Exercise 3 - Efficient Algorithms

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Ex. 1

Let X be a random variable describing the number of l-sequences with $l = 1 + \log_2 n$. Let X_i be a indicator random variable describing whethe an l-sequence begins at index/toss i or not.

Since the coin gets tossed $n + \log_2 n$ times and an l-sequence is $1 + \log_2 n$ long, the highest index/toss, where an l-sequence can start is n. Therefore

$$X = \sum_{i=1}^{n} X_i \quad \text{and} \quad \mathbb{E}(X) = \sum_{i=1}^{n} \mathbb{E}(X_i)$$

$$= \sum_{i=1}^{n} Pr(X_i)$$

The probability to get L consecutive of either heads or tails is

$$\underbrace{2}_{\text{head- or tail-sequence}} \cdot \left(\frac{1}{2}\right)^{L} = 2 \cdot \left(\frac{1}{2}\right)^{1+\log_2 n} = \left(\frac{1}{2}\right)^{\log_2 n} = \frac{1}{n}$$

$$\Rightarrow \mathbb{E}(X) = \sum_{i=1}^{n} \frac{1}{n} = n \cdot \frac{1}{n} = 1$$