Let X be a RV, with X being the total number of L-sequences. Let X_i be indicator variable where an L-sequence begins at position i.

~~Let $L = \log_2(n)$ be the number of trials~~

Because of the length L of an L-sequence, the largest index an L-sequence can start is n. Therefor

$$X = \sum_{i=1}^{n} X_i \quad \text{and} \quad \text{(X_i indicator variable)}$$

$$\mathbb{E}[X] = \sum_{i=1}^{n} \mathbb{E}[X_i] = \sum_{i=1}^{n} \Pr(X_i)$$

The probability to have L consecutive of either heads or tails is

$$2 \cdot \left(\tfrac{1}{2}\right)^{L} = 2 \cdot \left(\tfrac{1}{2}\right)^{1+\log_2(n)} = \left(\tfrac{1}{2}\right)^{\log_2(n)} = \tfrac{1}{n}$$

(heads or tail)

$$\Rightarrow \mathbb{E}[X] = \sum_{i=1}^{n} \tfrac{1}{n} = n \cdot \tfrac{1}{n} = 1$$

□