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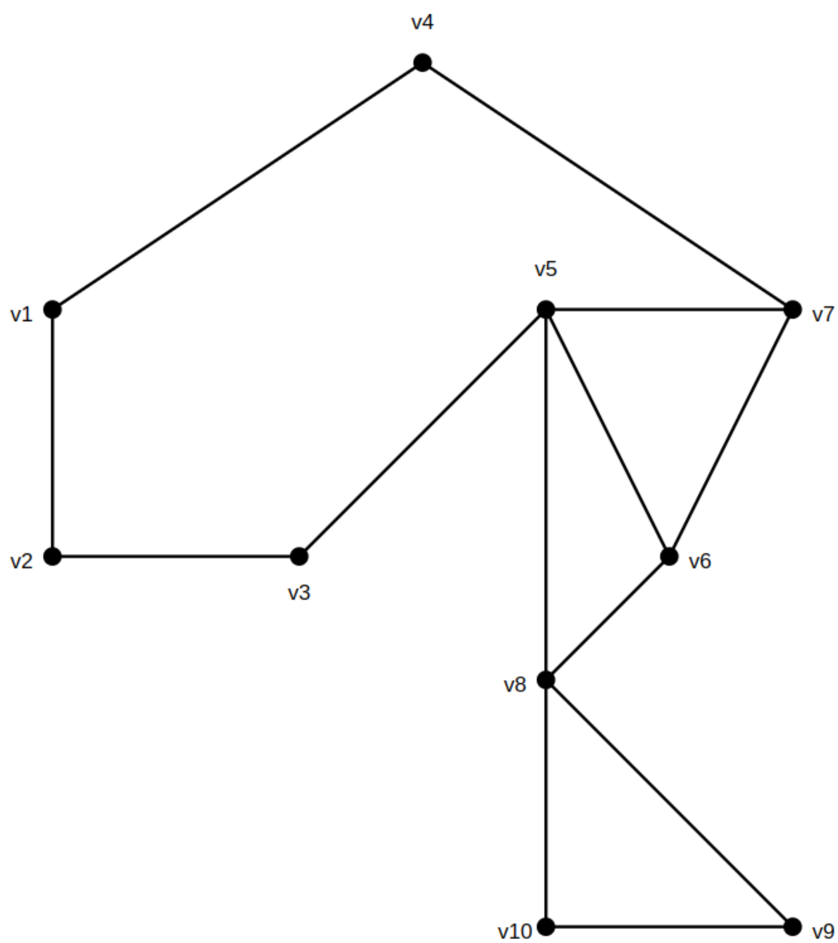
Exercise 02
Efficient Algorithms

Ex 1

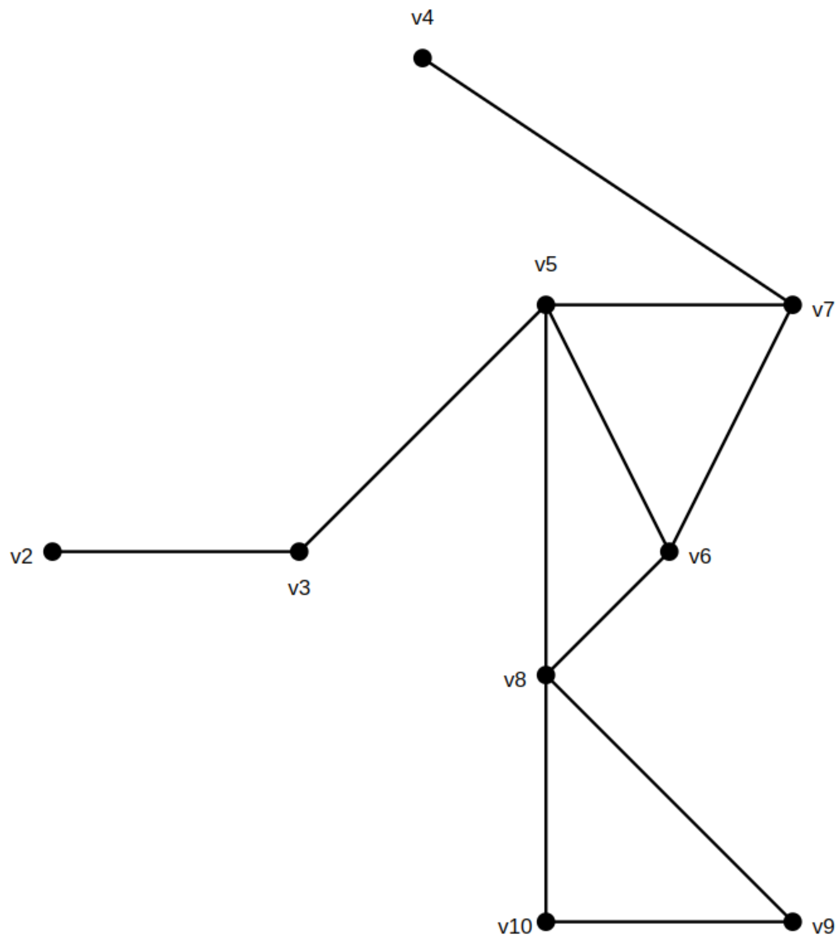
after 1. iteration:

- v11 was deleted.
- S and U are updated since $13/10 > 14/11$



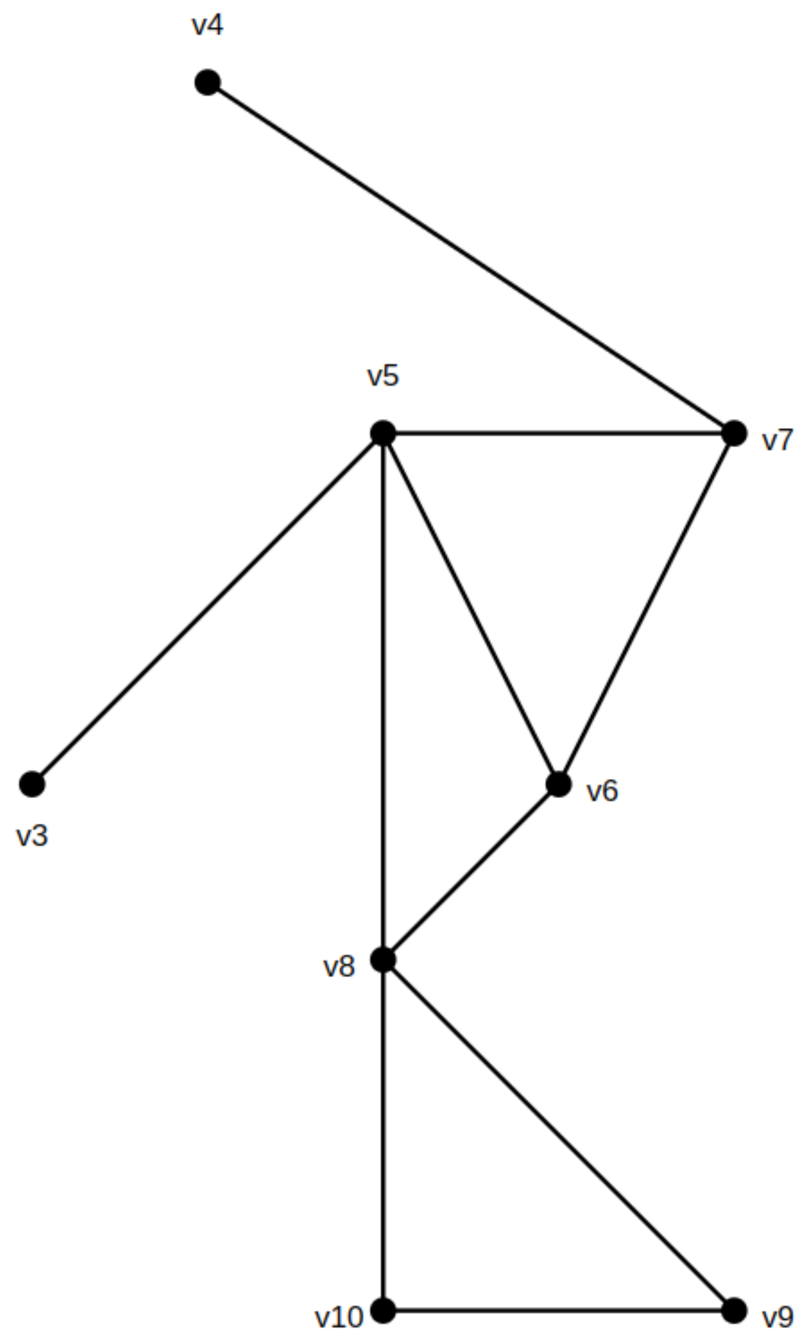
after 2. iteration:

- v1 was deleted.
- S and U are not updated since $11/9 < 13/10$



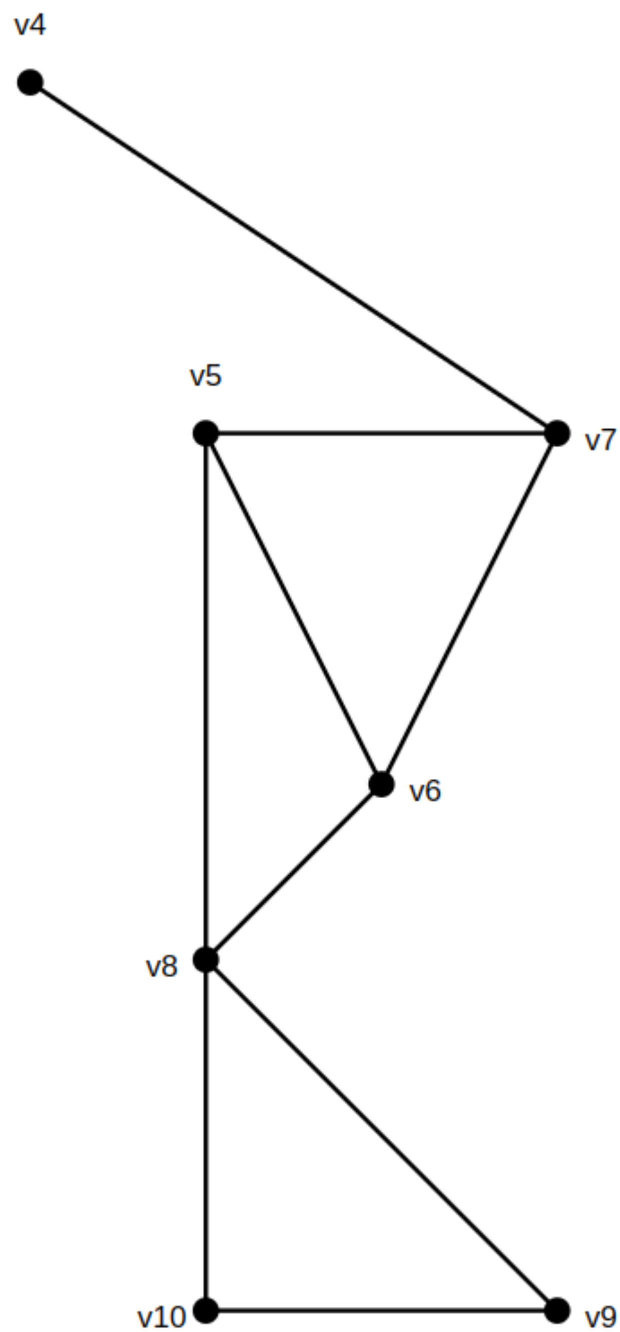
after 3. iteration:

- v2 was deleted.
- S and U are not updated since $10/8 < 13/10$


v4
v5
v7
v3
v6
v8
v10
v9

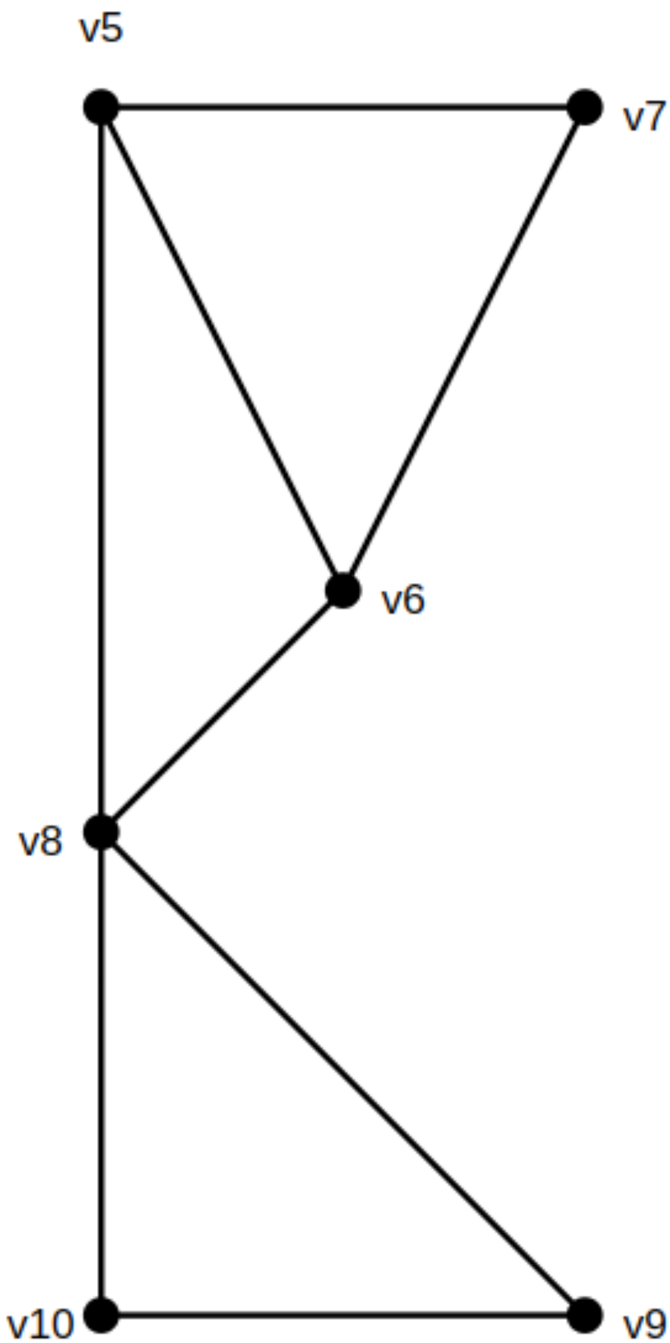
after 4. iteration:

- v3 was deleted.
- S and U are not updated since $9/7 < 13/10$


v4
v5
v7
v6
v8
v10
v9

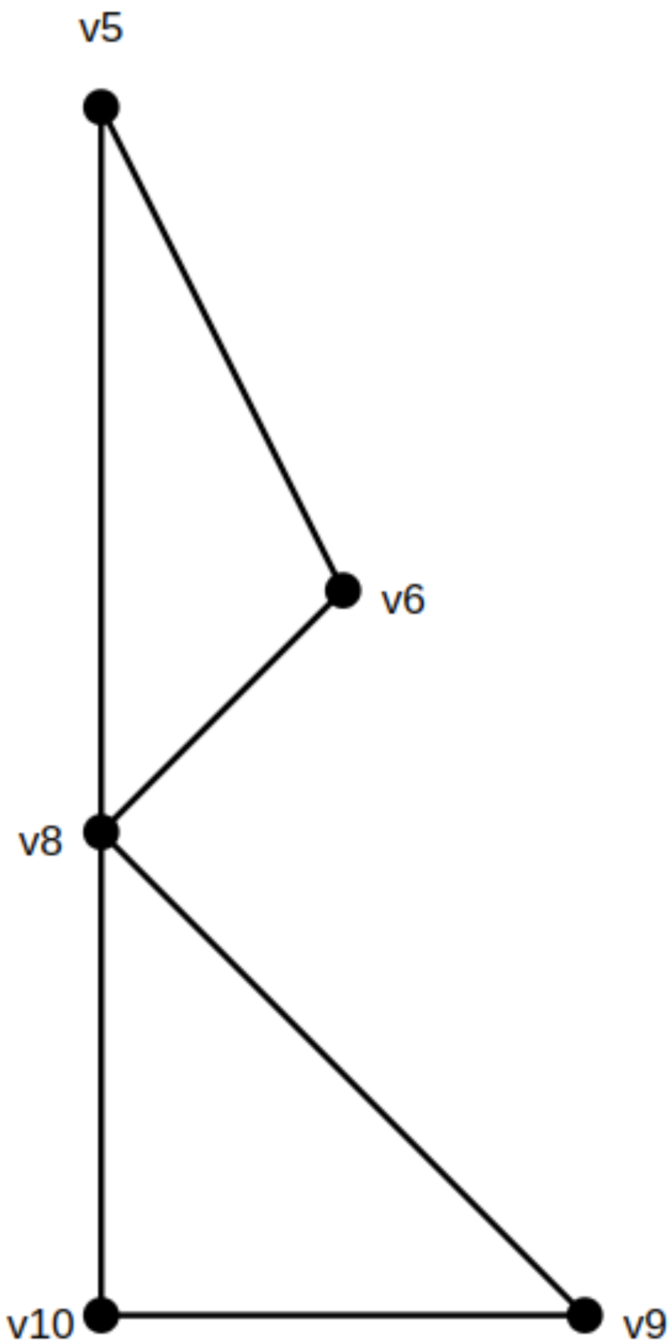
after 5. iteration:

- v4 was deleted.
- S and U are updated since $8/6 > 13/10$


v5
v7
v6
v8
v10
v9

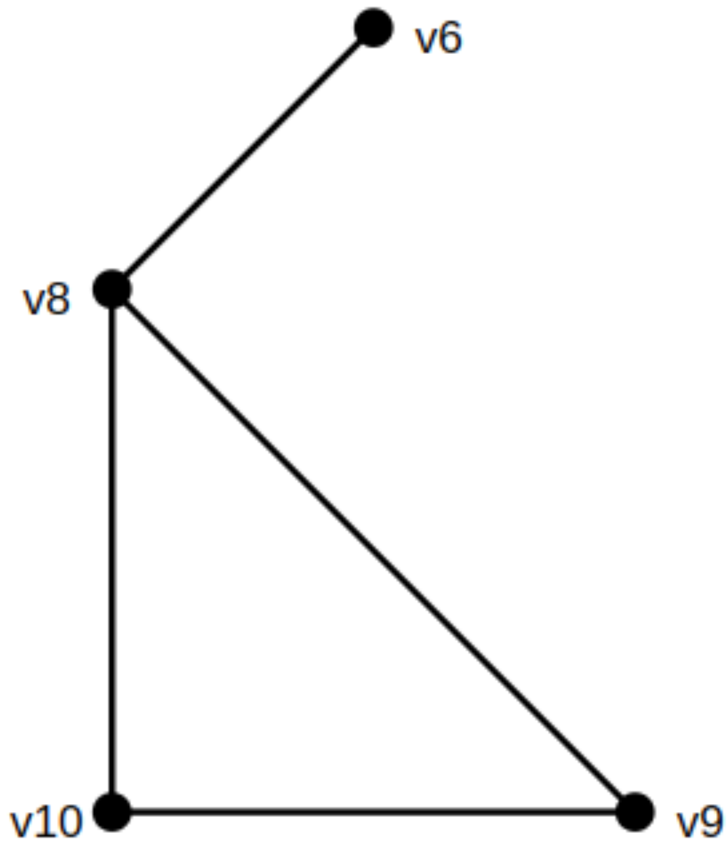
after 6. iteration:

- v7 was deleted.
- S and U arenot updated since $6/5 < 8/6$


v5
v6
v8
v10
v9

after 7. iteration:

- v5 was deleted.
- S and U are not updated since $4/4 < 8/6$



after 8. iteration:

- v6 was deleted.
- S and U are not updated since $3/3 < 8/6$

after 9. iteration:

- v8 was deleted.
- S and U are not updated since $1/2 < 8/6$

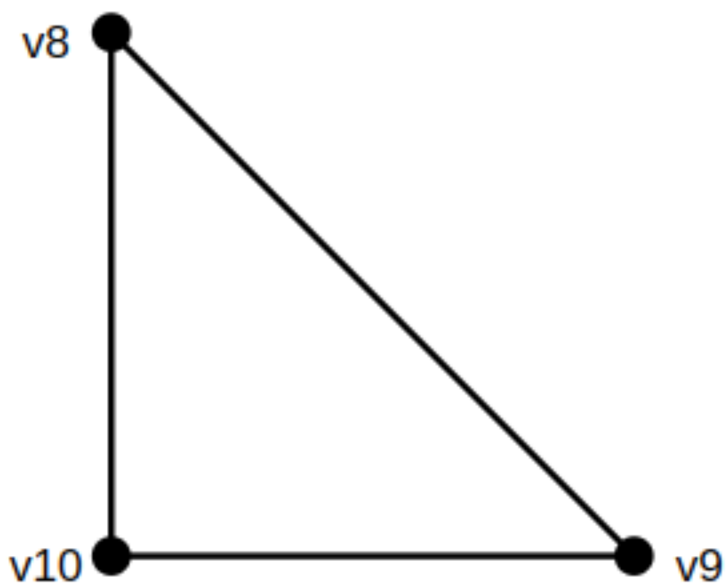
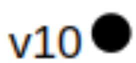


Figure 1: image

after 10. iteration:

- v9 was deleted.
- S and U are not updated since $0 < 8/6$



=> returned S is {v5, v6, v7, v8, v9, v10}

Ex. 2

Sonntag, 3. November 2024 21:51

a) $Pr(X=3) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \frac{25}{216} \approx 0,1157$

b) $Pr(X=k) = \underbrace{\frac{5}{6}}_{\substack{\text{first roll} \\ \neq 6}} \cdot \underbrace{\frac{5}{6}}_{\substack{\text{second roll} \\ \neq 6}} \cdot \ldots \cdot \underbrace{\frac{5}{6}}_{\substack{(k-1)\text{-th roll} \\ \neq 6}} \cdot \underbrace{\frac{1}{6}}_{\substack{k\text{-th roll} \\ = 6}} = \left(\frac{5}{6}\right)^{k-1} \cdot \frac{1}{6}$ for $k \in \mathbb{N}$

$$Pr(X \leq 3) = Pr(X=1) + Pr(X=2) + Pr(X=3)$$
$$= \frac{1}{6} + \frac{5}{36} + \frac{25}{216} = \frac{91}{216} \approx 0,4213$$

Let Y be the random variable, that denotes the number $X \leq 3$ results in ten attempts as described in the task.

$$Pr(Y=k) = \binom{10}{k} Pr(X \leq 3)^k (1 - Pr(X \leq 3))^{10-k}$$
for $k \in [0, 10]$

c) $Pr(Y=10) = Pr(X \leq 3)^{10} = \left(\frac{91}{216}\right)^{10} \approx 0,00018$

d) $E(Y) = \sum_{i=0}^{10} i \cdot Pr(Y=i)$

$$= \sum_{i=0}^{10} i \cdot \binom{10}{i} Pr(X \leq 3)^{i} (1 - Pr(X \leq 3))^{10-i}$$

Bernoulli-Distribution
expected value

$$= 10 \cdot Pr(X \leq 3) = \frac{910}{216} \approx 4{,}213$$

a)

Algorithm 1 Cluster Formation with Blackbox Function

$clusters \leftarrow \emptyset$
for each a in P **do**
    $currentCluster \leftarrow \{a\}$
    for each b in P **do**
        if $d(a, b) \leq r$ **then**
            $currentCluster \leftarrow currentCluster \cup \{b\}$
        end if
    end for
    $clusters \leftarrow clusters \cup currentCluster$
end for
$result \leftarrow \text{blackbox}(\mathcal{T} = clusters, N = k)$
return $result$

b)

This would not be possible since the distance to the cluster center is not known prior to running the algorithm in the standard k-center problem. Therefore there is no metric we could use to assign possible cluster points to the cluster centers.

c)

Algorithm 2 Cluster Formation with known OPT

$clusters \leftarrow \emptyset$
while $P \neq \emptyset$ **do**
    $currentCenter \leftarrow \text{some } c \in P$
    $currentCluster \leftarrow \emptyset$
    for each p in P **do**
        if $d(c, p) \leq 2OPT$ **then**
            $currentCluster \leftarrow currentCluster \cup \{p\}$
        end if
    end for
    $clusters \leftarrow clusters \cup currentCluster$
    $P \leftarrow P \setminus currentCluster$
end while
return $clusters$