Ex. 1

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given: random permutation $x_1, x_2, \dots, x_n$ of the numbers $\{1, 2, \dots, n\}$

a) There are $i!$ possible permutations of the first i numbers. Fixing the largest one at the end results in $(i-1)!$ permutations where $x_i > x_1, x_2, \dots, x_{i-1}$ holds. Therefore

$$Pr(x_i > x_1, x_2, \dots, x_{i-1}) = \frac{(i-1)!}{i!} = \frac{1}{i}$$

b) Let S be the number of rounds in which we draw a number, that is larger than all previous numbers:

$$S = \sum_{i=1}^{n} X_i \quad \text{with} \quad X_i = \begin{cases} 1, \text{ if } x_i > x_1, x_2, \dots, x_{i-1} \\ 0, \text{ else} \end{cases}$$

$$\mathbb{E}(S) = \mathbb{E}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \mathbb{E}(X_i)$$

$$= \sum_{i=1}^{n} \frac{1}{i} = H_n \text{ (Harmonic number)}$$