# Ex. 2

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a) $Pr(X=3) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \frac{25}{216} \approx 0{,}1157$

b) $$Pr(X=k) = \underbrace{\frac{5}{6}}_{\substack{\text{first roll}\\ \neq 6}} \cdot \underbrace{\frac{5}{6}}_{\substack{\text{second roll}\\ \neq 6}} \cdots \underbrace{\frac{5}{6}}_{\substack{(k-1)\text{-th roll}\\ \neq 6}} \cdot \underbrace{\frac{1}{6}}_{\substack{k\text{-th roll}\\ = 6}} = \left(\frac{5}{6}\right)^{k-1} \cdot \frac{1}{6} \quad \text{for} \quad k \in \mathbb{N}$$

$$Pr(X \leq 3) = Pr(X=1) + Pr(X=2) + Pr(X=3)$$
$$= \frac{1}{6} + \frac{5}{36} + \frac{25}{216} = \frac{91}{216} \approx 0{,}4213$$

Let $Y$ be the random variable, that denotes the number $X \leq 3$ results in ten attempts as described in the task.

$$Pr(Y=k) = \binom{10}{k} Pr(X \leq 3)^{k} (1 - Pr(X \leq 3))^{10-k}$$
for $k \in [0, 10]$

c) $Pr(Y=10) = Pr(X \leq 3)^{10} = \left(\frac{91}{216}\right)^{10} \approx 0{,}00018$

d) $$E(Y) = \sum_{i=0}^{10} i \cdot Pr(Y=i)$$

$$= \sum_{i=0}^{10} i \cdot \binom{10}{i} Pr(X \leq 3)^{i} (1 - Pr(X \leq 3))^{10-i}$$

Bernoulli-Distribution
expected value

$$= 10 \cdot Pr(X \leq 3) = \frac{910}{216} \approx 4{,}213$$