

Ex. 2

Donnerstag, 5. Dezember 2024

23:08

- Assumption:

$$Pr(X_1 \cap X_2 \cap \dots \cap X_n) = Pr(X_1) \cdot Pr(X_2 | X_1) \cdot \dots \cdot Pr(X_n | X_1, \dots, X_{n-1})$$

- Basis step with $n=1$:

$$Pr(X_1) = Pr(X_1) \quad \checkmark$$

- Induction step ($n \rightarrow n+1$):

$$\text{Suppose } Pr(X_1 \cap X_2 \cap \dots \cap X_n) = Pr(X_1)$$

$$Pr(X_2 | X_1) \cdot \dots \cdot Pr(X_n | X_1, \dots, X_{n-1}) \quad (*)$$

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$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

$$\Leftrightarrow Pr(A \cap B) = Pr(B) Pr(A|B) \quad (*)$$

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$$\Rightarrow P_r(X_1 \cap X_2 \cap \dots \cap X_{n+1})$$

$$= P_r((X_1 \cap X_2 \cap \dots \cap X_n) \cap X_{n+1})$$

$$\begin{aligned} & (*) \\ &= P_r(X_1 \cap X_2 \cap \dots \cap X_n) \\ &\quad \cdot P_r(X_{n+1} | X_1, X_2, \dots, X_n) \end{aligned}$$

$$\begin{aligned} & (*) \\ &= P_r(X_1) P_r(X_2 | X_1) \cdot \dots \\ &\quad \cdot P_r(X_n | X_1, X_2, \dots, X_{n-1}) \\ &\quad \cdot P_r(X_{n+1} | X_1, X_2, \dots, X_n) \\ &\quad \hookrightarrow \text{chain rule for } n+1 \quad \square \end{aligned}$$