EE6110: Project presentation

Paper:

A Family of Robust M-Shaped Error Weighted Least Mean Square Algorithms: Performance Analysis and Echo Cancellation Application

Paper review by:

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Abstract:

Least mean square (LMS) algorithm is a popular adaptive algorithm that works well in determining the coefficients of an LTI system under different conditions. Several variants of this algorithm are proposed in literature and they are broadly classified into three categories based on the properties of the error weighing function as:

- V-Shaped
- Λ-shaped
- M-shaped

A robust M-shaped algorithm has been proposed in the mentioned paper and simulations are conducted to test its performance.

LMS algorithm:

It is an iterative, statistical algorithm using which, we can estimate the coefficients of a linear model. For example, let's consider the LTI model with input-output relation as:

$$y_n = \boldsymbol{w}_{opt}^T \boldsymbol{x}_n + v_n$$

Where x_n = [x(n) x(n-1) … x(n-L+1)]^T , $\boldsymbol{w}_{opt}^T$ = [w_0, w_1, …, w_{L-1}] are the system filter taps. V_n is additive noise (Typically white gaussian). Estimation error at time 'n' is defined as:

$$e_n = y_n - w^T \boldsymbol{x}_n$$

‘**w**’ is the adaptive filter. The general LMS algorithm is given as:

$$w_{n+1} = w_n + uf(e_n)e_n x_n$$

Where $f(e_n)$ is known as the **Error weighing function.** For standard LMS algorithm, this function is unity. Depending on this function, LMS algorithms are divided into three categories:

- **V-shaped algorithms**:

  $f(e_n)$ is non-decreasing function, i.e., $f'(e_n) \geq 0$

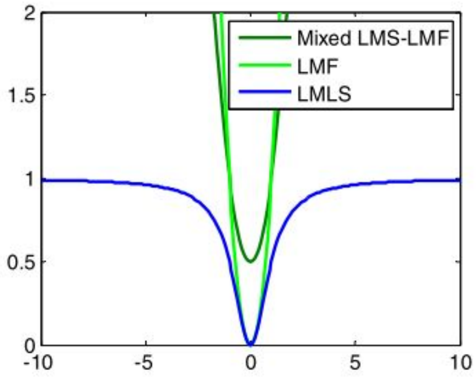
- **Λ-shaped algorithms**:

  $f(e_n)$ is non-increasing function, i.e., $f'(e_n) \leq 0$

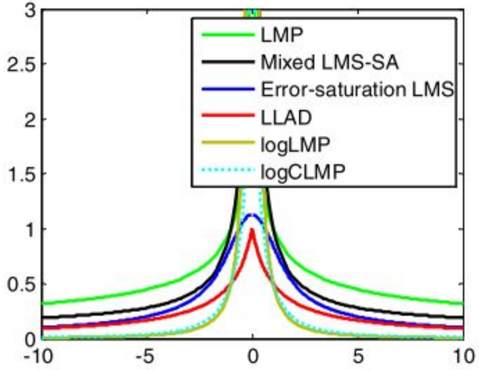
- **M-shaped algorithms**:

  $f(e_n)$ has mixed characteristics. Two sub-categories are present within M-shaped algorithms: Non-inverted M-shaped and inverted M-shaped.

V-shaped functions



Λ-shaped functions



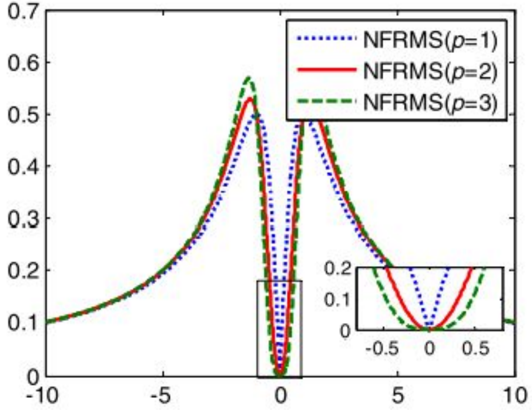
Robust M-shaped variant:

In this paper, the model proposed is known as the **“New family of Robust M-shaped functions (NFRMS)”**. The error weighing function for this model is given as:

$$f\left(e_n\right)=\frac{\left|e_n\right|^p}{\varepsilon+\left|e_n\right|^{p+1}}, \quad p>0$$

where ε is a positive design parameter. The plots of the function for ε=1 and different values of ‘p’ are given beside:

$$J\left(e_n\right)=\int_0^{e_n} \frac{x\left|x\right|^p}{\varepsilon+\left|x\right|^{p+1}} dx$$



NFRMS-LMS – Steady State Analysis:

For the proposed NFRMS-LMS algorithm with ***p=2*** and $\boldsymbol{v_n = w_{opt} - w_n}$:

i. If $|v_n|^3$ is sufficiently small such that $|v_n|^3 << \varepsilon$, the **steady state MSD** is -

$$MSD = \mu L E\left\{|\upsilon_n|^6\right\} / \left(6\varepsilon E\left\{|\upsilon_n|^2\right\}\right)$$

ii. If the noise term $|v_n|^3$ is sufficiently large *i.e* $\varepsilon << |v_n|^3$, the **steady state MSD** is -

$$MSD = \mu L\left(E\left\{|\upsilon_n|^{-2}\right\} + 2\varepsilon E\left\{|\upsilon_n|^{-4}\right\}\right)^{-1}$$

In case (i), for large noise, the steady-state MSD depends on the negative order moments of noise. In case (ii), small noise activates the the higher-order moments of steady-state MSD.

The steady-state MSD acts like a V-shaped or Λ-shaped function depending on the magnitude of noise

Acoustic Echo Cancellation (AEC):

Using the proportionate sign algorithm method and the proposed robust M-shaped function, the proportionate normalized robust M-shaped (PNRMS) algorithm is proposed for AEC. The update equation is given by -

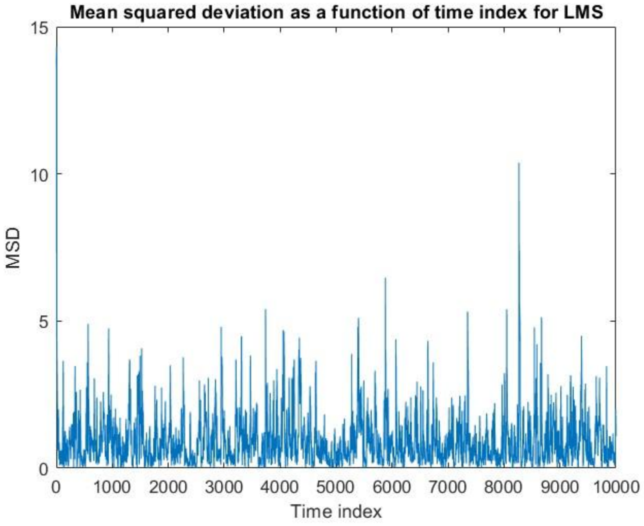
$$\boxed{\hat{\mathbf{w}}_{n+1} = \hat{\mathbf{w}}_n + \mu \frac{|e_n|^p}{\varepsilon \, \|\mathbf{x}_n\|_{G_n}^{p+1} + |e_n|^{p+1}} \frac{e_n G_n \mathbf{x}_n}{\|\mathbf{x}_n\|_{G_n}}}$$

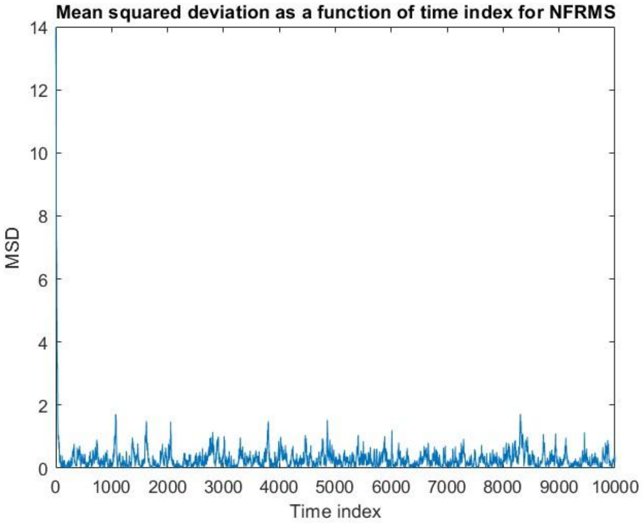
The stochastic cost-function from M-shaped variant is proportionately normalized by *a posteriori error* $\mathbf{e}_\mathbf{p}$ and *a proportionate matrix* $\mathbf{G}_\mathbf{n}$ such that -

$$\min J_{AC} = \left\| \hat{\mathbf{w}}_{n+1} - \hat{\mathbf{w}}_n \right\|_{G_n}^2 + \mu \int_0^{\frac{e_{p,n}}{\|\mathbf{x}_n\|_{G_n}}} \frac{x\,|x|^p}{\varepsilon + |x|^{p+1}} dx$$

MATLAB Simulation Analysis - System identification:

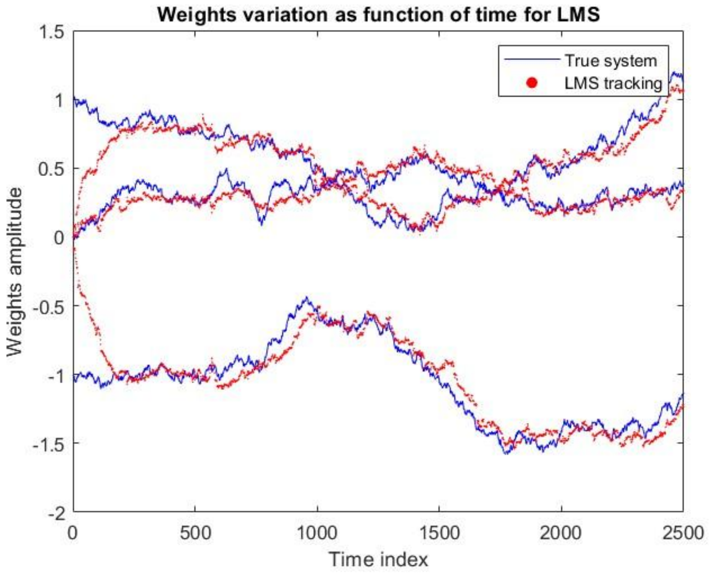
The NFRMS algorithm is tested for system identification and tracking. The system is a causal LTI filter with taps {1,2,3}. A random gaussian signal of variance 0.5 is sent. The output is also corrupted by a laplace noise of variance 5.

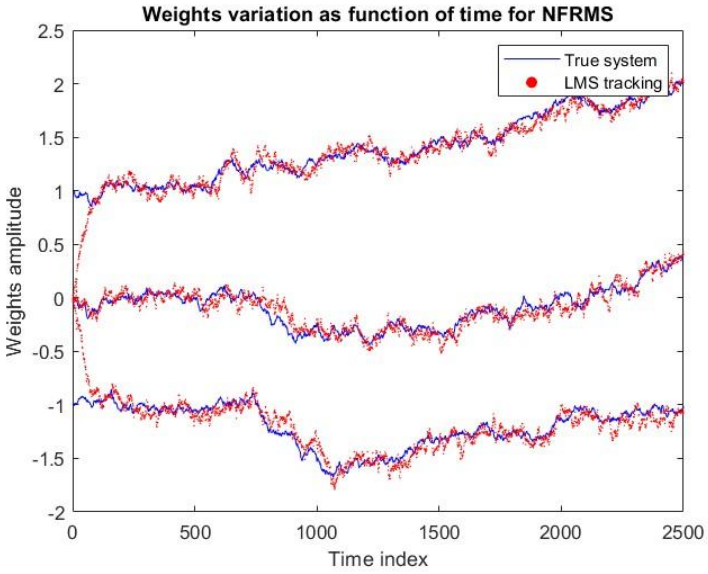




MATLAB Simulation Analysis - System tracking:

In this problem statement, the system is again a causal LTI filter but now the taps change with time (Random walk model). A laplace noise of variance 0.1 is used to model the measurement noise.





MATLAB Simulation Analysis - Noise distribution:

Three algorithms - LMLS, LLAD and NFRMS, are tested for three different mixtures of noise: 20% uniform and 80% laplace, 50% uniform and 50% laplace and 80% uniform and 20% laplace (LMLS is a V-shaped algorithm and LLAD is Λ-shaped).