EE5175: Image Signal Processing - Lab 4 report

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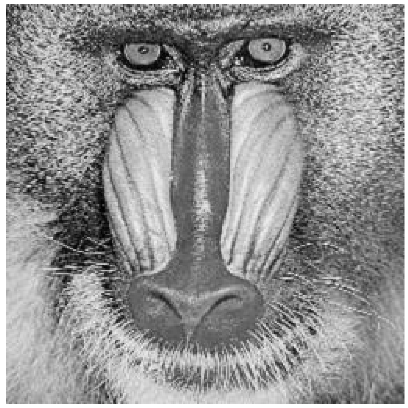
Contents

1 Space-invariant blurring

Aim: Given an image, the blurred versions of it must be generated using Gaussian kernels of different standard deviations.



Input image

2 2-D convolution:

Convolution is a mathematical operation where two signals are taken as inputs and a third signal is produced as an output which captures the overlap between relatively shifted versions of the two input signals. It is denoted as $(*)$. Mathematically, it's given by the following formula:

$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$

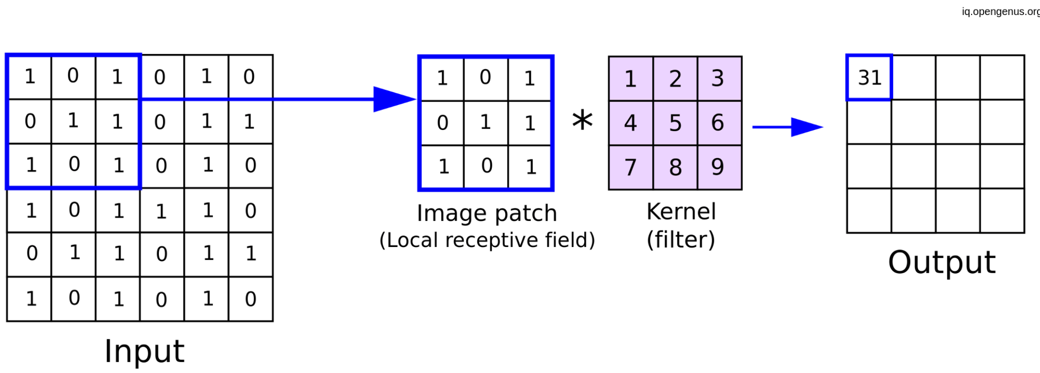
Similar to the continuous variable definition, we have a definition for convolution of discrete time signals:

$$f[x] * g[x] = \sum_{k=-\infty}^{\infty} f[k] \cdot g[x-k]$$

All the definitions provided above are for signals which depend only on one variable. However, images are 2-D signals, i.e., they depend on two independent variables m and n (Row index, Column index). For such 2-D signals, convolution is similarly defined:

$$y[m,n] = x[m,n] * h[m,n] = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} x[i,j] \cdot h[m-i, n-j]$$

Pictorially, it is represented as given below:

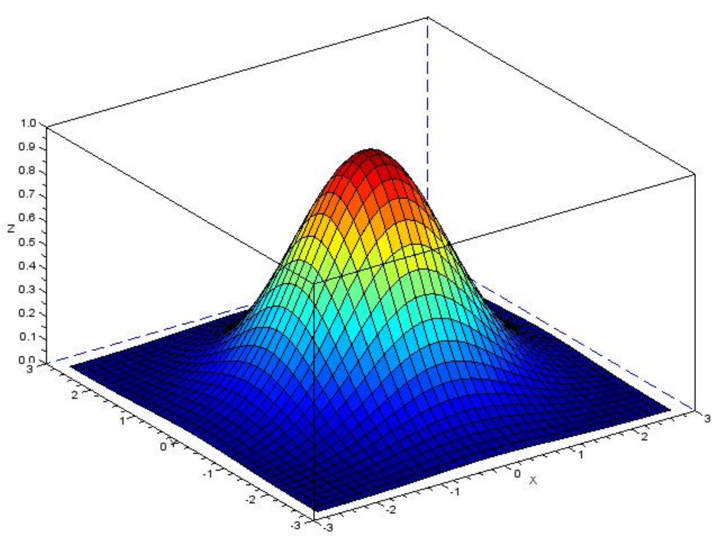


Any **linear, space-invariant (LSI)** system can be thought as a convolution system with a special signal known as impulse response. Let us say the input to the system is $x[m,n]$ and the impulse response of the system is $h[m,n]$. Then the output $y[m,n]$ of the system is given by the convolution formula given above. Usually, the impulse response $h[m,n]$ is called as a **filter** or **kernel** since they perform a filtering operation on the image signal. Blurring is also an LSI operation and hence, can be defined by the kernel matrix. But the blurring kernels must satisfy the following property:

$$\sum_{m=0}^{L_x-1}\sum_{n=0}^{L_y-1} h[m,n] = 1$$

where L_x, L_y are the kernel dimensions. This property must be satisfied since blurring doesn't add or remove intensity.

In this assignment, Gaussian kernels are to be used. Gaussian kernels are kernels whose values are given by the 2-D Gaussian function. The pictorial representation is given below:



In theory, the Gaussian kernel has infinite support. But for practical purposes, we truncate the kernel to dimensions $\lceil 6\sigma+1 \rceil \text{X} \lceil 6\sigma+1 \rceil$ where σ is the standard deviation of the kernel.

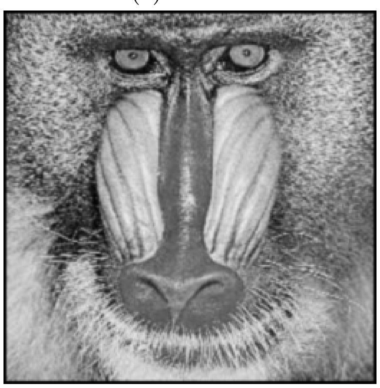
3 Outputs generated by different Gaussian kernels:

Gaussian kernels of standard deviations 1.6, 1.2, 1.0, 0.6, 0.3 and 0 were used to blur the input image. For $\sigma = 0$, there won't be any blur and hence, the output will be the input itself. The results are given below:

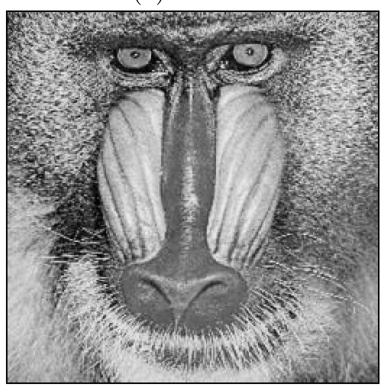
(a) $\sigma = 1.6$

(b) $\sigma = 1.2$

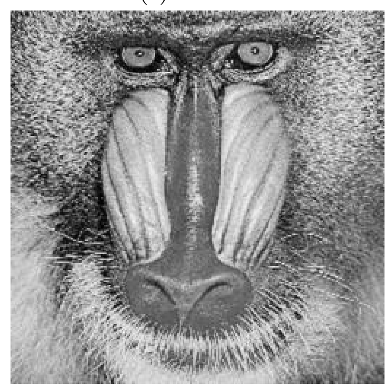
(c) $\sigma = 1.0$



(d) $\sigma = 0.6$



(e) $\sigma = 0.3$



(f) $\sigma = 0$

4 Observations and Conclusions

1. As expected, the blur is more for higher values of σ and the image is very sharp for low σ.
2. From images (a) to (e), there are dark borders around the image. These are generated because of the convolution happening at the image edges. To perform valid convolution, the input image is zero padded. The convolution which happens at these edges will not provide a high intensity value since the signal values are mostly zero here. These low intensity output values show up as dark borders around the image.