Ex. 3

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a)

Let A be the given sequence and n its length.

```
1    randomPermutation(A, n)
2        B = []

3        for i in 1,...,n do
4            index = RANDOM(n-i+1)
5            B[i] = A[index]  # append A[index] to list B
6            swap(A[index], A[n-i+1])  # swap appended element with last element of A

7        return B
```

b) Line 1-2: O(1)
Line 3-6: O(n)
Line 7: O(1)

=> running time is $O(n)$

c) In each iteration an index of a nomber gets selected, that hasn't been appended to B yet. The number gets appended to B and in A it gets swapped with the last number, that hasn't been picked yet.

The probability for a nombe to be selected for the i-th position is $\frac{1}{n-i+1}$.

Therefore the probability for any output permutation is

$$\prod_{i=1}^{n} \frac{1}{n-i+1} = \frac{1}{n} \cdot \frac{1}{n-1} \cdot \ldots \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{n!}$$