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6. Home Exercises

Efficient Algorithms

Ex. 1 *(4 + 4 Points)*

You are given a sequence (of unknown length) of objects with sizes in $(0, 1]$ one at a time. These objects should be placed into boxes of size 1. Multiple objects may be placed inside a single box as long as the sum of their sizes does not exceed the size of the box. The objective is to minimize the number of boxes that you use.

a) Design a 2-competitive online algorithm for this problem.

b) Show that your algorithm is indeed 2-competitive.

Ex. 2 *(4 + 4 points)*

A hungry cow wants to go to a field of grass. She is currently on a path that she knows it leads her to the field. Sadly the cow forgot in which direction to go and has very bad senses. She can only recognize the field when she is standing exactly on it. Assume that the field is $m \geq 1$ meters away from the current starting position of the cow.

a) Design a c-competitive deterministic strategy for some constant c, i.e., the cow needs to walk at most $c \cdot m$ meters to find the field.

Hint: The best deterministic strategy needs at most $9 \cdot m$ meters.

b) Show that your strategy is indeed c-competitive.

Ex. 3 *(4 Points)*

A professor is hiring a research assistant. There are n applicants for the position, and the value of n is known. The applicants can be ranked from 1 (best) to n (worst) without ties. They are interviewed sequentially in any arbitrary order. The professor has to make a decision of either accepting or rejecting just after interviewing a candidate and the decision is irrevocable. The decision is based on the relative ranks of the applicants interviewed so far. The objective is to hire the best candidate of the whole group for the position. Thus we want to maximize the payoff which is defined to be one if the best applicant is chosen and zero otherwise. Prove that for any deterministic strategy, the worst payoff is always zero.