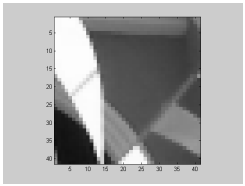
Pattern Matching Using Correlation

- Correlation: Finding one image within another

Template



Image

Pattern Matching Using Correlation

- A good match has low least-squares error
- A good match between image **a** and template **p** is given in terms of metric:

$$d = \Sigma(\mathbf{a} - \mathbf{p})^2$$

$$d = \Sigma\mathbf{a}^2 - 2\Sigma\mathbf{ap} + \Sigma\mathbf{p}^2$$

- For a good match
  - d is small
  - Implies $\Sigma\mathbf{ap}$ is large

Correlation

- Shift **p** over all possible locations of **a** and compute match Σ**ap**
- This method is called:
  - Unnormalized Correlation
  - Unnormalized Cross Correlation
  - Matched Filtering
  - Template Matching
- Summation is done over support of **p**

Correlation

- Problems
  - if **a** is large over the template support, Σ**ap** is also large, even though a good match may not exist
  - if **a** and **p** have a large number of zeros in them, (good match of zeros), Σ**ap** will not reflect that match

Correlation

- Solution
  - Correlation may be normalized in some manner

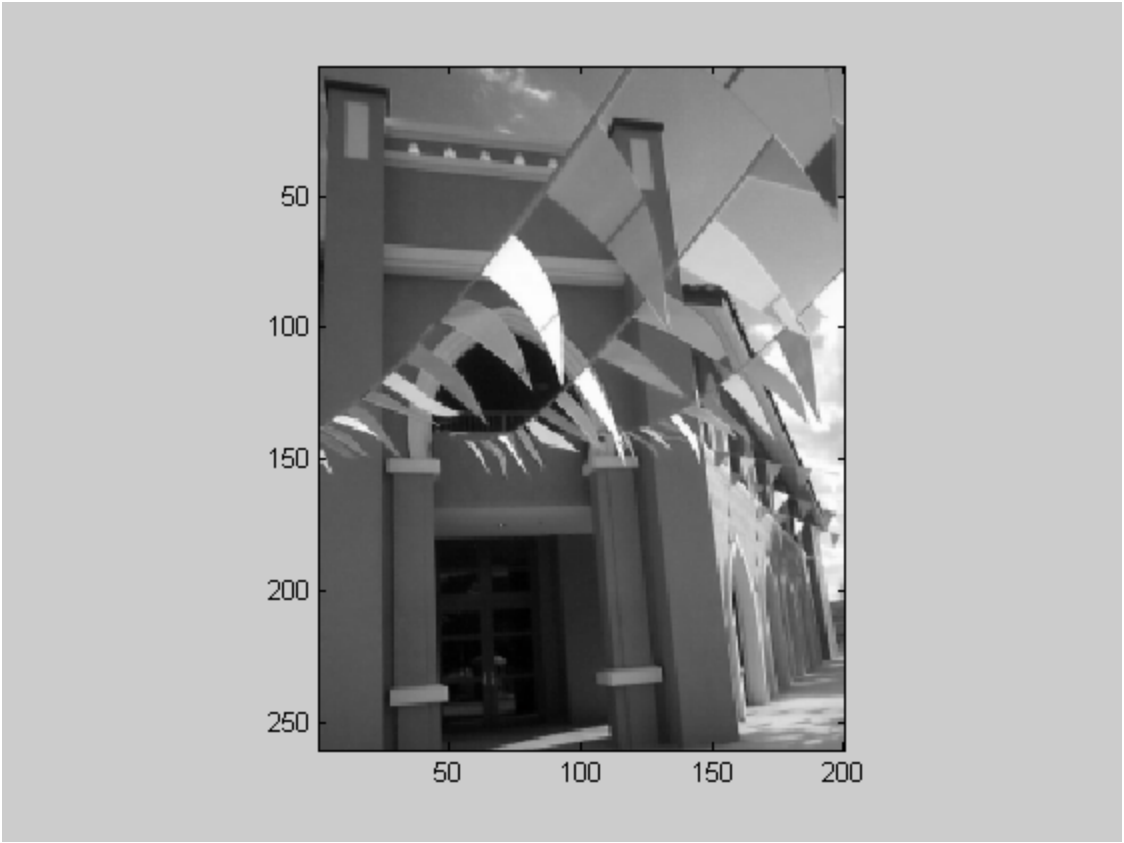
$$c = 1/\alpha \ \Sigma \mathbf{ap}$$

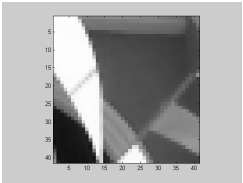
  - where $\alpha = \Sigma \mathbf{a}$ (over support of **p**)
  - In another method:

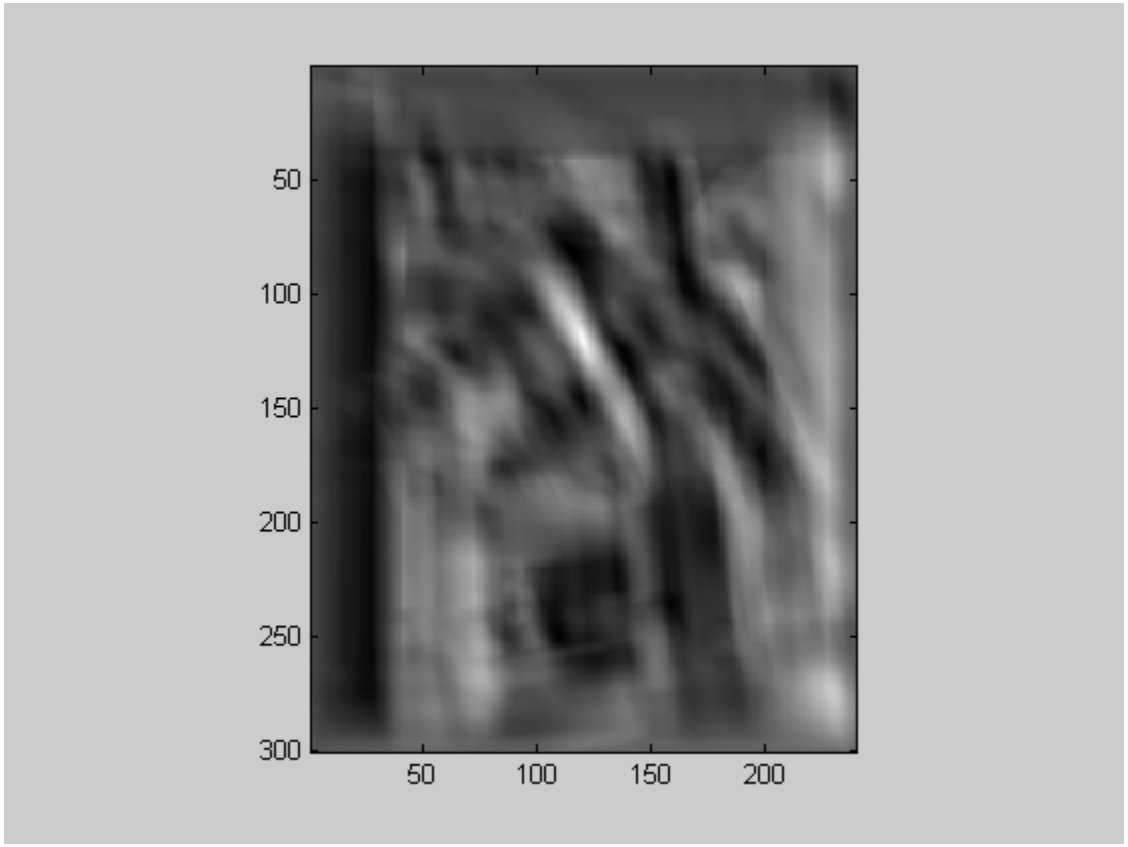
$$\alpha = \left(\sum \mathbf{a}\right)^{\frac{1}{2}} \left(\sum \mathbf{p}\right)^{\frac{1}{2}}$$

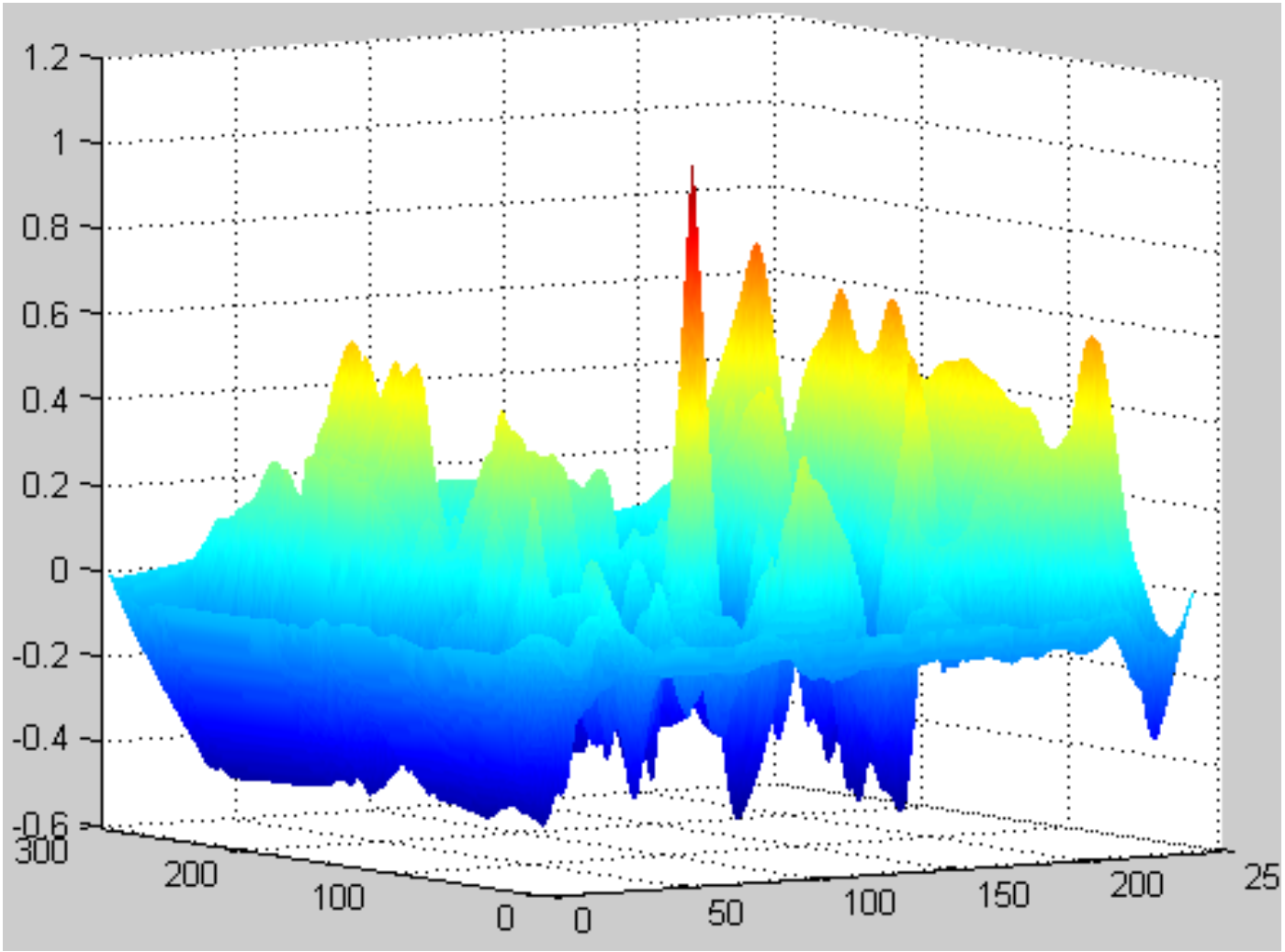
  - results in normalized correlation between 0 and 1
    - Alternate normalization schemes may normalize between –1 and 1
    - 1 indicates perfect match
    - -1 indicates –ve correlation (inverted image)
    - 0 indicates no correlation at all

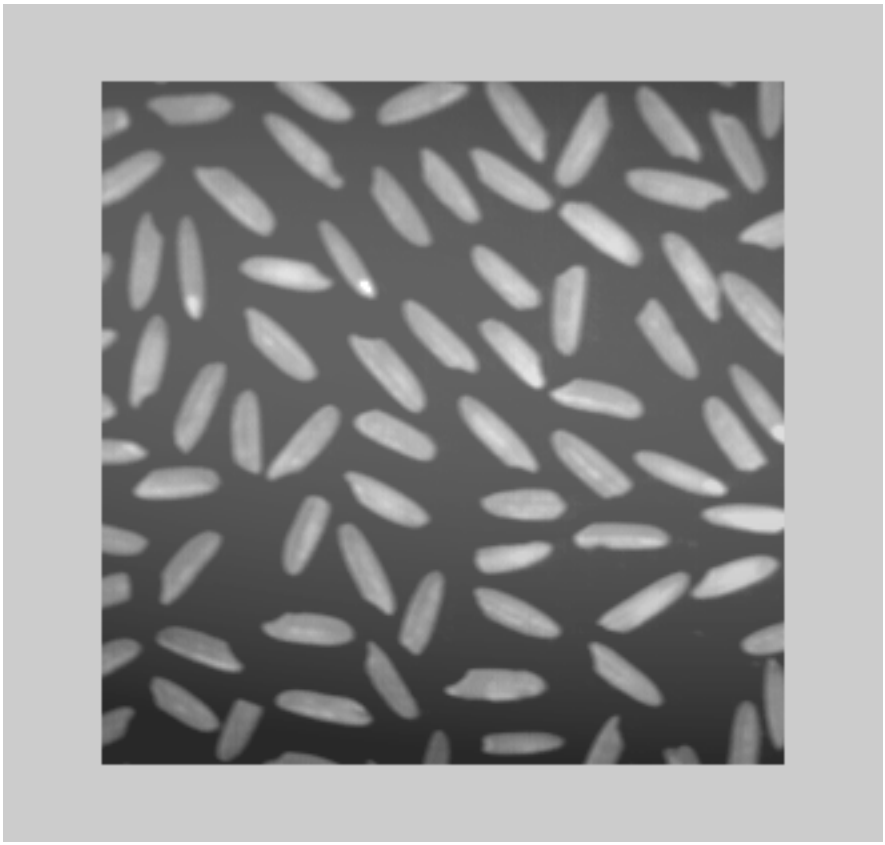
Example

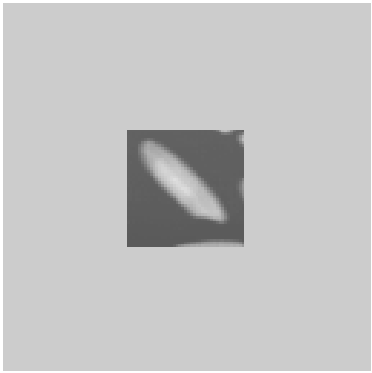


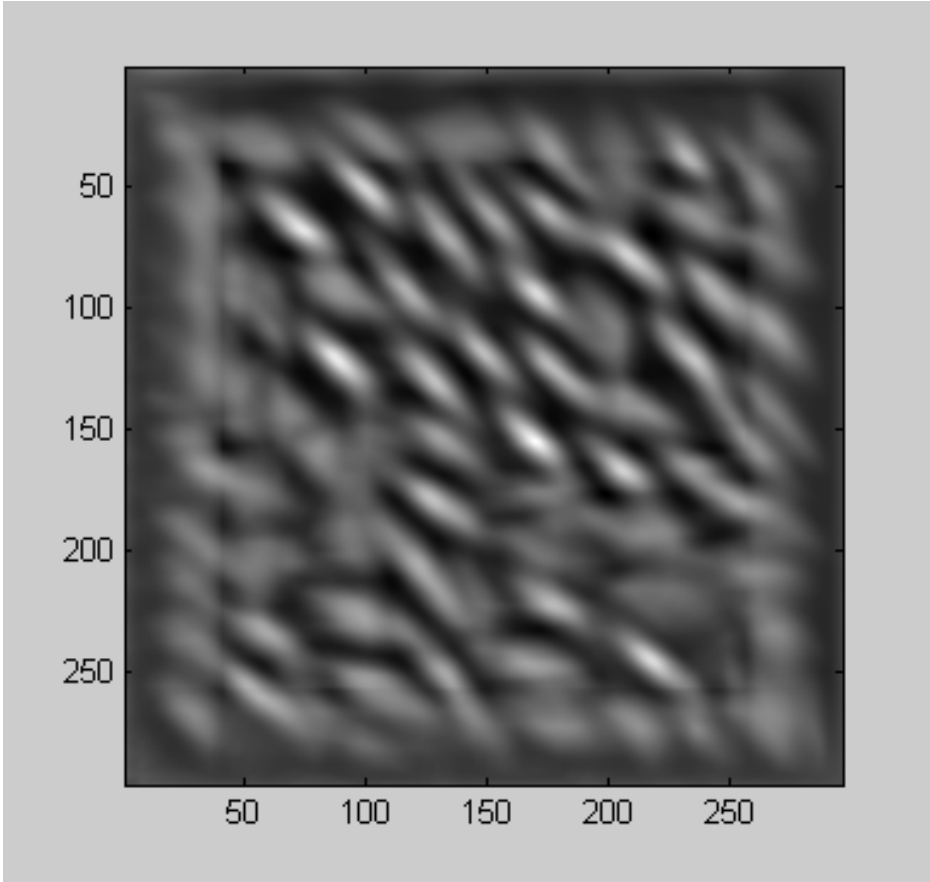



50
100
150
200
250
300
50
100
150
200


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
300
200
100
0
0
50
100
150
200
25






50
100
150
200
250
50
100
150
200
250

Correlation Methods

- Sum of Squared Differences (SSD)

$$(U(x,y),V(x,y)) = \arg\min_{\substack{u=0\ldots-8 \\ v=0\ldots8}} \sum_{i=0}^{-7}\sum_{j=0}^{7}\left(f_k(x+i,y+j) - f_{k-1}(x+i+u,y+j+v)\right)$$

- Minimum Absolute Difference (MAD)

$$(U(x,y),V(x,y)) = \arg\min_{\substack{u=0\ldots-8 \\ v=0\ldots8}} \sum_{i=0}^{-7}\sum_{j=0}^{7}\left|\left(f_k(x+i,y+j) - f_{k-1}(x+i+u,y+j+v)\right)\right|$$

Correlation Methods

- Maximum Matching Pixel Count (MPC)

$$T(x,y;u,v) = \begin{cases} 1 & \text{if } |f_k(x,y) - f_{k-1}(x+u,y+v)| \le t \\ 0 & \text{Otherwise} \end{cases}$$

$$(U(x,y),V(x,y)) = \arg\max_{\substack{u=0,\ldots,-8 \\ v=0,\ldots,8}} \sum_{i=0}^{-7} \sum_{j=0}^{7} T(x+i,y+j;u,v)$$

- Cross Correlation

$$(U,V) = \arg\max_{\substack{u=0,\ldots,-8 \\ v=0,\ldots,8}} \sum_{i=0}^{-7} \sum_{j=0}^{7} \left(f_k(x+i,y+j)\right).\left(f_{k-1}(x+i+u,y+j+v)\right)$$

Correlation Methods

- Normalized Cross Correlation

$$(U,V) = \arg\max_{\substack{u=0\ldots-8 \\ v=0\ldots8}} \frac{\sum_{i=0}^{j=-7}\sum_{j=0}^{7}\left(f_k(x+i,y+j)\right).\left(f_{k-1}(x+i+u,y+j+v)\right)}{\sqrt{\sum_{i=0}^{-7}\sum_{j=0}^{7} f_{k-1}(x+i+u,y+j+v).f_{k-1}(x+i+u,y+j+v)}}$$

- Mutual Correlation

$$(U,V) = \arg\max_{\substack{u=0\ldots-8 \\ v=0\ldots8}} \frac{1}{64\sigma_1\sigma_2}\sum_{i=0}^{-7}\sum_{j=0}^{7}\left(f_k(x+i,y+j)-\mu_1\right).\left(f_{k-1}(x+i+u,y+j+v)-\mu_2\right)$$

Use of Correlation

- Correlation provides motion vector at the location
- Translation only model
- If deviation of image from translation-model is large, correlation of whole image will fail
- Correlation over small areas may still work

Correlation using Pyramids

- Same idea as that of Lucas-Kanade with pyramids
- Find the best match at the highest level
- At the next level, search only around the answer from the previous level
- Another efficient way to do correlation for large windows is to use the FFT (not part of this course)