EE5175: Image Signal Processing - Lab 11 report

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1 K-means clustering

Aim: To perform K-means clustering on two given images with fixed and random initialization for the cluster centroids. Number of clusters $K = 3$ and Euclidean distance is the distance measure to be used. The data points to be clustered are the pixel color intensity vectors, i.e., $[r\ g\ b]$. Five iterations of the algorithm has to be performed. The input images `car.png` and `flower.png` are shown below:

Input image 1

Input image 2

2 K-means clustering algorithm:

The K-means clustering algorithm is an unsupervised learning algorithm which is used to cluster unlabelled datasets into different clusters. K denotes the number of clusters which is usually pre-defined. It is an iterative algorithm where the cluster centroids are updated in every iteration until convergence is reached.

Let the data points be denoted as x_n where $n = 1, 2, ...N$ (N is the size of the dataset), the cluster centroids after iteration r be denoted as $\mu_1(r), \mu_2(r)...\mu_K(r)$, cluster index of n^{th} data point after iteration r be $k(n, r)$ and the l^{th} cluster be $C_l(r)$. Then the algorithm **(Steps for next iteration)** is given as follows:

$$k(n, r+1) = \arg\min_p ||x_n - \mu_p(r)||^2 \quad (n = 1, 2, ...N)$$

$$C_l(r+1) = \{x_n : k(n, r+1) = l\}$$

$$\mu_l(r+1) = \sum_{x \in C_l(r+1)} \frac{x}{|C_l(r+1)|} \quad (l = 1, 2, ...K)$$

where $|S|$ denotes cardinality of the set S.

Basically, each data point is assigned a cluster index based on its closest cluster centroid. After such an assignment is done for all data points, new clusters are formed by grouping data points with same cluster indices. Once such clusters are formed, new cluster centroids are computed. These steps are iteratively done till convergence is achieved.

A key factor which decides the performance and the convergence of the algorithm is the initialization of the centroids. Usually, when nothing is known about the data, random initialization is done to prevent any bias. But if we have some idea about the data, pre-defined initialization can also be done.

In this particular assignment, both pre-defined and random initializations are tested. The pre-defined cluster centroids are:

$$C_1(0) = [255\ 0\ 0] \ \ (Red)$$

$$C_2(0) = [0\ 0\ 0] \ \ (Black)$$

$$C_3(0) = [255\ 255\ 255] \ \ (White)$$

Random initialization is done 30 times for each image and a cost (C) is computed as follows for each such initialization:

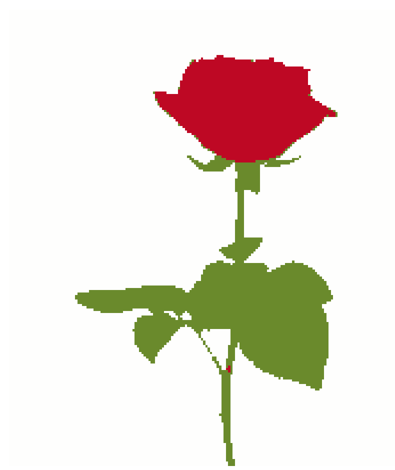
$$C = \sum_{n=1}^{N} ||x_n - \mu_{k(n)}||$$

where $k(n)$ denotes the cluster index and μ_l denotes the cluster mean at the end of the process. Once the costs are computed, the output corresponding to the highest and lowest costs are obtained. To visualize the clustering, each data point is replaced with its cluster centroid and the output image is plotted.

3 Results on the given images:

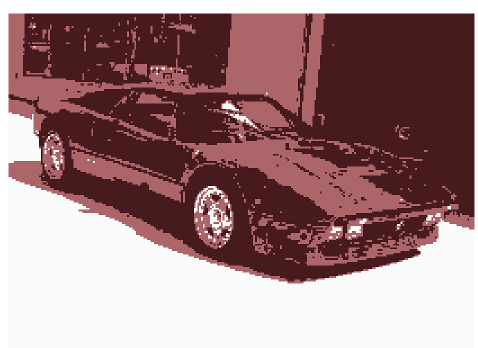
The results obtained on the given input images are shown below:

Clustering image 1 with fixed initialization

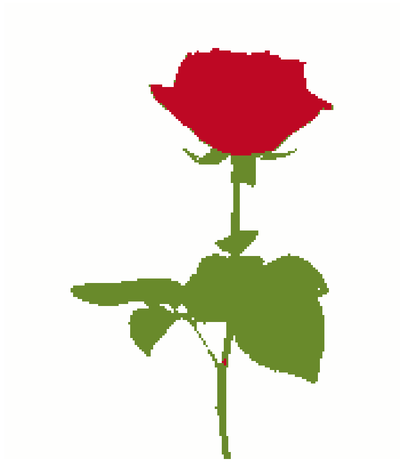


Clustering image 2 with fixed initialization

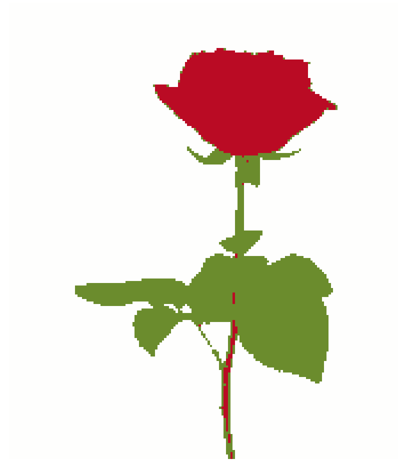
Random output 1 with maximum cost



Random output 1 with minimum cost



Random output 2 with maximum cost

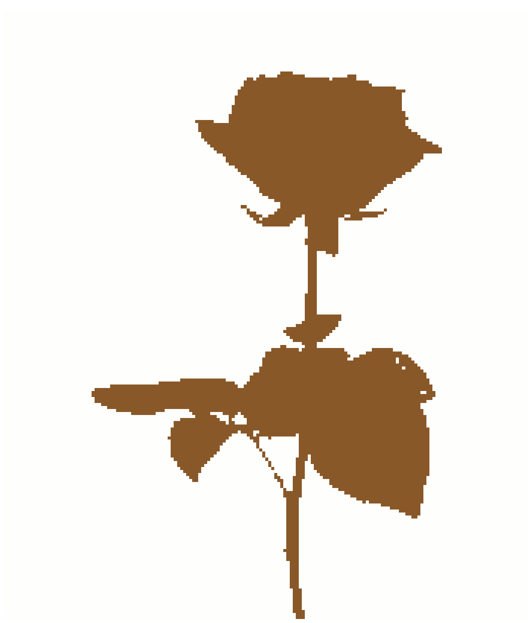


Random output 2 with minimum cost

It seems like fixed initialization has worked fairly well since in both the images, we predominantly have only three distinct colors and hence, three clusters. Red and white are present in both of them and hence, two of the initial cluster centroids are already fairly correct. The last cluster centroid got adjusted accordingly in the respective images.

For the randomly initialized versions, again both the images have produced convincing results. There are not much differences in the outputs of the second image but clear difference is present in the outputs of second images. The output with maximum cost has all 3 clusters and captures most of the details present in the original image. The minimum cost output seems to have only 2 clusters in the end but has considerably more details than the maximum cost output.

Sometimes, the maximum cost output of the second image has only 2 clusters. The output is given below:



It seems like the colors red and green have mixed to produce this mixed output.

4 Observations and Conclusions

1. Fixed initialization works well in the given problem statement since the images have only 3 clusters and the initial cluster centroids are similar to the colors already existing in the images.

2. Random initialization too produced convincing results. While maximum cost outputs are convincing, the minimum cost outputs have finer detail at the expense of cluster loss.

3. The cost defined in this assignment is in a way a direct measure of the intra-class variances. Hence, the cost will be high if there are fewer clusters and vice-versa. This explains why the maximum cost output of second image sometimes only has two clusters (It mainly depends on the initialization though).