EE5175: Image Signal Processing - Lab 7 report

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March 17, 2023

Contents

1 DFT, Magnitude-Phase Dominance, and Rotation Property

Aim: To obtain the 2D DFT matrices of two given images, reconstruct two more images by mixing the magnitude and phase of the obtained DFT matrices. Also, verify the rotation property of 2D DFT for an anti-clockwise rotation of 90°.

(a) Fourier

(b) Fourier transform

(c) Peppers

2 2D Discrete Fourier transform:

Similar to the 1D discrete Fourier transform which is used to find the spectral content of 1D signals, we have a 2D DFT operation for 2D discrete signals such as images. Let's assume an image signal is given as $f(m,n)$ where $m = 0,1,2,...,M-1$ and $n = 0,1,2,...,N-1$. Then the 2D DFT of the image signal is defined as:

$$F(k,l) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)exp(-j2\pi(\frac{km}{M} + \frac{ln}{N}))$$

where $k = 0,1,2,...,M-1$ and $l = 0,1,2,...,N-1$. They denote the frequency index in x and y respectively. 2D DFT is an important transform in image processing since it allows us to understand the spectral content of the image. If the image is sparse in frequency domain, storing it becomes easier by employing its sparsity.

By brute force method, the complexity of this transform will be O(M^2N^2) since there are MN DFT coefficient and to compute each coefficient, MN multiplication operations are required. However, we can reduce the complexity tremendously by using the FFT (Fast Fourier Transform) algorithm. We can re-write the 2D DFT equation as:

$$P(m,l) = \sum_{n=0}^{N-1} f(m,n)exp(-j2\pi\frac{ln}{N})$$

$$F(k,l) = \sum_{m=0}^{M-1} P(m,l)exp(-j2\pi\frac{km}{M})$$

Essentially we've split the 2D DFT as combination of two 1D DFTs which can be found using the FFT algorithm. The first equation corresponds to DFT of each row. Once the DFT vector of each row is computed, for the new matrix (whose rows are the above computed DFT vectors), DFT of each column is taken. The output matrix will be the required 2D DFT matrix.
The complexity of this method will be O($MN\log M\log N$) since now each row DFT computation will take $logN$ computations (M rows are there) and each column DFT computation will take $logM$ computations (N columns are there).

3 Rotated 2D DFT:

Rotated 2D DFT is obtained by simply rotating the input image co-ordinate vector. Let us assume that the input image is of dimensions NXN. Let us define the following:

$$\underline{k} = [k, l]^T$$

$$\underline{m} = [m, n]^T$$

$$R = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

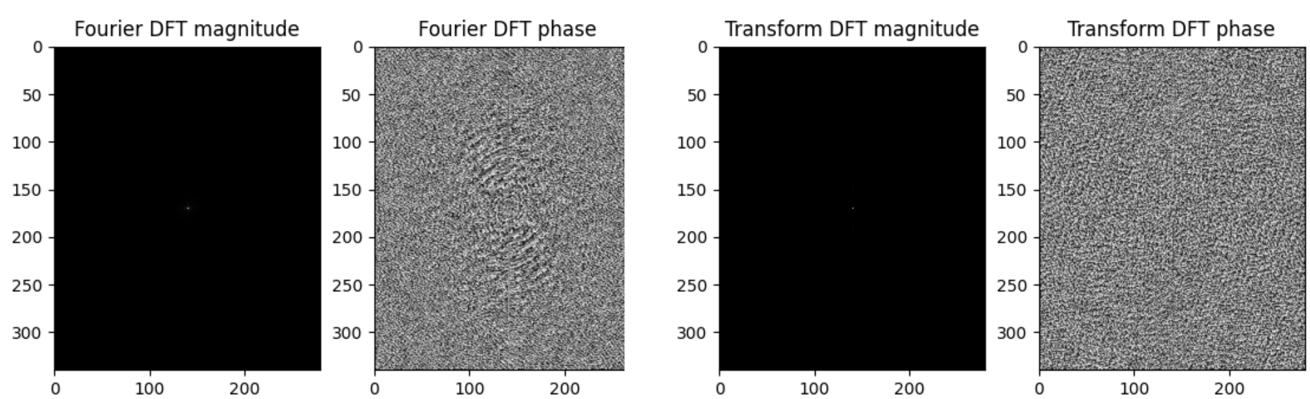
where $\underline{m}$ is the input image co-ordinate vector, $\underline{k}$ is the frequency co-ordinate vector and R is the rotation matrix (Anti-clockwise direction). With these definitions in hand, the rotated 2D DFT transform is given as:

$$F(k, l) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(m, n) exp(-j \frac{2\pi}{N} \underline{k}^T R \underline{m})$$

We can easily prove that 2D IDFT of this DFT matrix will give the rotated version of the input image. This works well for rotation angles that are multiples of 90°since such rotations preserve the X and Y axes as horizontal and vertical axes. For any other rotation angle, the output image obtained will not replicate the actual rotated input image.

4 Results on the given images:

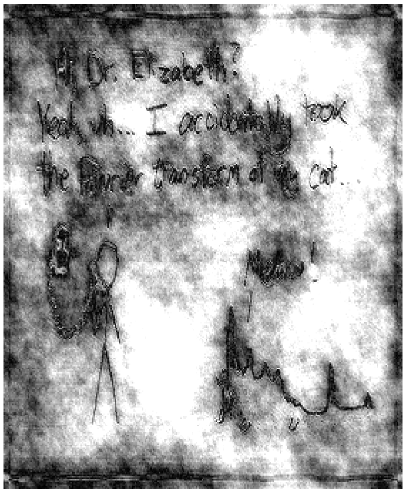
The 2D DFT matrices were obtained for the `fourier.png` and `fourier_transform.png` images and the plots are shown below:



(a) Fourier DFT

(b) Transform DFT

It seems like most of the frequency components are contained around the DC component. High frequency components are very negligible compared to the low frequency components. Also, as asked in the assignment, Magnitude-Phase mixing was done to obtain two new images. Those images are shown below:



(a) Fourier magnitude - Transform phase

(b) Fourier phase - Transform magnitude

Phase dominates the output since in both the images, the one which contributed the phase term is only visible in the image. In the first image, it is not so clear but in the second image, it's very much visible. The image of Fourier can be clearly seen. And finally, the output corresponding to IDFT of the rotated DFT is shown below:

Rotated peppers

As expected, the IDFT of the rotated DFT produces a rotated version of the input image. Note that the rotation is in anti-clockwise direction as expected.

5 Observations and Conclusions

1. For both `fourier.png` and `fourier_transform.png`, the frequency content is heavily concentrated around the DC frequency. It can be explained from the fact that there are no sudden oscillations in the images. The images are continuous and smooth at most of the pixels.

2. Phase seems to have a dominant effect over magnitude when mixing is done. The image which contributed phase is only visible in the mixed image. Magnitude just has a distorting effect.

3. The IDFT of a rotated 2D DFT gives the rotated version of the input image as expected and has been verified for the `peppers_small.png` image.