Optimization Model

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Abstract

Here is a basic optimization model about our project. Some assumption are made in this model, such as how to transfer driving condition to driver's performance and estimate risk. The point is to find out whether we could build an optimization model, then combine with statistical result from other group to provide an decision- making model to improve safety.

Model description

We model this by using dynamic programming knowledge. There is a route between origin and destination. Besides of that, we have weather, traffic or other information related to our driving condition in advance. We can simulate these information or get prediction from historical data. Our problem is to give guidance of driver's decision, such as when should driver stop to take a rest or avoid bad ahead.

Variables

Here are some variables:

- x_t: number of miles from origin at each time epoch. We use this to represent location.
- w_t: random information about driving condition at time t, such as weather information, traffic condition, etc..So w actually is a n-dimensional variable. n is different information we have. For each location, w will vary as time.
  We will divide the whole route into different segments. For each segment, it will share same part of w. For example, in our simple example, we assume all location within each segment will have same weather and traffic information.
  In the future, we can have more specific details about w. For weather information, we could have format like this:
  Segment A: Cloudy for next 24 hours;
  Segment B: 12 am - 6 am 90% rain; 6 am - 8 am 100% rain; 8 am - 10 am 60% rain; ...
  For traffic condition, we could have traffic flow or density. Besides of that, we could also use the number of driving since last stop as one variable in w.
  Besides of that, we can also have variable related driving history, such as number of hours spending on the road since last rest. We can add more variables into our model if these variable has significant relation ship to driver's performance.
- $g(w)$: driver's performance which is a parameter to evaluate risk for driver. Here assumption is we only care about g at the beginning of each time period, and use this to calculate cumulated risk for this period.
- a_t: action we can choose at each time epoch and usually it is from 0 to speed limit;
- c_t: risk for each time period;

- T: total time for the whole route;
- Δ t: length of time for each time period.

Dynamic programming model

After all these, we can define a dynamic model as follows:

- Horizon: T;
- State: (x_t, g_t), location and driver's performance are used as state for each time; t means each stage
- Action: a_t and $a_t \in$ [0, speed limit]
- Transition:
$$(x_{t+1}, g_{t+1}) = \begin{cases} x_{t+1} = F(x_t, a_t) \\ g_{t+1} = G(g_t, x_t, w_{t+1}) \end{cases} \tag{1}$$
Here are some explanation about formulas in transition function. $g_{t+1} = G(g_t, a_t, w_{t+1})$ and $g_0 = G(w_0)$. This means at stage 0, driver's performance will be decided by driver's condition. At stage 1, driver's performance will be decided by performance, action from last stage and condition in current stage.
- Cost: $c_t = \phi(g_t, a_t, w_t)$. Here cost means risk for each time period will relate to action at time epoch, driver's performance and condition at the beginning for each time period. Actually, driver's condition may change if we decide to move forward, since it is possible for driver to transfer to next segment. A better method to evaluate cumulated risk for each period may help us improve this.
- Objective: Min $\sum_{t=0}^{T}$.

Example

In real life, we could have information about driving condition either from historical data or simulation and our problem is how could we decide our actions for the whole route to minimize total risk. For instance, we know probability of heavy rain in segment C is 90% and right now we are in segment B. Should we take a rest to avoid bad weather ahead or continue to drive?
To better illustrate this model, we give a simple example. At first, we need define all parameters in our model:

1. At first, w has only two dimension, weather and traffic.

$$\text{w} = 0.7\text{*m} + 0.3\text{n} + 0.2$$

Table 1: Driving Condition

Mile	Time after 8 am	0-0.5	0.5-1	1-1.5	1.5-2
0-25	Weather(m)	0	1	0	1
	Traffic(n)	1	1	0	0
25-50	Weather(m)	1	1	0	0
	Traffic(n)	0	0	1	1
50-75	Weather(m)	1	0	1	0
	Traffic(n)	0	1	0	1
75-100	Weather(m)	0	0	0	0
	Traffic(n)	0	0	0	0

$$m = \begin{cases} 1 & \text{bad weather} \\ 0 & \text{good weather} \end{cases} \tag{2}$$

$$n = \begin{cases} 0 & \text{bad traffic} \\ 1 & \text{good traffic} \end{cases} \tag{3}$$

2. $g_{t+1} = G(g_t, a_t, w_{t+1})$ and $g_0 = G(w_0)$

$$G(g_t, a_t, w_{t+1}) = \begin{cases} g_t + w_{t+1} * a_t * 0.02 * \Delta t & a_t > 0 \\ g_t - 0.1 * \Delta t & a_t = 0 \end{cases} \tag{4}$$

3. $c_t = \phi(g_t, a_t, w_t) = \Delta t * 0.002 + 0.5 * g_t * a_t * o.5 * w_t^2$
4. $\Delta t = 0.5$ hour
5. From origin to destination, the length is 100 miles and there are 4 segments for whole route. We will leave at 8 am and we already have prediction of weather and traffic information. Since drive has to arrive within 2 hours, so we don't list driving condition after that. To simplify our problem, we assume each segment share same traffic and weather information, and each segment has same distance.
6. Action set only contain [0, 55, 75].
7. Objective function: $p * \sum_{c_t} + q * T$. p and q are weights for total cost and time; T means total time for whole route.

Result

I program this in python and also solve it in excel. Here is the result for different weights used.

Table 2: Computation result

p	q	total time	total risk	actions	obj
1	1	1.833333333	0.335703125	[75, 0, 75, 75]	2.169036458
1	5	1.833333333	0.335703125	[75, 0, 75, 75]	2.169036458
1	10	1.333333333	1.018859375	[75, 75, 75]	14.35219271
1	25	1.333333333	1.018859375	[75, 75, 75]	34.35219271
1	50	1.333333333	1.018859375	[75, 75, 75]	67.68552604
1	100	1.333333333	1.018859375	[75, 75, 75]	134.3521927
5	1	1.833333333	0.335703125	[75, 0, 75, 75]	3.511848958
10	1	1.833333333	0.335703125	[75, 0, 75, 75]	5.190364583
25	1	1.833333333	0.335703125	[75, 0, 75, 75]	10.22591146
50	1	1.833333333	0.335703125	[75, 0, 75, 75]	18.61848958
100	1	1.833333333	0.335703125	[75, 0, 75, 75]	35.40364583

Some general conclusion

From result we can find that, only when we change weight for total time a lot, optimal solution will change. It is easy to understand that if we want to finish the whole route as soon as possible, we will take action like high speed no matter how risky it is. If we consider risk cost and time cost have similar weight or more weight on risk cost, the best action may always avoid bad weather to make sure low risk. For example, when driver's performance and driving condition increase risk a lot, driver will choose to stop to refresh performance and avoid bad weather ahead.
Besides of that, we found that action in optimal solution is stop or high speed. So we changed our risk formula, $c_t = \Delta t * 0.5 * g_t * a_t^2 * w_t + \Delta t$. Optimal solution will be [75,0,75,55]. So if risk increase dramatically when speed is high, optimal solution will also contain action between 0 and speed limit.

Future data analysis result we may need

Right now parameters in our model are not true, but this will change optimal solution significantly. In the future, it is better for us to have some real parameters from data analysis result. There are two important ones we need: the one is about how to transfer driving condition to driver's performance which is G in our model; the other one is about how to evaluate risk in our model. If we could have data analysis result about these two parameters, optimization model can get a solid result.

Further Study

Firstly, since we have finished our model, we could apply some algorithm to solve our problem. Value iteration and policy iteration are two common methods. For our case, value iteration maybe easier to solve. The main reason for difficulty of policy iteration is to assign action to states. Our state has two elements, one is about location and the other one is about driver's performance. When we assign our action to each state, actually it has three things we should take into consider: location, arriving time and driver's performance at each location and different time. So if we need assign action for each state, we need discretized our driver's performance and combine with two-dimension variable. For action and time, because we have divided into segments and time periods, we will have discrete variable.

Our next step is to see whether we could get result if we use these two algorithms and compare results with research tree methods like before. This part of work is based on result from analysis group. Besides of that, we have lots of real data from company, such as weather, driving...We can use some ADP methods directly into these data to see how is the result.