally with time and the circumference of the loop is equal to the wavelength at that frequency. The aim is to find the z-component of the magnetic field $\vec{B}$ due to this circular loop on the z-axis and also to find how it varies with respect to the z value. The current distribution is given as:

$$I = \frac{4\pi}{\mu_0} cos(\phi) exp(j\omega t)$$

where ϕ is the polar angle in cylindrical coordinates (r, ϕ, z). Also, it is asked to assume that the magnetic field should be found for z values in the interval $[1, 1000]$ cm with a stepsize of 1 cm (However in the code, the number of points to be present in the z-axis can be provided by the user as a command line argument)

2 Theory:

2.1 Generalised vector potential:

Vector potential is a mathematical tool used to compute magnetic field. Vector potential is not a real physical quantity that can be measured. However, for solving the differential equations related to a particular current distribution, it is easier to solve for the vector potential rather than solving for the field directly. Vector potential $\vec{A}$ is defined as:

$$\vec{B} = \vec{\nabla} X \vec{A}$$

Also, we don't have a single solution of $\vec{A}$ for a given $\vec{B}$ because of the property of the curl operator, i.e., $\vec{\nabla} X \vec{\nabla} f = 0$ (Curl of a gradient is zero). Hence, to solve for $\vec{A}$ we need to know its divergence as well which is usually taken as zero, i.e., $\vec{\nabla}.\vec{A} = 0$. With these two equations for $\vec{A}$,a differential equation can be formed from the Maxwell's equations which is:

$$(\nabla^2 - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2})\vec{A} = -\mu_0 \vec{J}$$

For a sinusoidally varying current distribution, the equation reduces to the following form:

$$(\nabla^2 + k^2)\vec{A}(\vec{r}) = -\mu_0 \vec{J}(\vec{r})$$

where:

- $\vec{A} = \vec{A}(\vec{r}) exp(j\omega t)$
- $\vec{J} = \vec{J}(\vec{r}) exp(j\omega t)$

- $k = \omega\sqrt{\mu_0\epsilon_0}$ (Wave number)

The general solution for $\vec{A}(\vec{r})$ is given as:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r'})e^{-jkR}dv'}{R}$$

where $\vec{R} = \vec{r} - \vec{r'}$ and R is just its magnitude. $\vec{r'}$ is a vector to denote the location of the current conducting material and dv' indicates the infinitesimal volume at that point. The integral is a volume integral. This integral, for the linear current distribution which we have, transforms as follows:

$$\vec{A}(r,\phi,z) = \frac{\mu_0}{4\pi}\int \frac{I(\phi')\hat{\phi'}.e^{-jkR}dl'}{R}$$

We know $dl' = ad\phi'$ where a is the radius of the loop. Writing this integral in discrete form and also substituting the value of I will yield:

$$\vec{A}(x,y,z) = \sum_{l=0}^{N-1} \frac{cos(\phi_l')^{-jkR}a(\Delta\phi')\hat{\phi_l'}}{R} \tag{1}$$

This is the formula we need to find the z component of the magnetic field along the z-axis. The variable N denotes the number of divisions we have in the interval of ϕ which is $[0, 2\pi]$. By default, it is asked to be taken as 100 in the assignment but in the code, the user can enter it as a command line argument. Also, it needs to be even so that π will also be one of the angles.

2.2 Finding $\vec{B}$ from $\vec{A}$ using the curl operator:

We know that:

$$\vec{B} = \vec{\nabla} X \vec{A}$$

The z-component of the magnetic field along the z-axis would be then given by:

$$B_z(0,0,z) = (\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})_{(x,y,z)=(0,0,z)}$$

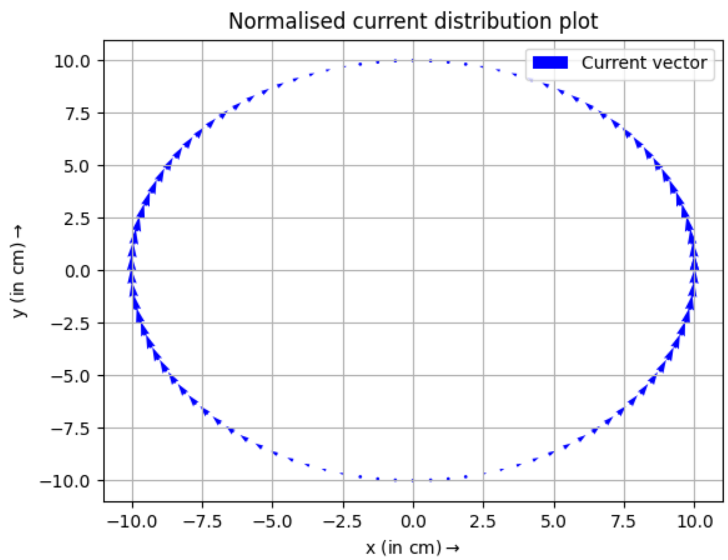
In discrete form, these derivative terms can be written as:

$$B_z(0,0,z) = \frac{A_y(\Delta x,0,z) - A_y(-\Delta x,0,z)}{2\Delta x} + \frac{A_x(0,-\Delta y,z) - A_x(0,\Delta y,z)}{2\Delta y} \tag{2}$$

This is the formula we need to find magnetic field's z-component along the z-axis from the vector potential. To use this formula, we also need an interval in the x and y axes. So, in this assignment, a 3×3 mesh along the x-y plane is made with the mesh points seperated by 1 cm in both the axes. However, in the code, the number of mesh points to be present in the axes can be given as command line arguments (They should be odd in order to ensure $x = 0$ and $y = 0$ are one of the mesh points).

2.3 Important complications to be taken care of in the code:

For this particular distribution, the z-component of the magnetic field will be zero along the z-axis. The reason is because of the current distribution we have. Let us see the current distribution:



The quiver plot of the current distribution

We know that:

$$B_z(0,0,z) = (\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})_{(x,y,z)=(0,0,z)}$$

Hence for the distribution we have, we can write the following equations:

$$[\frac{\partial A_y}{\partial x} = -\int_0^{2\pi} \frac{cos^2(\phi^{'})ad\phi'}{R^2} e^{-jkR}(1+jkR)\frac{\partial R}{\partial x}]_{(x,y,z)=(0,0,z)}$$

$$[\frac{\partial A_x}{\partial y} = +\int_0^{2\pi} \frac{cos(\phi^{'})sin(\phi^{'})ad\phi'}{R^2} e^{-jkR}(1+jkR)\frac{\partial R}{\partial y}]_{(x,y,z)=(0,0,z)}$$

We then have:

$$\frac{\partial R}{\partial y}|_{(x,y,z)=(0,0,z)} = \frac{y-y^{'}}{R}|_{(0,0,z)} = \frac{-asin(\phi^{'})}{R}$$

$$\frac{\partial R}{\partial x}|_{(x,y,z)=(0,0,z)} = \frac{x-x^{'}}{R}|_{(0,0,z)} = \frac{-acos(\phi^{'})}{R}$$

$$R|_{(0,0,z)} = \sqrt{a^2+z^2}(Constant)$$

Clearly both the partial derivatives will become zero after substituting the values of $\frac{\partial R}{\partial x}$ and $\frac{\partial R}{\partial y}$. Hence, the z-component of the magnetic field along the z-axis is zero theoretically. However, we are not actually computing an integral but approximating it using a Riemann summation which is given below:

$$\vec{A}(x,y,z) = \sum_{l=0}^{N-1} \frac{cos(\phi_l^{'})e^{-jkR}a(\Delta\phi^{'})(-sin(\phi_l^{'})\hat{x}+cos(\phi_l^{'})\hat{y})}{R}$$

Clearly the x-component A_x is zero along the y-axis and y-component A_y is an even function along the x-axis. Hence, the partial derivatives $\frac{\partial A_y}{\partial x}$ and $\frac{\partial A_x}{\partial y}$ are zero (Only those two contribute to B_z). However, in the summation formula of $\vec{A}$, if we include the contribution due to angle 2π, we will be double counting the contribution due to the current element present at $\phi = 0$. Because of this, we will have unequal A_y value at $+\Delta x$ and $-\Delta x$ and hence a non-zero $\frac{\partial A_y}{\partial x}$.

This is an important issue which needs to be taken care in the code. Infact, in my code, the user can decide whether to include 2π as the last element or not based on a flag value. This is the first command line argument. If it is zero (Default case), then 2π won't be taken into account. If it is non-zero, then 2π will be taken into account. This complication is later discussed in this report.

Note: 2π should not be included since we have already included the current element at $\phi = 0$. However without 2π, the magnetic field $B_z(z)$ would be zero and the plot we would get will be due to computer error. Hence for finding the trend of $B_z(z)$ with respect to z, 2π is included. Including 2π is not correct but for the sake of finding the trend, it is done in my code. The correct plot refers to the computer precision error plot I got when 2π was not included.

3 Pseudocode:

The pseudocode for this assignment code is given below:

```
1. INIT Flags to zero (Flag, k_flag)
2. INIT radius and wave number (a and k)
3. INIT Number of divisions variables (Nx, Ny, Nz, N_angle)
```

```
OBTAIN command line arguments
PRINT error if arguments are not valid

INIT x,y,z intervals and stepsizes
INIT angle interval and stepsize
DETERMINE volume, radial, tangential and current vectors arrays

IF k_flag is not zero
    SET k as zero (Static case)

PLOT current distribution
PLOT radial vectors
PLOT tangential vectors

FUNCTION: Calc
    INPUT: Index of the current vectors array
    CALCULATE distances for that particular index
    CALCULATE Vector potential contribution from that index
    RETURN vector potential
ENDFUNCTION

IF flag is zero
    SET number of iterations (Niter) as N_angle (Excluding 2pi)
ELSE
    SET Niter as N_angle+1 (Including 2pi)
ENDIF

FOR index in the set {0,1,2,...Niter}
    CALL calc with input as index RETURNING vector potential
    INCREMENT vector potential by the returned values
ENDFOR

COMPUTE indices at which x=0, y=0
COMPUTE partial derivatives of the curl formula for all z
COMPUTE magnetic field for all z

CREATE the matrices needed for least squares solution
OBTAIN least squares fit
PLOT z vs original Bz
PLOT z vs BZ obtained from fit

PRINT least squares solution values
```

4 Assignment codes and plots:

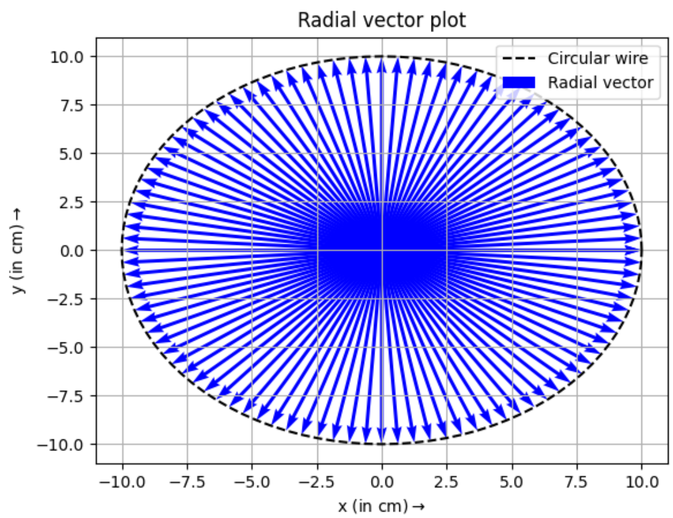
4.1 Creating volume array and current, radial and tangential vector arrays:

```
a = 10; k = 1/a
Nx = 3; Ny = 3; Nz = 1000; N_angle = 100

if k_flag!=0:
    k = 0

xval = p.linspace(-1,1,Nx); del_x = xval[1]-xval[0]
yval = p.linspace(-1,1,Ny); del_y = yval[1]-yval[0]
zval = p.linspace(1,1000,Nz); del_z = zval[1]-zval[0]

angle_vals = p.linspace(0,2*p.pi,N_angle+1)
del_angle = angle_vals[1]-angle_vals[0]

x_vals = a*p.cos(angle_vals)
y_vals = a*p.sin(angle_vals)

phi_cap_x_vals = -p.sin(angle_vals)
phi_cap_y_vals = p.cos(angle_vals)

dl_x_vals = a*del_angle*phi_cap_x_vals
dl_y_vals = a*del_angle*phi_cap_y_vals

current_x_vals = p.cos(angle_vals)*phi_cap_x_vals
current_y_vals = p.cos(angle_vals)*phi_cap_y_vals

yy, zz, xx = p.meshgrid(yval,zval,xval)
```

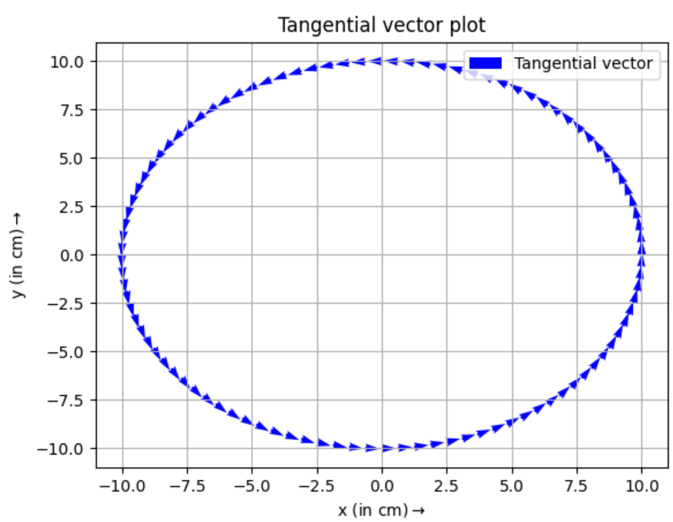
In this code snippet, the $3 \times 3 \times 1000$ volume grid is created. But before doing that, three 1D python arrays are created using the **Linspace** command. These arrays contain the x,y,z interval values. These 1D arrays are passed into the **Meshgrid** command in the order y,z,x. The outputs we get are yy,zz and xx.

- yy = 3D grid which contains the y-coordinate values in it
- zz = 3D grid which contains the z-coordinate values in it
- xx = 3D grid which contains the x-coordinate values in it

Also, the angle interval $[0,2\pi]$ is splitted into 101 equidistant points. In this manner, there will be now 101 elements with a stepsize of $\frac{2\pi}{100}$. Depending on the flag value the user entered, the last element 2π may or may not contribute in the computation of vector potential. Once the angle array is formed, it is used to generate the radial vector arrays $\vec{r'}$= (x_vals, y_vals), tangential vector arrays $\hat{\phi}'$= (phi_cap_x_vals, phi_cap_y_vals), infinitesimal wire element vector arrays $\vec{dl'}$= (dl_x_vals, dl_y_vals) and finally the normalised current vector arrays $\vec{I}$= (current_x_vals, current_y_vals). The plots of radial and tangential vectors are:



The radial vectors are properly enclosed within the circle



The tangential unit vectors are correctly oriented

Also based on the k_flag value, the wave number k is modified. If k_flag=0, then there will be no change, i.e., $k = \frac{1}{a}$. However, if k_flag$\neq$ 0, then k is set to zero (Static case).

4.2 Function $calc(l)$:

This function takes in the index of the angle interval array as the input, calculates the distances $R_{ijk} = |\vec{r}_{ijk} - \vec{r}_l^{'}|$ for all ijk triplets and then computes the vector potential term:

$$\vec{A}_{ijkl} = \frac{cos(\phi_l^{'})e^{-jkR}a(\Delta\phi^{'})(-sin(\phi_l^{'})\hat{x} + cos(\phi_l^{'})\hat{y})}{R}$$

for the input l and returns the x and y components. This function acts as a helper function to calculate the total vector potential. The following code snippet contains this function:

```
def calc(l):
    ang = angle_vals[l]
    xl = x_vals[l]; yl = y_vals[l]
    Rl = p.sqrt(((xx-xl)**2)+((yy-yl)**2)+(zz**2))

    Ax_l = p.cos(ang)*p.exp(-1j*k*Rl)*dl_x_vals[l]/Rl
    Ay_l = p.cos(ang)*p.exp(-1j*k*Rl)*dl_y_vals[l]/Rl

    return Ax_l, Ay_l
```

Once the input l is obtained, the corresponding $x_l^{'}$, $y_l^{'}$ and $\phi_l^{'}$ values are obtained. Based on these values, the $R_{ijk}(l)$ matrix is computed by using the xx, yy, zz matrices returned by the meshgrid command. Once this matrix is computed, the vector potential terms Ax_l and Ay_l are computed and returned.

4.3 Computing the vector potential and magnetic field:

Once we have the calc function, we can use it to compute the total vector potential. For this purpose, a for loop is used. The following code snippet does that:

```
Ax = p.zeros((Nz,Ny,Nx))
Ay = p.zeros((Nz,Ny,Nx))

if flag==0:
    Niter = N_angle
else:
    Niter = N_angle+1

for l in range(Niter):
    Ax_l, Ay_l = calc(l)
    Ax = Ax+Ax_l; Ay = Ay+Ay_l

zero_ind_x = int(Nx/2); zero_ind_y = int(Ny/2)

Bz1 = (Ay[:,zero_ind_y,zero_ind_x+1]-Ay[:,zero_ind_y,zero_ind_x-1])/(2*del_x)
Bz2 = (Ax[:,zero_ind_y-1,zero_ind_x]-Ax[:,zero_ind_y+1,zero_ind_x])/(2*del_y)
Bz = Bz1+Bz2
```

Before proceeding on with the for loop, first the flag variable is used to set the *Niter* value. If flag=0, Niter=N_angle, i.e., the last current element (ϕ=2π) is not included in the vector potential computation. The magentic field will be zero in that case. However, if flag$\neq$ 0 then Niter = N_angle+1, i.e., the last current element is also included and hence, magnetic field will not be zero.

In the for loop, the function $calc(l)$ is called and the input is the index. The returned paramaters are assigned to variables Ax_l and Ay_l respectively. These temporary variables are added with the 3D arrays Ax and Ay which are initialised with zeros. After the for loop is executed, the zero indices of x and y are calculated since we are finding magnetic field along the z-axis (0,0,z). After those indices are determined, the field is calculated according to the formula:

$$B_z(0,0,z) = \frac{A_y(\Delta x,0,z) - A_y(-\Delta x,0,z)}{2\Delta x} + \frac{A_x(0,-\Delta y,z) - A_x(0,\Delta y,z)}{2\Delta y}$$

4.4 Curve fitting:

As already discussed, if we don't consider 2π as a current element angle then the z-component of the magnetic field will be theoretically zero. Hence, the plot we obtain will be only due to the computer precision error. However, if we include 2π then we will have a proper plot of $B_z(0,0,z)$ as a function of z. Hence, in order to find the dependence of the field on z, we are fitting a mathematical model for it. The model is:

$$B_z(z) = cz^b$$

In logarithmic form, we can write this as:

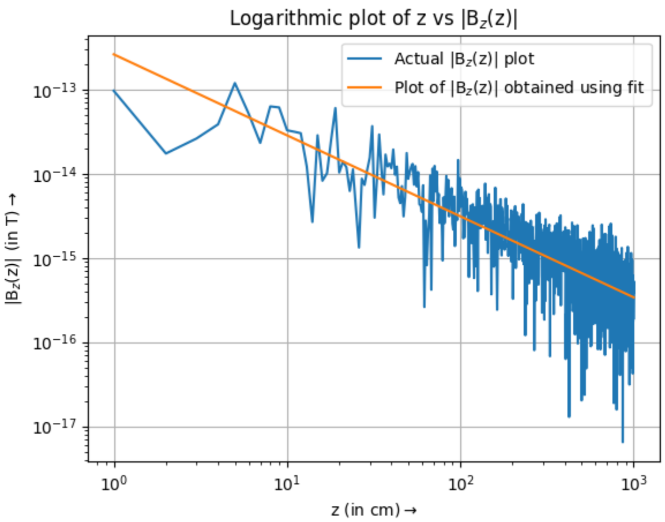
$$log(B_z(z)) = log(c) + b.log(z)$$

$$\begin{pmatrix} 1 & log(z_1) \\ 1 & log(z_2) \\ 1 & log(z_3) \\ .. & .. \\ .. & .. \\ 1 & log(z_{1000}) \end{pmatrix} \begin{pmatrix} log(c) \\ b \end{pmatrix} = \begin{pmatrix} log(B_z(z_1)) \\ log(B_z(z_2)) \\ log(B_z(z_3)) \\ .. \\ .. \\ log(B_z(z_{1000})) \end{pmatrix} \text{(Matrix form of the equation. Mx = N)}$$

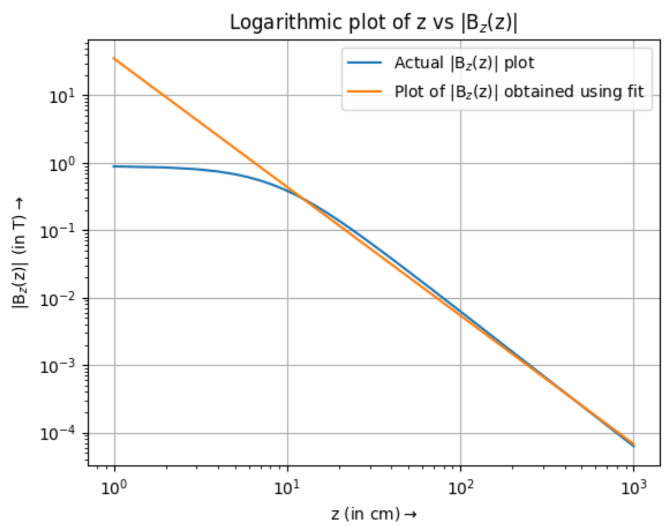
We can use the scipy.linalg.lstsq function for finding the log(c) and b values. The following code snippet implements this least squares fitting algorithm:

```
M = p.c_[zval**0,p.log(zval)]
N = p.log(abs(Bz))
x = sp.lstsq(M,N)[0]
c = p.exp(x[0])
b = x[1]
Bz_fit = c*(zval**b)
```

The matrix M has two columns in which the first column is filled with ones and the second column is filled with logarithms of z values. The matrix N is made up of one column which is filled with logarithms of absolute values of $B_z(z)$**.** These matrices are passed into the scipy.linalg.lstsq function and that function returns the parameters $log(c)$ and b. Taking exponent of log(c) will give us c. Now, let us look at the plots:



This is when 2π is not included. As expected, we get zero $B_z(z)$ and the plot we get is due to computer precision error



In this case, 2π is included and hence, we have a proper decreasing trend

Also, the parameters for the above provided plot are:

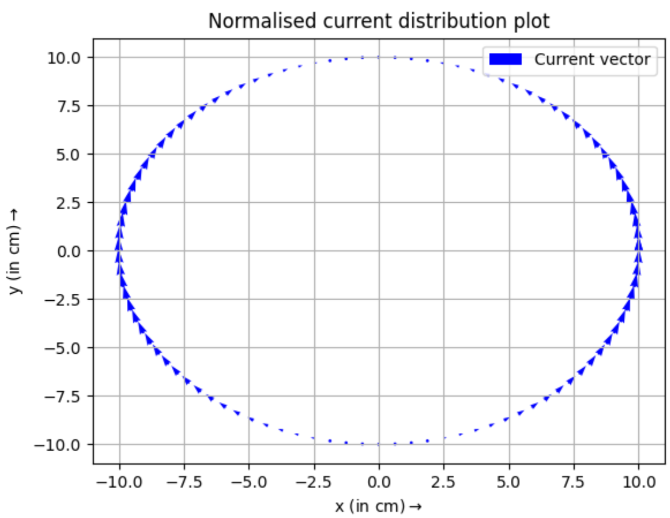
- Estimated value of c: 0.353
- Estimated value of b: -1.906

4.5 Effect of x, y, angle stepsizes and time depedence of current on the structure of the plot:

In order to know how the stepsizes will affect the magnetic field value, we need to know what exactly happens when we include the current element at $\phi = 2\pi$ in the vector potential computation. We already say:

$$\vec{A}(x,y,z) = \sum_{l=0}^{N-1} \frac{cos(\phi_l^{'})e^{-jkR}a(\Delta\phi^{'})(-sin(\phi_l^{'})\hat{x} + cos(\phi_l^{'})\hat{y})}{R}$$

For finding $B_z(0,0,z)$, we need to know A_y at $\pm\Delta x$ and A_x at $\pm\Delta y$. So, let's try to understand what would be the values by looking at the current plot:



For finding A_x on the y-axis, consider the y-axis as a mirror (Axis of reflection). Clearly, At any point on the y-axis, A_x must be zero since an actual point-reflection point pair will provide net zero x-component. For example consider, point $\phi = 60^o$. The reflection of this point would be $\phi = 120^o$. Both these current elements have the same current magnitude, are equidistant from the y-axis point (0,y,z). Hence their x-components get cancelled and thus A_x is zero.

Now for finding A_y along the x-axis, we need to set x-axis as the axis of reflection. Now, if an actual point-reflection point pair is considered, the x-component will be zero and y-component will be non-zero. One example of such a pair is $\phi = 30^o$ and $\phi = 330^o$(or -30^o). Also, by symmetry around the y-axis we can say A_y value will be same at both $+x$ and $-x$. This is the reason why we get a perfect zero for $B_z(0,0,z)$ since the partial derivatives $\frac{\partial A_y}{\partial x}$ and $\frac{\partial A_x}{\partial y}$ are zero because of the above mentioned reasons.

However, when we consider the contribution due to current element at $\phi = 2\pi$, things become different. We already considered the contribution at $\phi = 0$. So when we consider the contribution due to $\phi = 2\pi$, we are actually double counting the contribution due to that element. And that tiny element contributes differently at different x values. On y-axis, the contribution due to that element will be symmetrical and hence, $\frac{\partial A_x}{\partial y}$ will still be zero. However, the term $\frac{\partial A_y}{\partial x}$ will become significant now. The new formula for this term would be:

$$\frac{\partial A_y}{\partial x} = \frac{A_y(+\Delta x, 0, z, \phi = 2\pi) - A_y(-\Delta x, 0, z, \phi = 2\pi)}{2\Delta x}$$

These vector potential terms now have contribution only from $\phi = 2\pi$ since the vector potential summation from $\phi = 0$ to $\phi = 2\pi - \Delta\phi$ would've got cancelled. Now, let's see what these terms are:

$$A_y(+\Delta x, 0, z, \phi = 2\pi) = \frac{cos^2(2\pi)e^{-jkR_1}a(\Delta\phi)}{R_1}$$

$$A_y(-\Delta x, 0, z, \phi = 2\pi) = \frac{cos^2(2\pi)e^{-jkR_2}a(\Delta\phi)}{R_2}$$

Hence:

$$\frac{\partial A_y}{\partial x} = \frac{a(\Delta\phi)(\frac{e^{-jkR_1}}{R_1} - \frac{e^{-jkR_2}}{R_2})}{2\Delta x}$$

where:

- $R_1 = \sqrt{(\Delta x - acos(2\pi))^2 + (0 - asin(2\pi))^2 + z^2} = \sqrt{(\Delta x - a)^2 + z^2}$
- $R_2 = \sqrt{(-\Delta x - acos(2\pi))^2 + (0 - asin(2\pi))^2 + z^2} = \sqrt{(\Delta x + a)^2 + z^2}$

In order to know the trend, let's actually use the integral formula of A_y and find its partial derivative with respect to x. After doing this, we have:

$$\frac{\partial A_y}{\partial x} = -a(\Delta\phi)\frac{(e^{-jkR})(1 + jkR)}{R^2}\frac{\partial R}{\partial x}|_{(x,y,z)=(0,0,z)}$$

$$\frac{\partial R}{\partial x} = \frac{x - a}{R}|_{(x,y,z)=(0,0,z)}$$

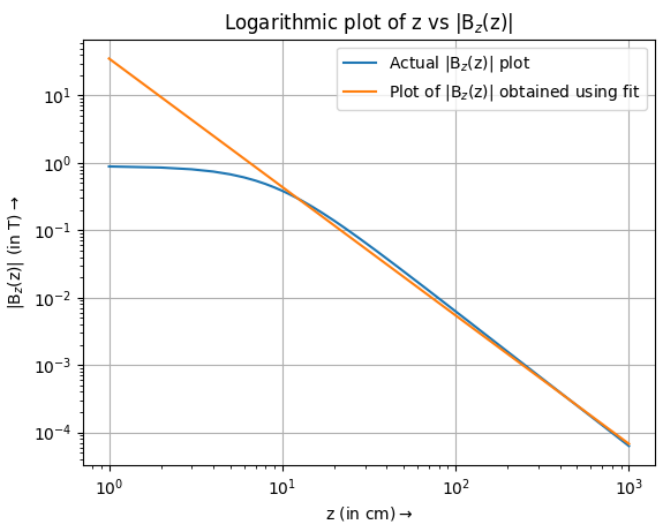
After substituting,we have:

$$\frac{\partial A_y}{\partial x} = a^2(\Delta\phi)\frac{(e^{-jkR})(1 + jkR)}{R^3}|_{(x,y,z)=(0,0,z)} \quad (3)$$

This is the important equation we need to know. It tells us how the magnetic field's z-component will vary with respect to the stepsizes we choose. Now from the formula, we can see that it is complex. Hence, for plotting, we will anyway take the absolute value. Hence the complex exponential won't contribute anything to the magntiude. The magnitude will be:

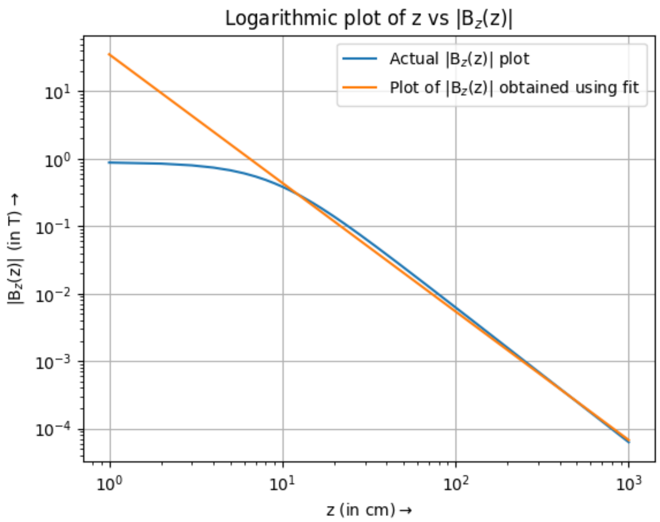
$$|B_z(z)| = a^2(\Delta\phi)\frac{\sqrt{1+k^2R^2}}{R^3}|_{(x,y,z)=(0,0,z)}$$

Clearly, the magnitude does not vary with respect to the stepsize Δx. This is evident from the formula. However, for this to be valid, our stepsize should be much less than the radius a and it is comparitively very less (Maximum $\Delta x = 1$ and radius $a = 10$). This is evident from the below plots:



This is the plot for stepsize $\Delta x = 1cm$ and $\Delta\phi = 2\pi/100$

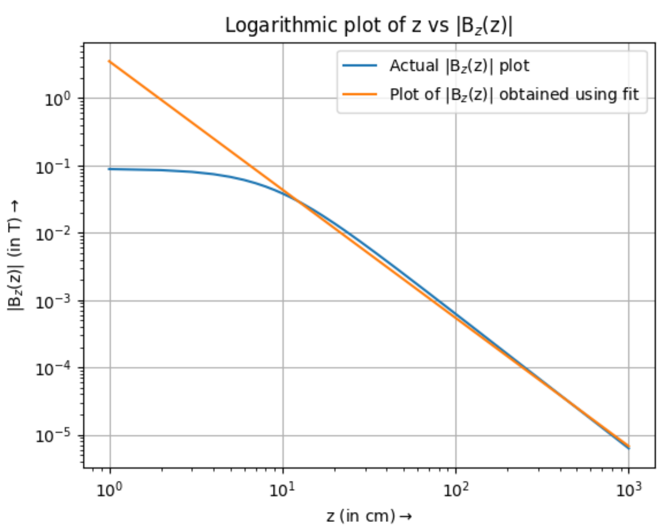
- Estimated value of c: 35.275
- Estimated value of b: -1.906



This is the plot for stepsize $\Delta x = \frac{1}{7}cm$ and $\Delta\phi = 2\pi/100$

- Estimated value of c: 35.288
- Estimated value of b: -1.906

And also from the magnitude expression, we can see that the magnitude is directly proportional to the angle stepsize $\Delta\phi$. Hence, decreasing it will lead to the magnitude getting decreased. This is evident from the plot given below:



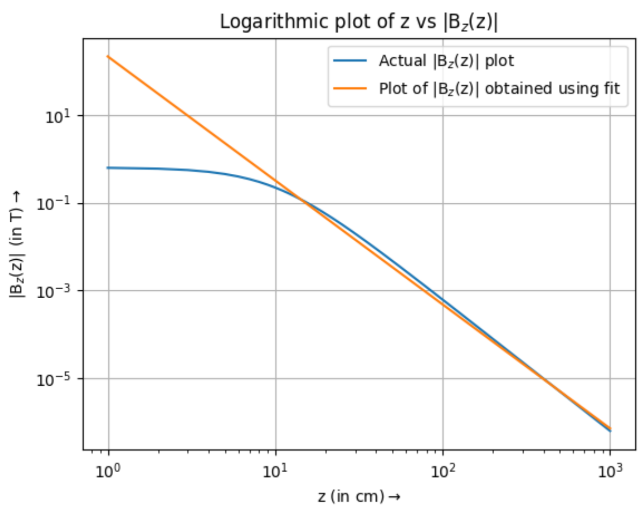
This is the plot for stepsize $\Delta x = 1cm$ and $\Delta\phi = 2\pi/1000$. As expected, the plot for $\Delta\phi = 2\pi/100$ just got displaced below by one logarithmic unit to produce this plot

- Estimated value of c: 3.527
- Estimated value of b: -1.906

In all the above cases, we had a time-varying current and also the power term b was $-1.96 \approx -2$. However, in the static case, we will have a different power term. This is because of the expression of the magnitude:

$$|B_z(z)| = a^2(\Delta\phi)\frac{\sqrt{1+k^2R^2}}{R^3}|_{(x,y,z)=(0,0,z)}$$

with $R = \sqrt{z^2+a^2}$. Hence, for high values of z and non-zero k, the magnitude will vary with respect to z as z^{-2}. However, if k=0, then the magnitude will vary with respect to z as z^{-3}. This is evident from the plot below:

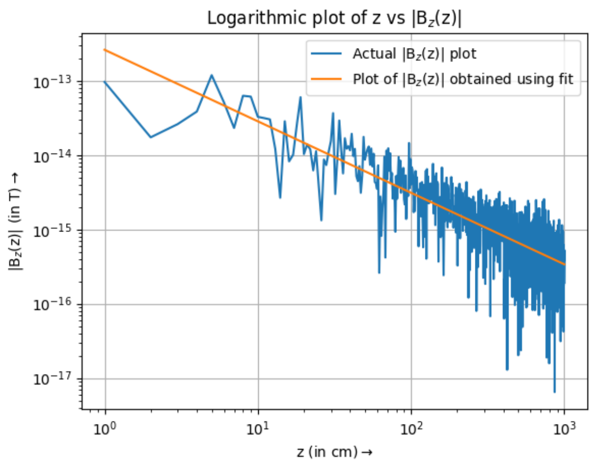


This is the plot for stepsize $\Delta x = 1cm$ and $\Delta\phi = 2\pi/100$ and k=0

- Estimated value of c: 216.134
- Estimated value of b: -2.826

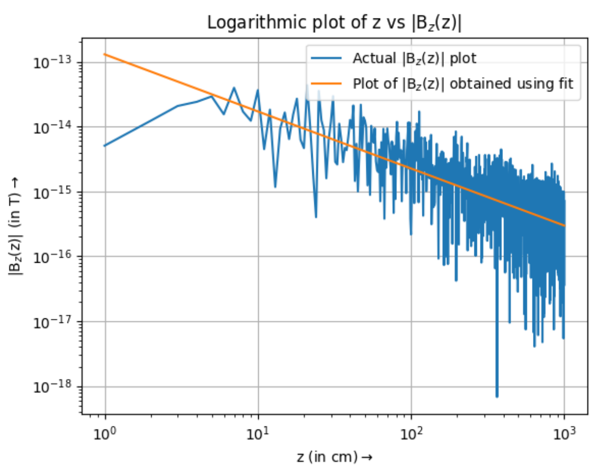
Clearly, $b = -2.826 \approx 3$ as expected.

Finally, let's also see how the time dependence of current affects the plot when we don't include 2π:



This is the plot for stepsize $\Delta x = 1cm$ and $\Delta\phi = 2\pi/100$ and 2π is excluded. Also, it's for the time-varying current

- Estimated value of c: 2.605373e-13
- Estimated value of b: -0.962



This is the plot for stepsize $\Delta x = 1cm$ and $\Delta\phi = 2\pi/100$ and 2π is excluded. Also, it's for the time-independent current

- Estimated value of c: 1.298025e-13
- Estimated value of b: -0.880

Clearly, we don't have a clear trend with respect to z because the computer precision error is too random.

5 Conclusions:

1. When the integral limits are correctly set, i.e., from $\phi = 0$ to $\phi = 2\pi - \Delta\phi$, we get the z-component of magnetic field equal to zero as expected. The plot we get is also because of computer precision error and is not a proper plot.

2. However, if the contribution from $\phi = 2\pi$ is also included, we have a proper plot with $B_z(z)$ decreasing smoothly with respect to z. The decay rate is around -2 as expected for a time varying current.

3. When the stepsize Δx and Δy are changed, we don't have those many changes in the plot. From this observation, we can conclude that the x,y axes stepsizes don't determine the $B_z(z)$ value. As long as they are very much less than the radius of the circular loop, the $B_z(z)$ values would be proper. That's the only constraint we have on these stepsizes.

4. Since the field gets its contribution only from the current element present at $\phi = 2\pi$, it's length matters. Intuitively, we would expect the magnitude of $B_z(z)$ to directly depend on $\Delta\phi$. And it does actually directly depends on $\Delta\phi$ and this is evident from the plots.

5. Finally, when we have a static current distribution, our k becomes zero. Now, we would expect the decay rate to be equal to -3 in theory. The plots and estimated results also reflect that.