

Decision Tree For Regression

(Basic, Advanced Concepts and Its Applications)

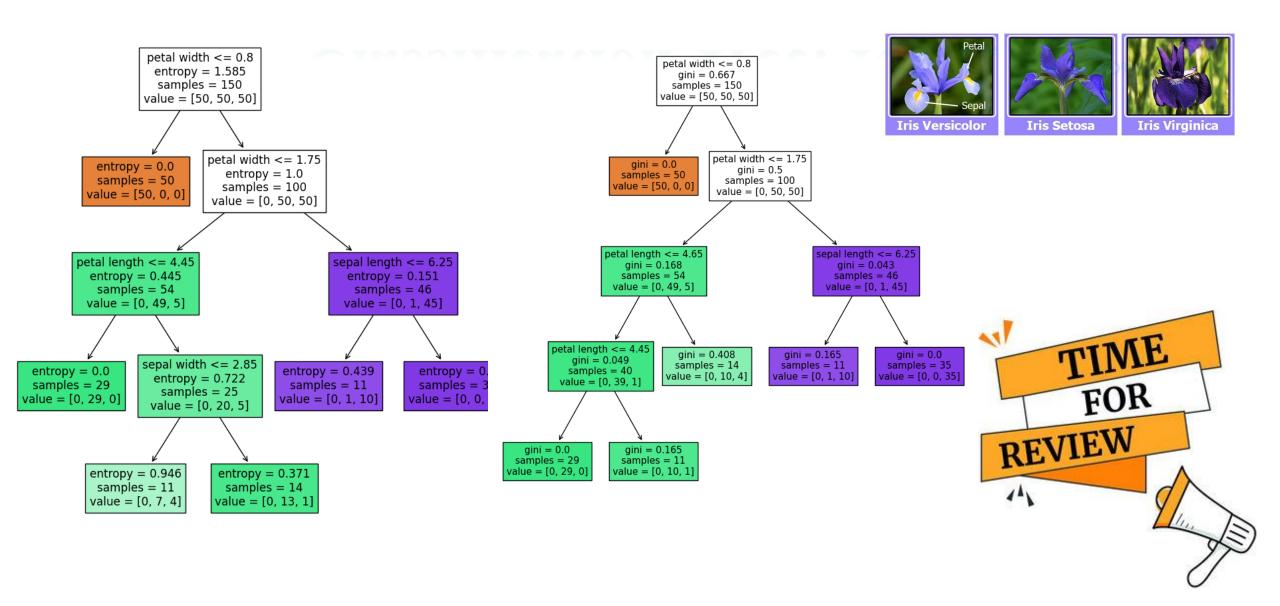
Vinh Dinh Nguyen PhD in Computer Science

Outline



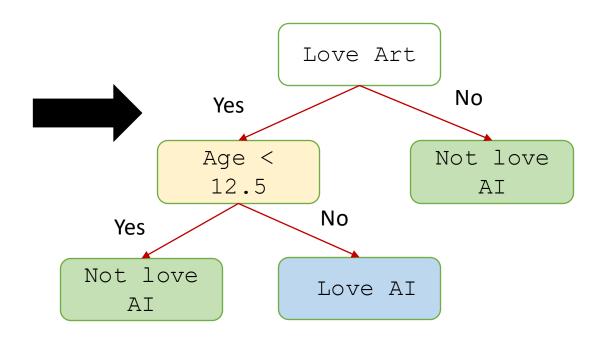
- Classification Tree: Review
- **Regression Tree: Motivation**
- Regression Tree: Clearly Explain
- **Regression Tree: Overfitting Problem**
- **Examples**



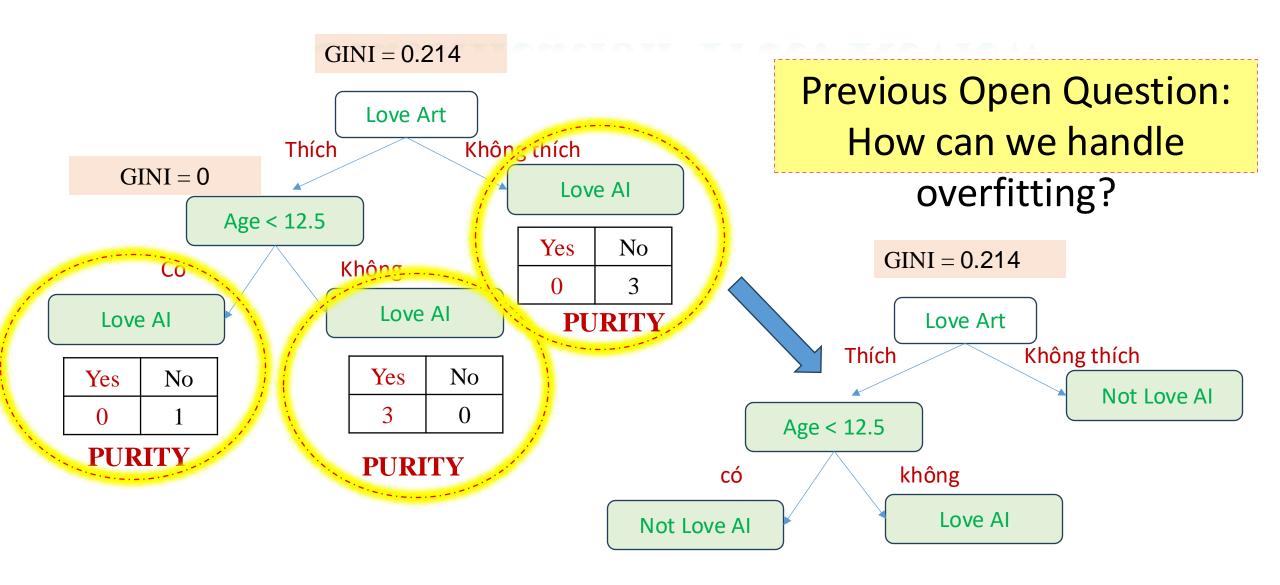




•				
Love Math	Love Art	Age	Love AI	
Yes	Yes	7	No	
Yes	No	12	No	
No	Yes	18	Yes	
No	Yes	35	Yes	
Yes	Yes	38	Yes	
Yes	No	50	No	
No	No	83	No	
	Y		Υ	
	Features	L	Labels	







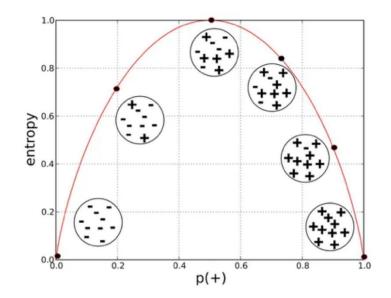


Evaluation Metric: Review

When should I use Gini Impurity as opposed to Information Gain (Entropy)

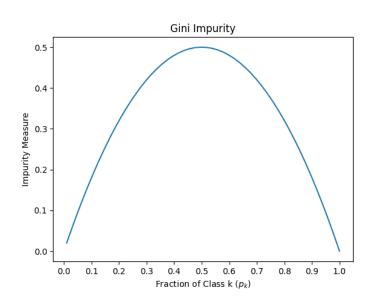
Entropy – Information Gain

$$GAIN_{iphi} = Entropy(p) - \left(\sum_{j=1}^{k} \frac{n_{j}}{p_{j}} Entropy(i)\right)$$



GNI IMPURITY

$$Gini = 1 - \sum_{i=1}^{C} (p_i)^2$$





GINI Vs. Entropy

Laura Elena Raileanu and Kilian Stoffel compared both in "<u>Theoretical comparison between the gini index and information gain criteria</u>". The most important remarks were:

- •It only matters in 2% of the cases whether you use gini impurity or entropy.
- •Entropy might be a little slower to compute (because it makes use of the logarithm).

Study the behavior of the Gini Index and Information Gain, to give an exact mathematical description of the situations when they are choosing the same test to split on and when they are choosing different tests.

Found that they disagree only in 2% of all cases, which explains why most previously published empirical results concluded that it is not possible to decide which one of the two tests performs better

Published: May 2004

Theoretical Comparison between the Gini Index and Information Gain Criteria

Laura Elena Raileanu & Kilian Stoffel

Annals of Mathematics and Artificial Intelligence 41, 77–93 (2004) | Cite this article

2960 Accesses 395 Citations Metrics

Abstract

Knowledge Discovery in Databases (KDD) is an active and important research area with the promise for a high payoff in many business and scientific applications. One of the main tasks in KDD is classification. A particular efficient method for classification is decision tree induction. The selection of the attribute used at each node of the tree to split the data (split criterion) is crucial in order to correctly classify objects. Different split criteria were proposed in the literature (Information Gain, Gini Index, etc.). It is not obvious which of them will produce the best decision tree for a given data set. A large amount of empirical tests were conducted in order to answer this question. No conclusive results were found. In this paper we introduce a formal methodology, which allows us to compare multiple split criteria. This permits us to present fundamental insights into the decision process. Furthermore, we are









Iris Setosa

Sepal length | Sepal width | Petal length | Petal width | Class 5.1 3.51.4 0.2setosa 4.93.0 0.21.4 setosa 4.73.21.3 0.2setosa 5.95.1 virginica 150 3.0 1.8



entropy = 0.946

samples = 11

value = [0, 7, 4]

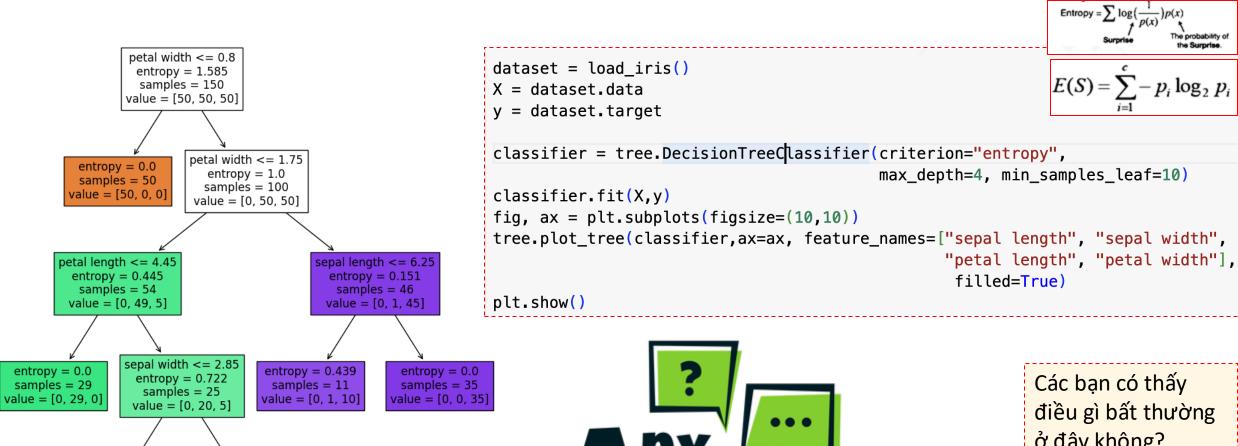
entropy = 0.371

samples = 14

value = [0, 13, 1]

Iris Flower Classification (Entropy)

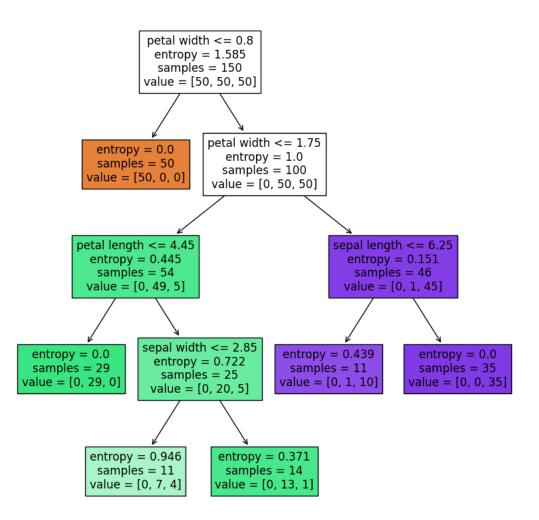
uestions

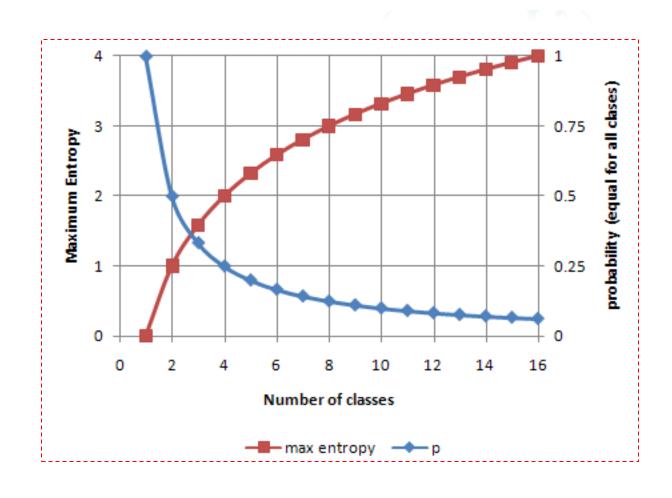


ở đây không?



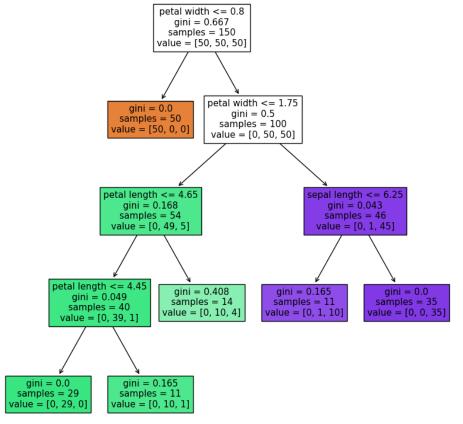
Iris Flower Classification (Entropy)







Iris Flower Classification (GINI)

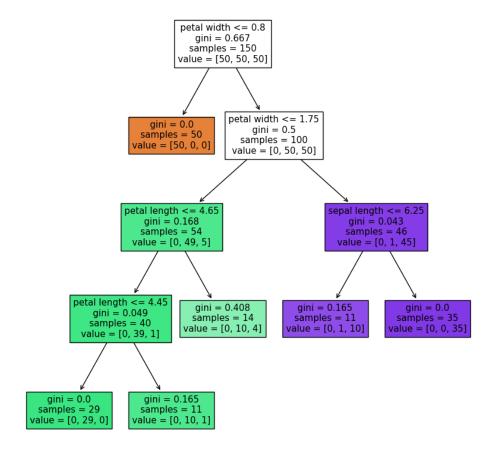


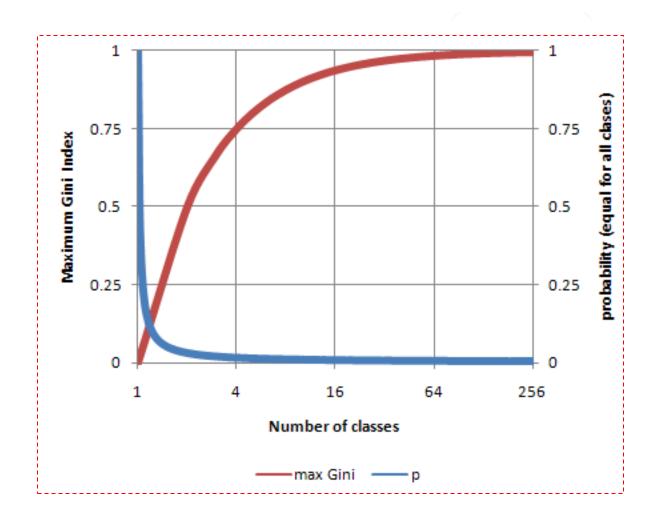


Các bạn có thấy điều gì bất thường ở đây không?



Iris Flower Classification (GINI)



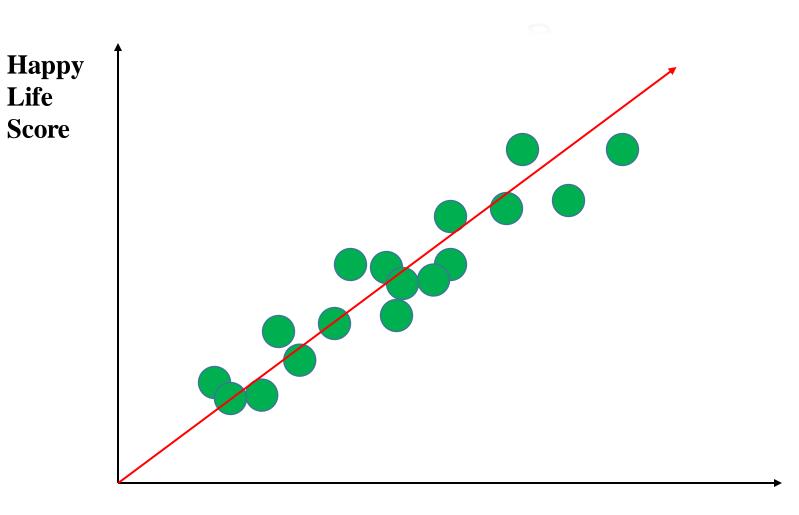


Outline



- **Regression Tree: Motivation**
- > Regression Tree: Clearly Explain
- **Regression Tree: Overfitting Problem**
- **Examples**

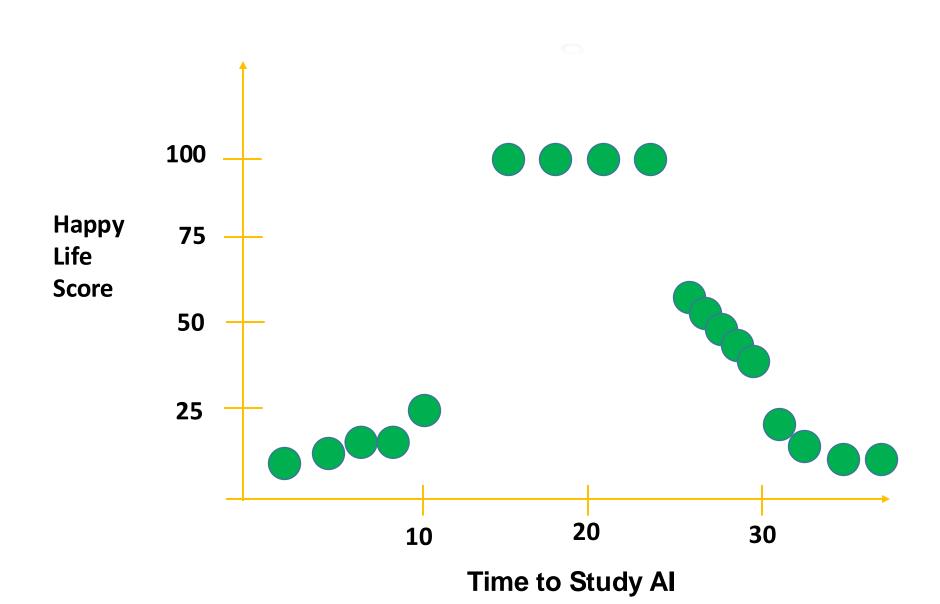
Linear Regression



Time to Study AI

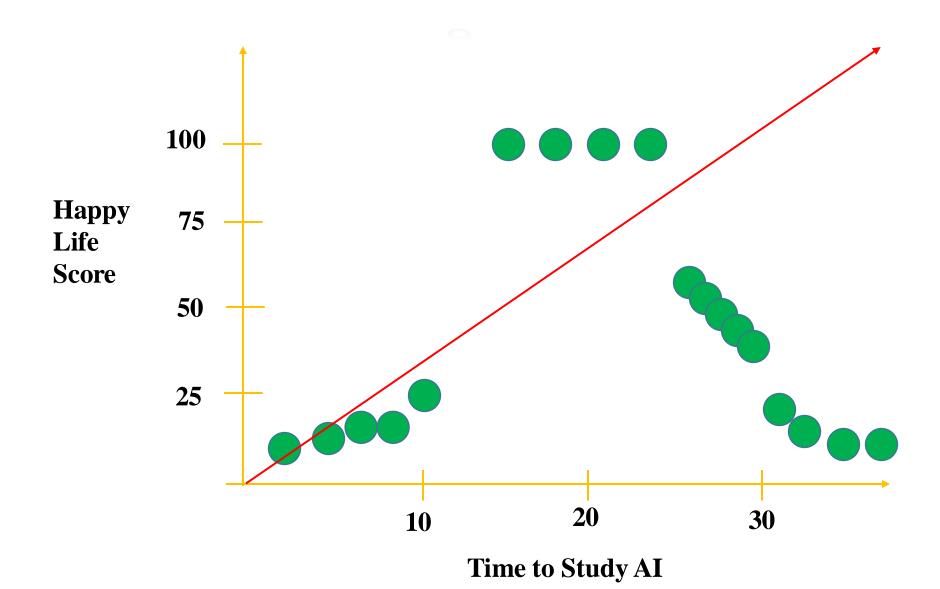


Linear Regression





Linear Regression: Problem





Case Study





Supposing that, you want to research and develop a new vaccine to cure the Covid-19



Case Study





Unit	Age	Sex	Effect (%)
10	25	Female	98
20	73	Male	0
35	54	Female	100
5	12	Male	44

Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với liều lượng dùng cố định (unit), tuổi (age) và giới tính (sex) của bệnh nhân.

Tiêm 5 đơn vị vaccine, 12 tuổi, giới tính nam



Hiệu quả vaccine:

Can we use Decision Tree for solving this research?



Outline

- **Classification Tree: Review**
- **Regression Tree: Motivation**



- **Regression Tree: Clearly Explain**
- **Regression Tree: Overfitting Problem**
- **Examples**



Which Node Should be the Root?





Unit(đơn vị)	Effect (hiệu quả) (%)
10	98
20	0
35	100
5	44
	•••

Tiêm 5 đơn vị vaccine





Hiệu quả vaccine:

Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với từng liều lượng (unit) dùng trên bệnh nhân.



Which Node Should be the Root?





Age	Effect (hiệu quả) (%)
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	•••

Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với tuổi (age) của bệnh nhân.

12 tuổi





Hiệu quả vaccine:



Which Node Should be the Root?





Sex	Effect (hiệu quả) (%)
Female	98
Male	0
Female	100
Male	44
	•••

Giới tính Male



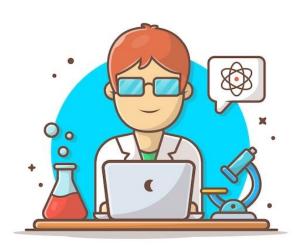


Hiệu quả vaccine:

Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với giới tính (sex) của bệnh nhân.







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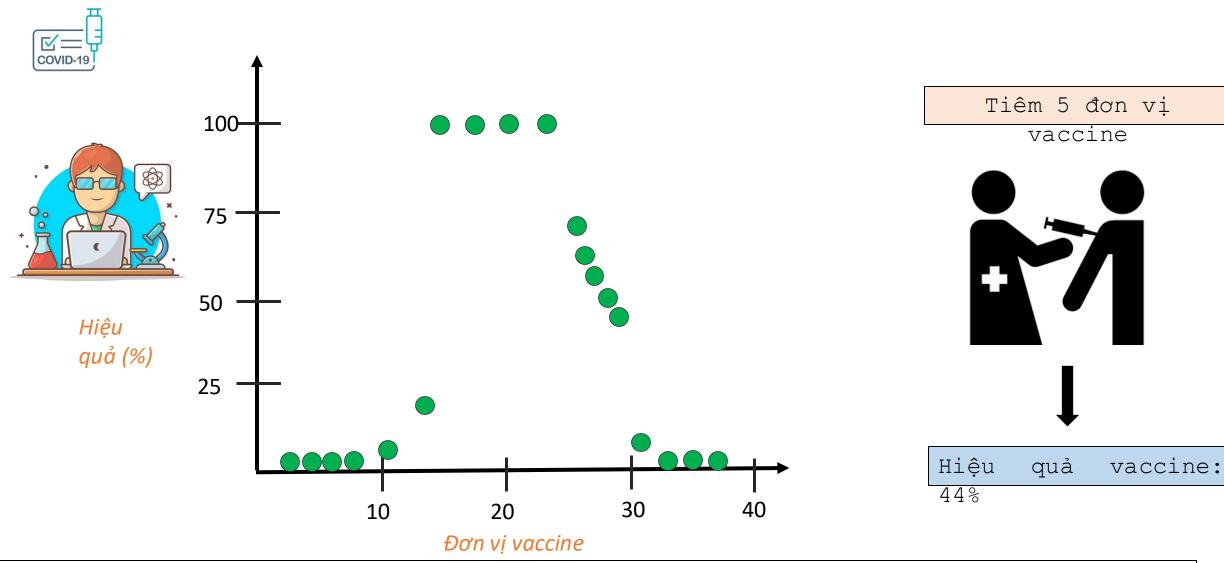
Tiêm 5 đơn vị vaccine





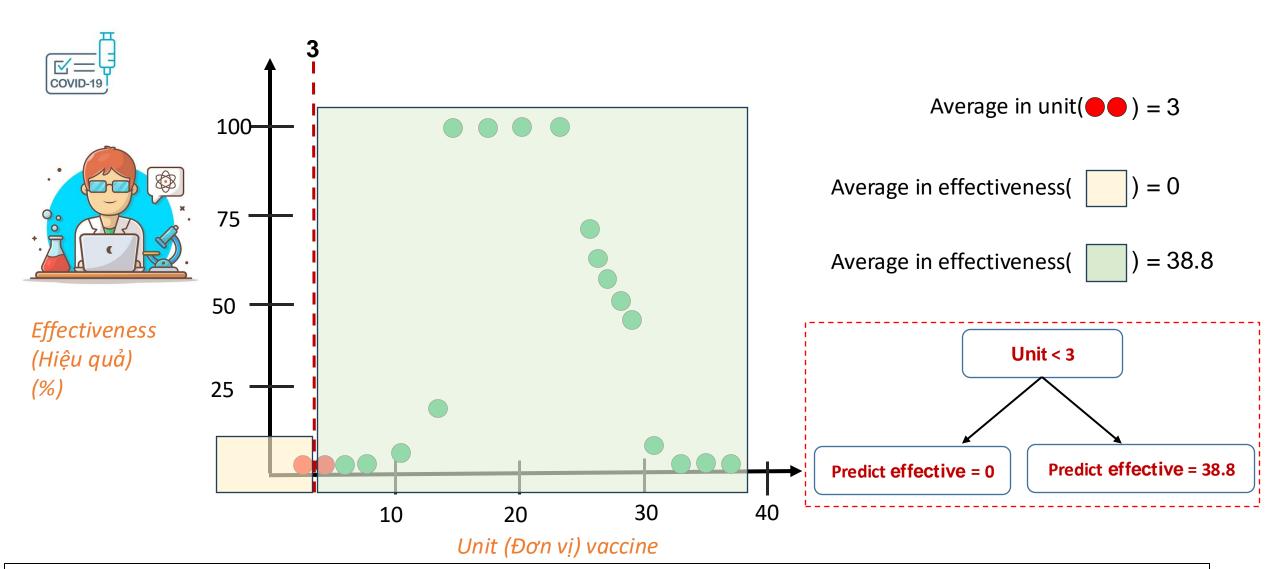
Hiệu quả vaccine:





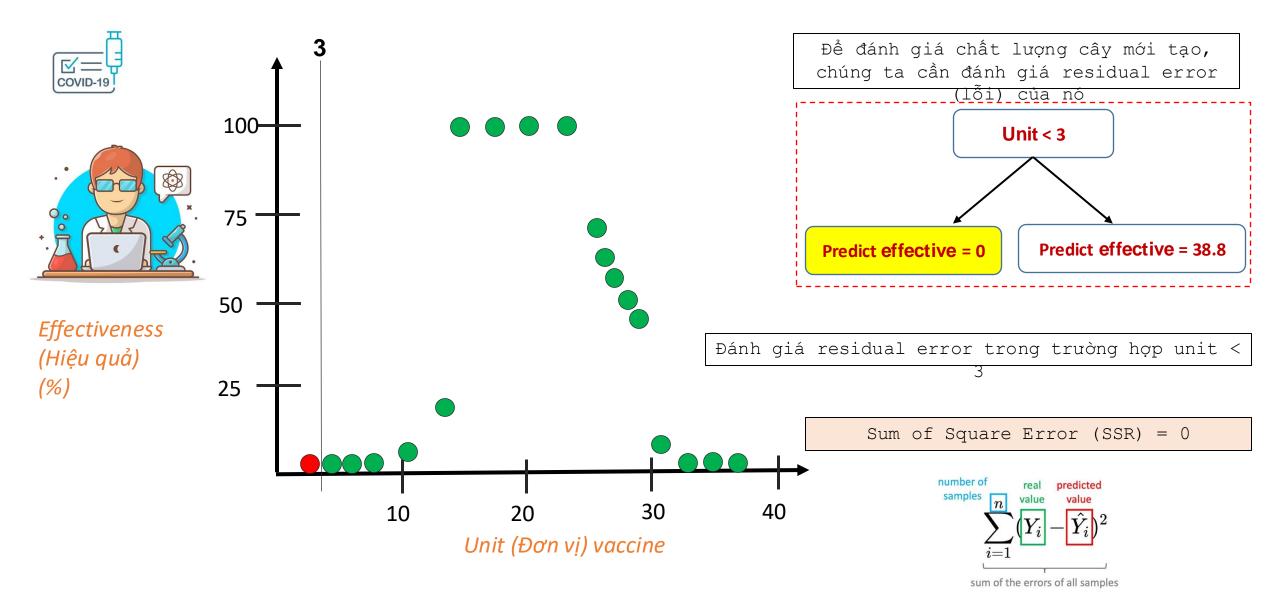
Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với từng liều lượng dùng (unit) trên bênh nhân.



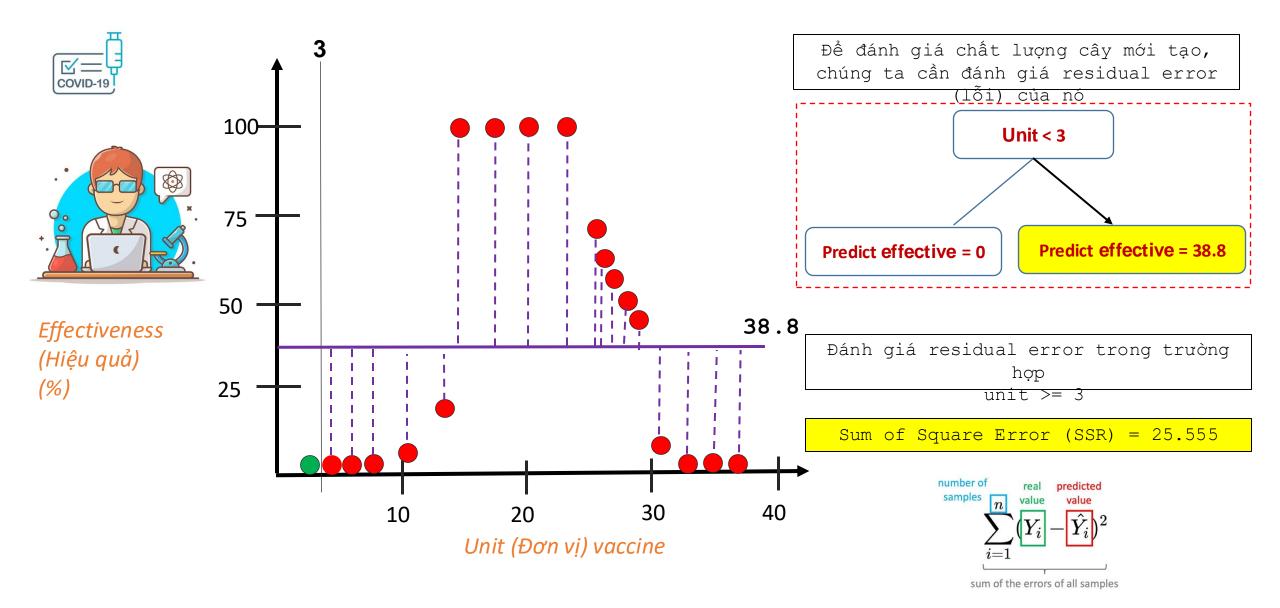


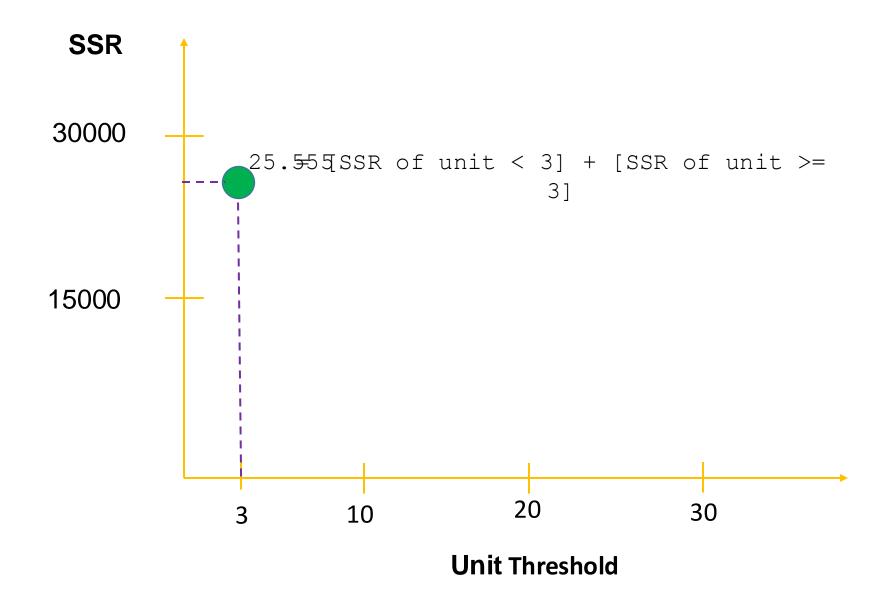
Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với từng liều lượng dùng (unit) trên bênh nhân.



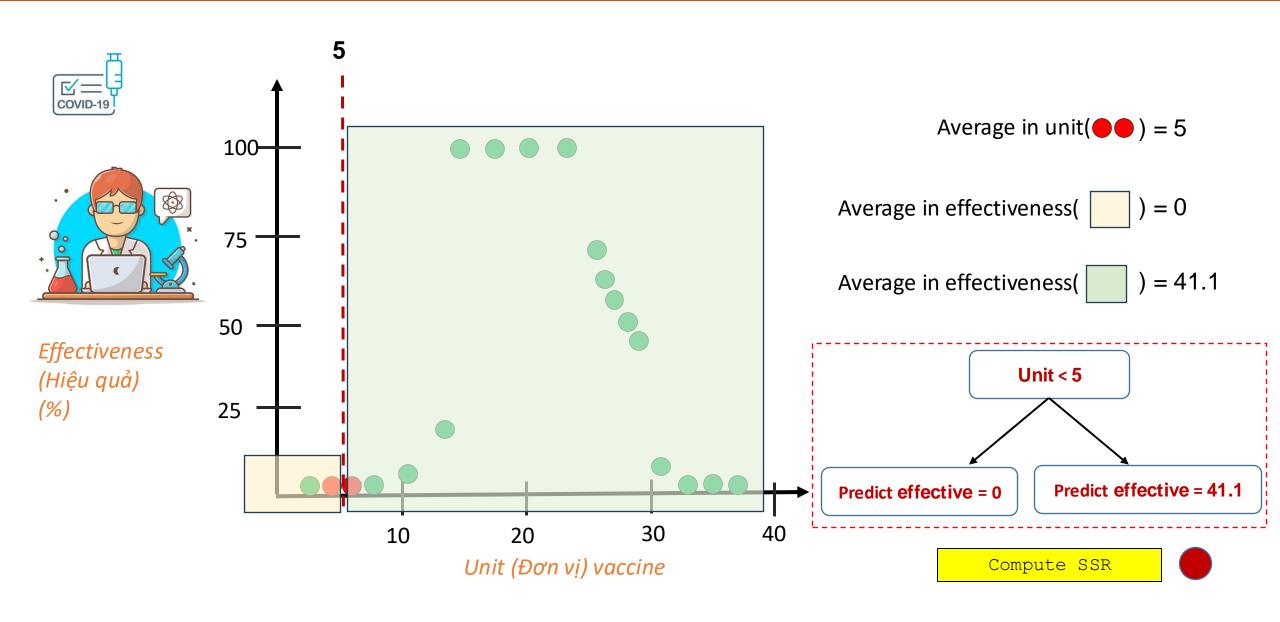




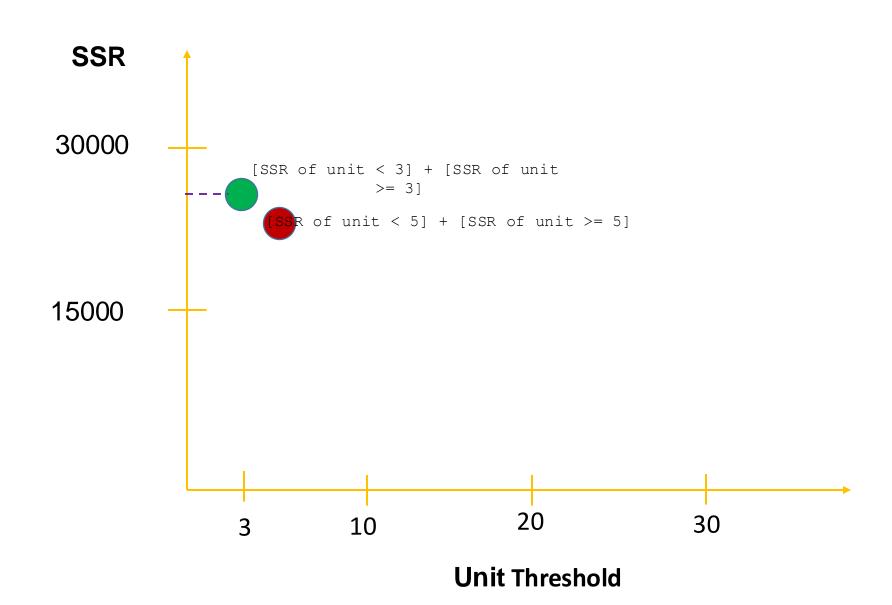




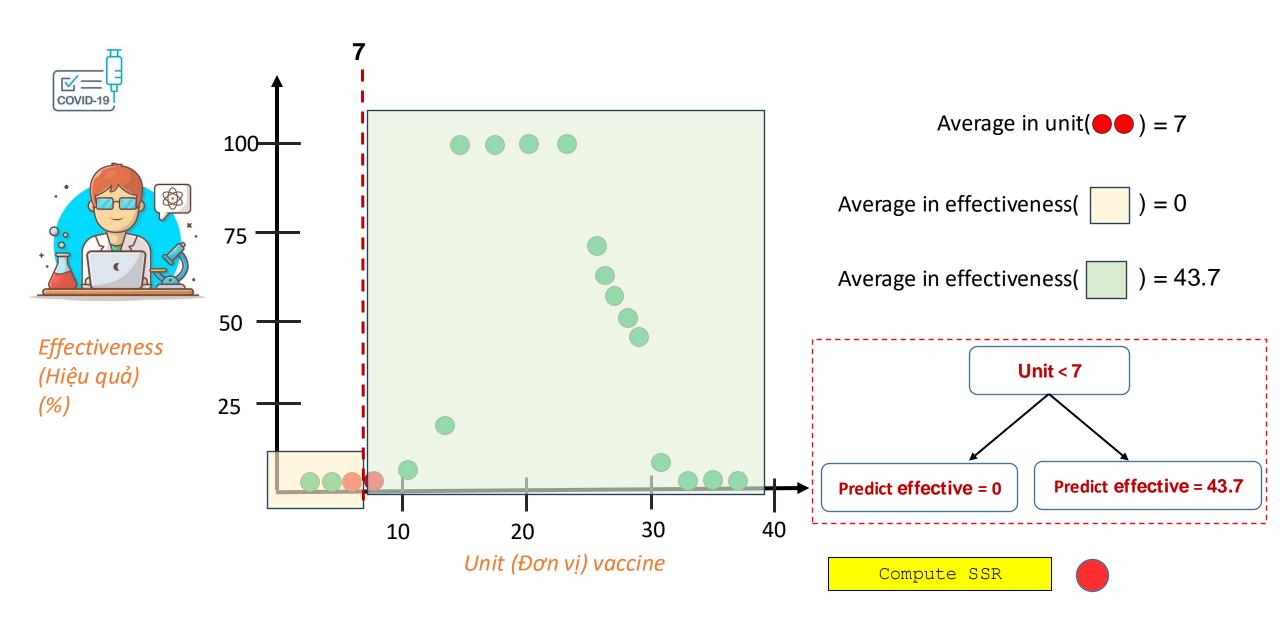




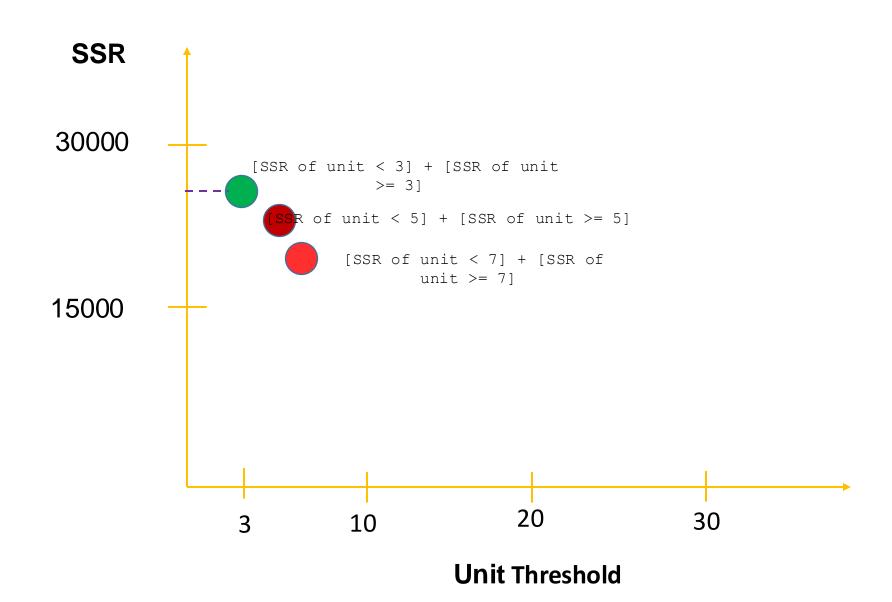




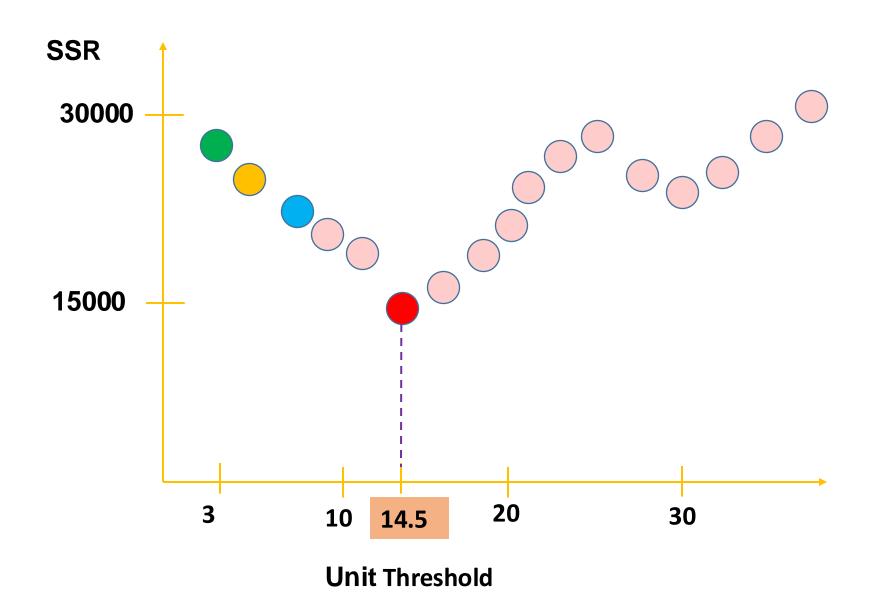




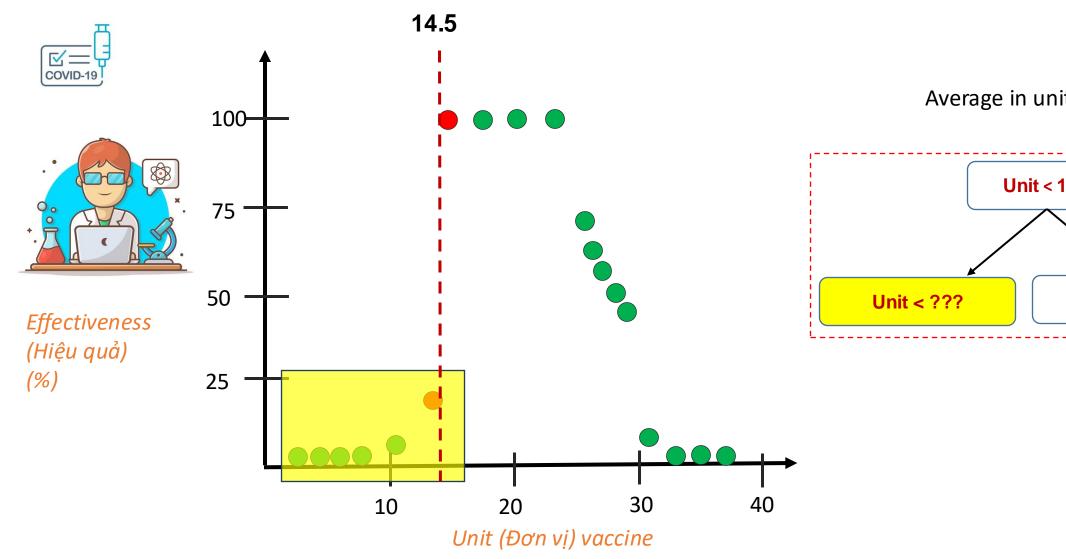




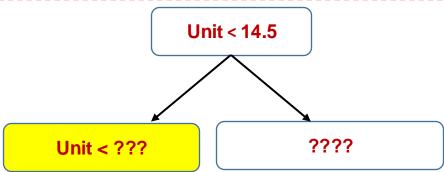




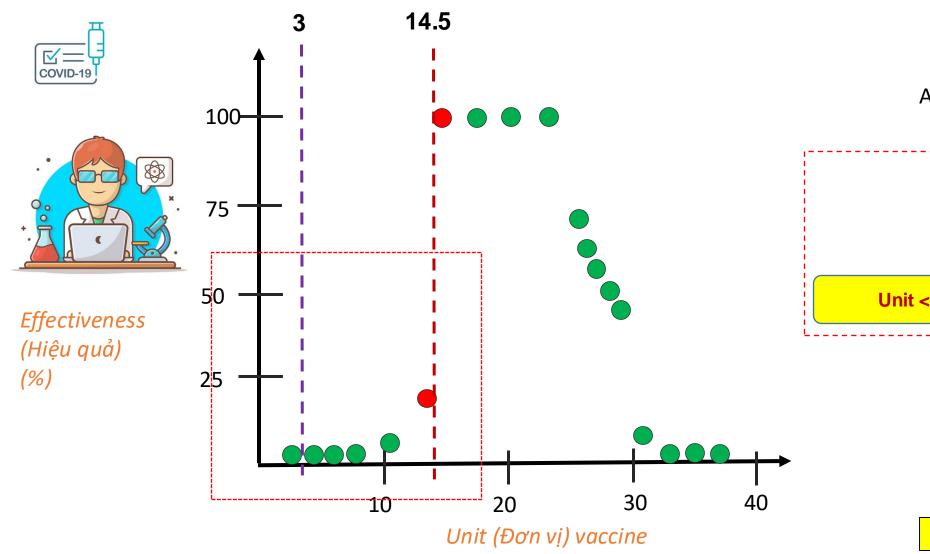




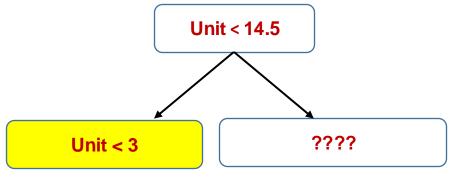
Average in unit(\bigcirc) = 14.5





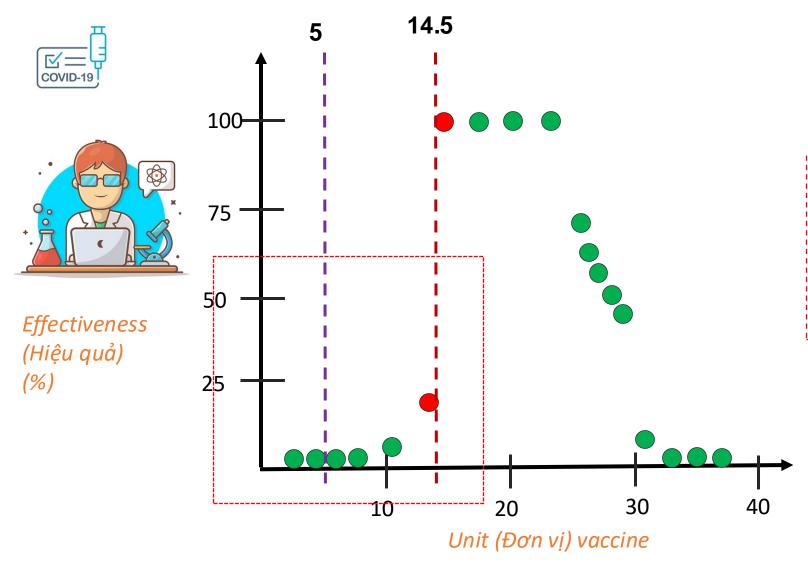


Average in unit(\bigcirc) = 14.5

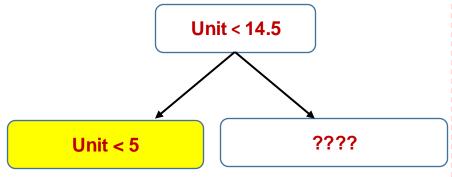


Compute SSR for unit 3



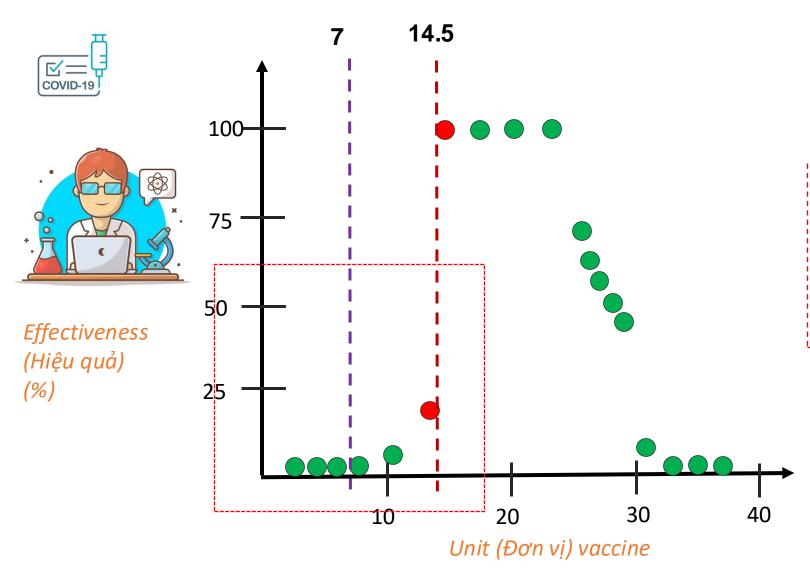


Average in unit(\bigcirc) = 14.5

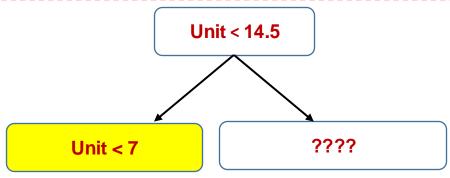


Compute SSR for unit



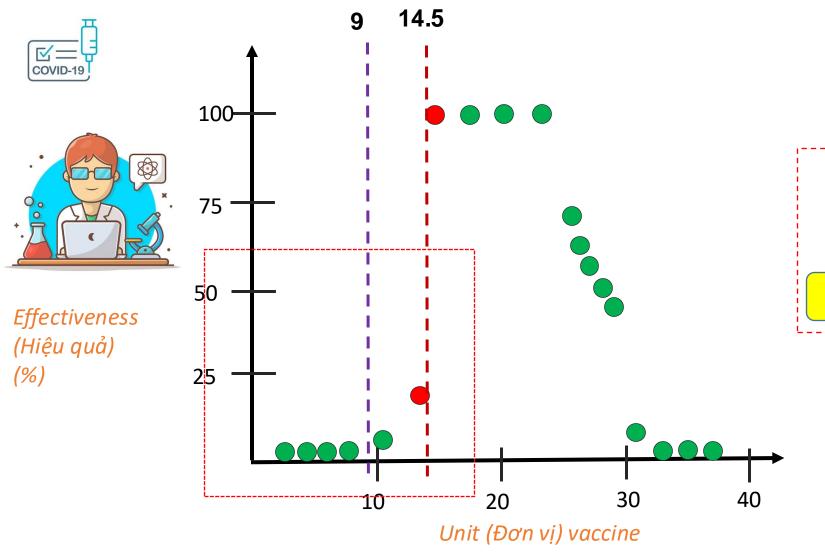


Average in unit(\bigcirc) = 14.5

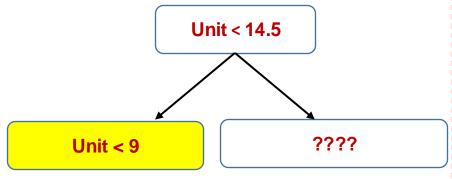


Compute SSR for unit

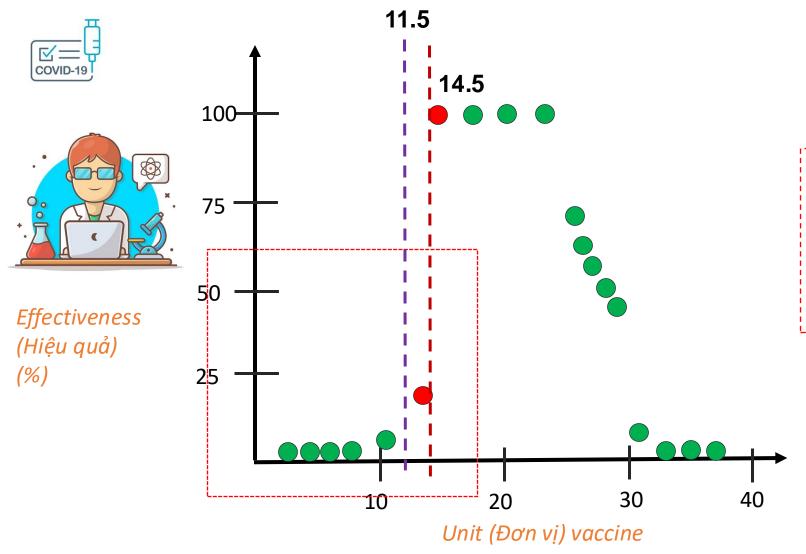




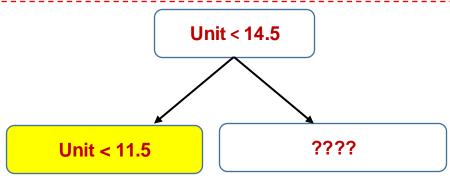
Average in unit(\bigcirc) = 14.5





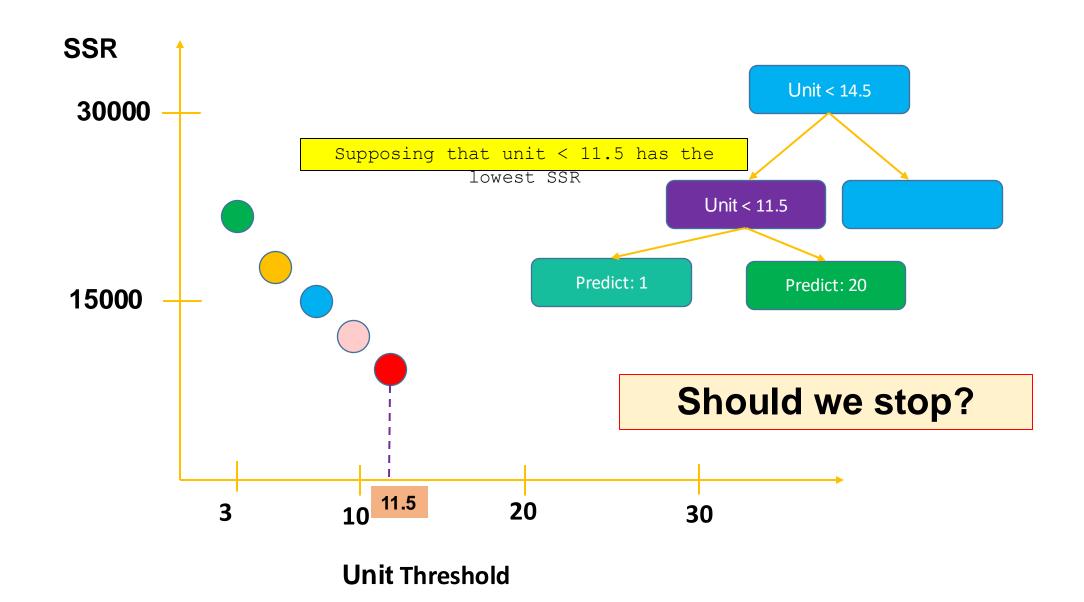


Average in unit(\bigcirc) = 14.5

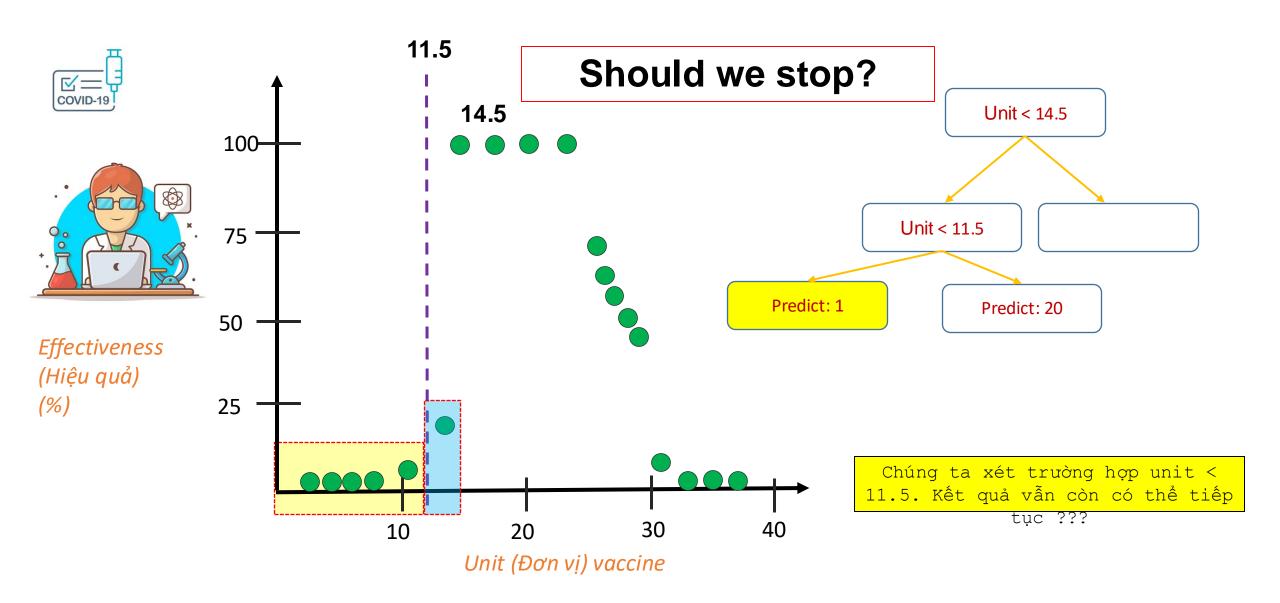


Compute SSR for unit 11.5

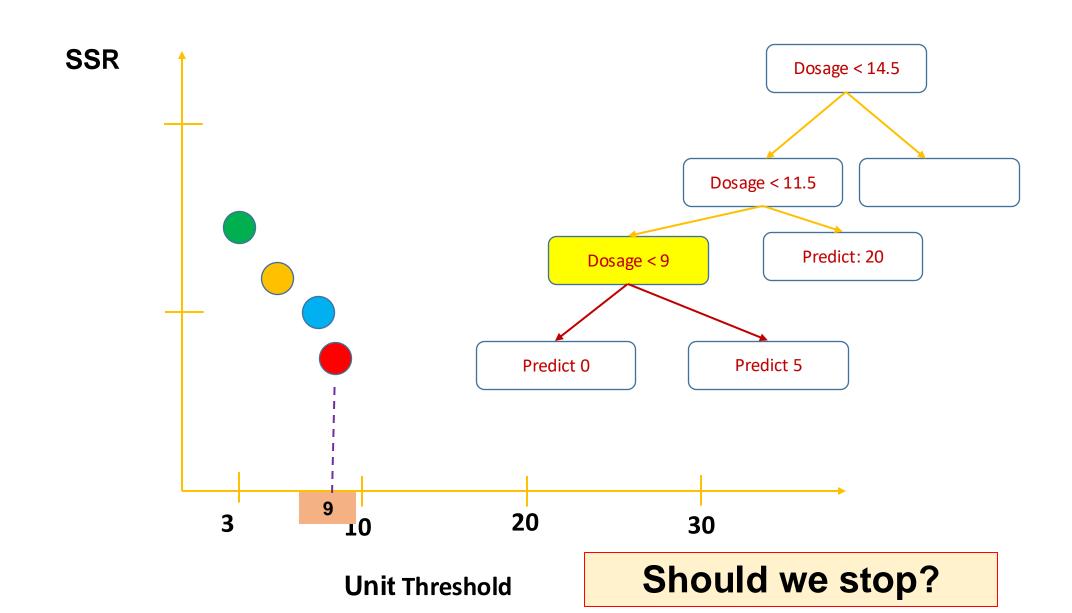




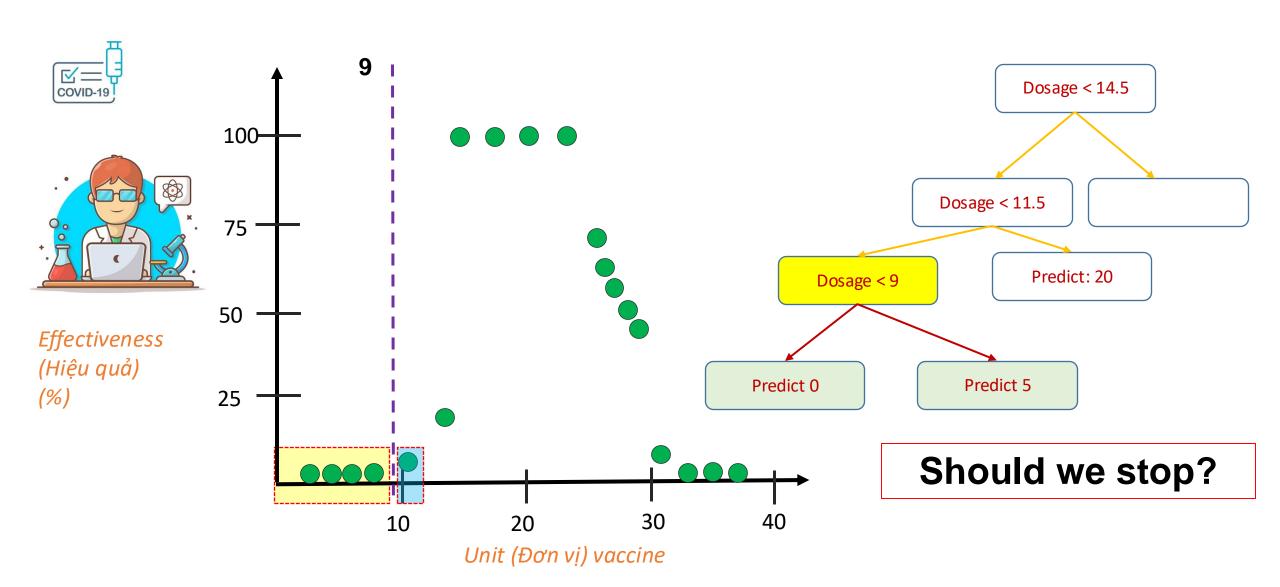






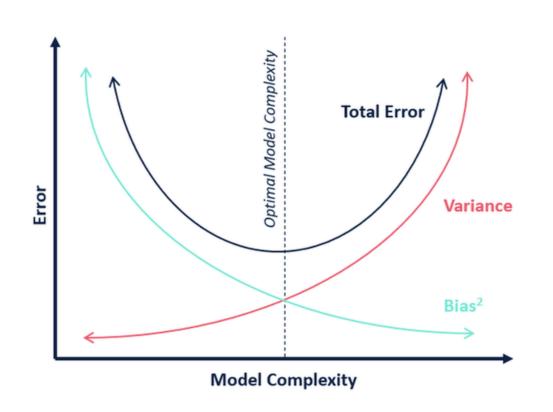






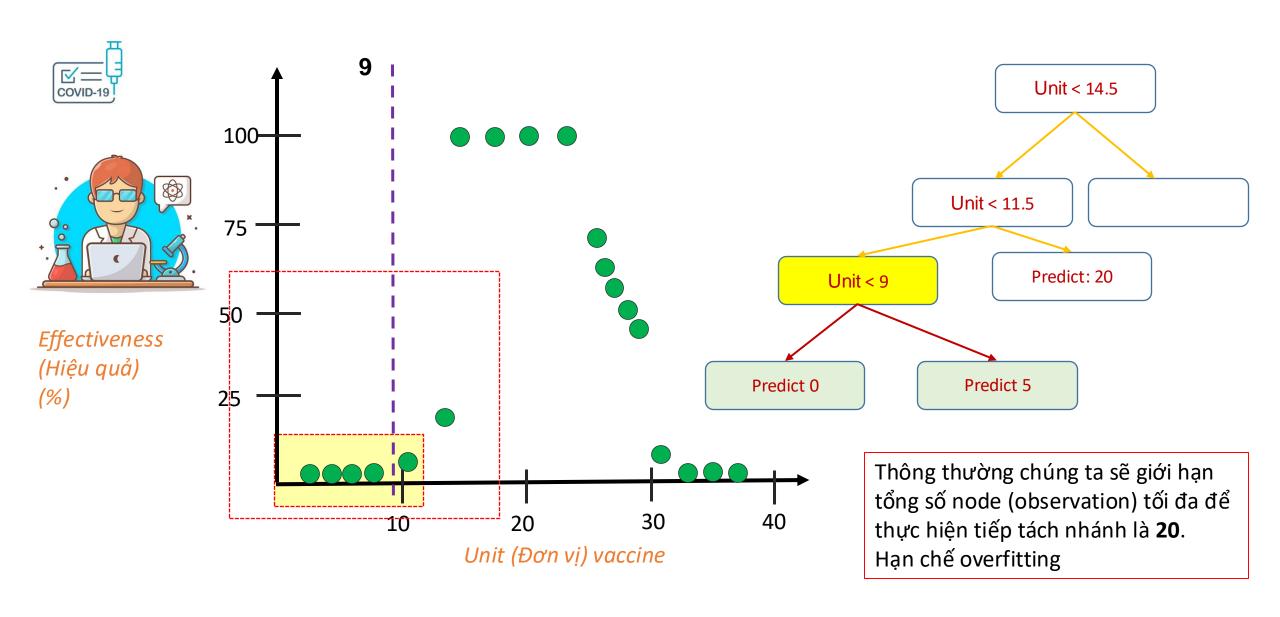


Overfitting Problem

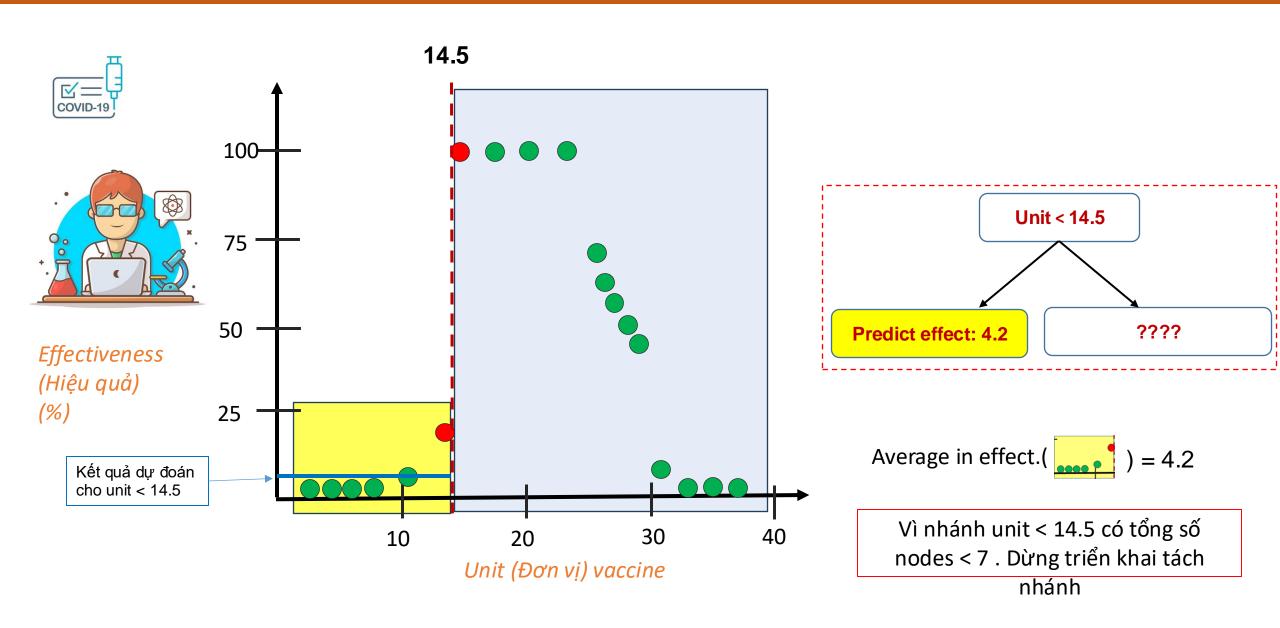




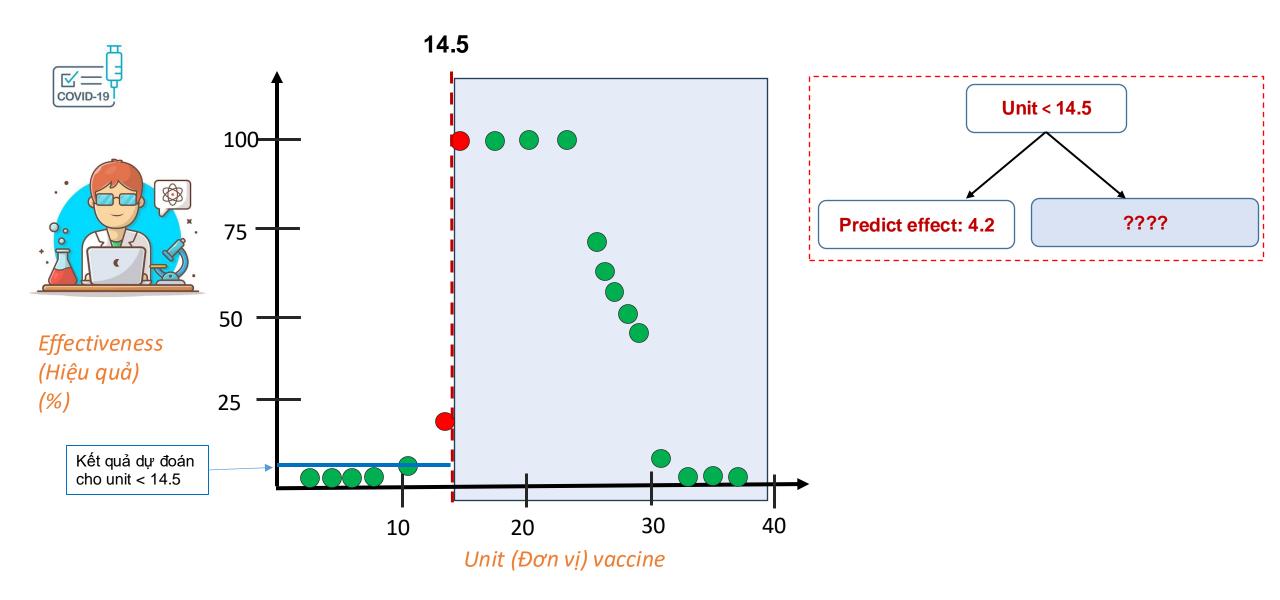




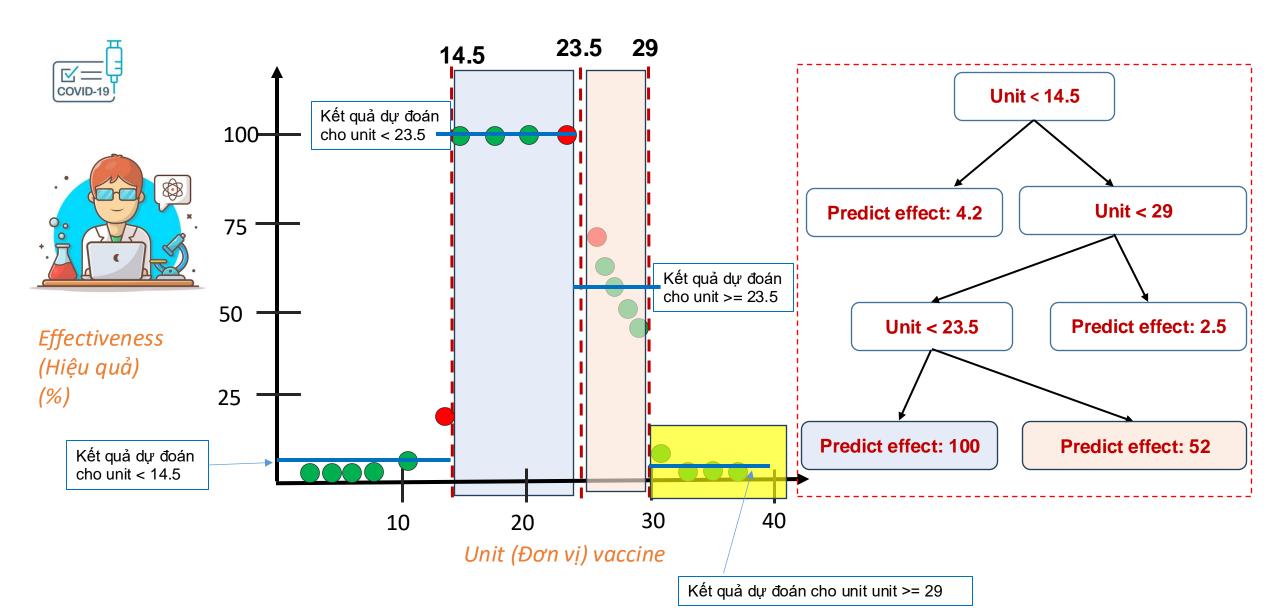




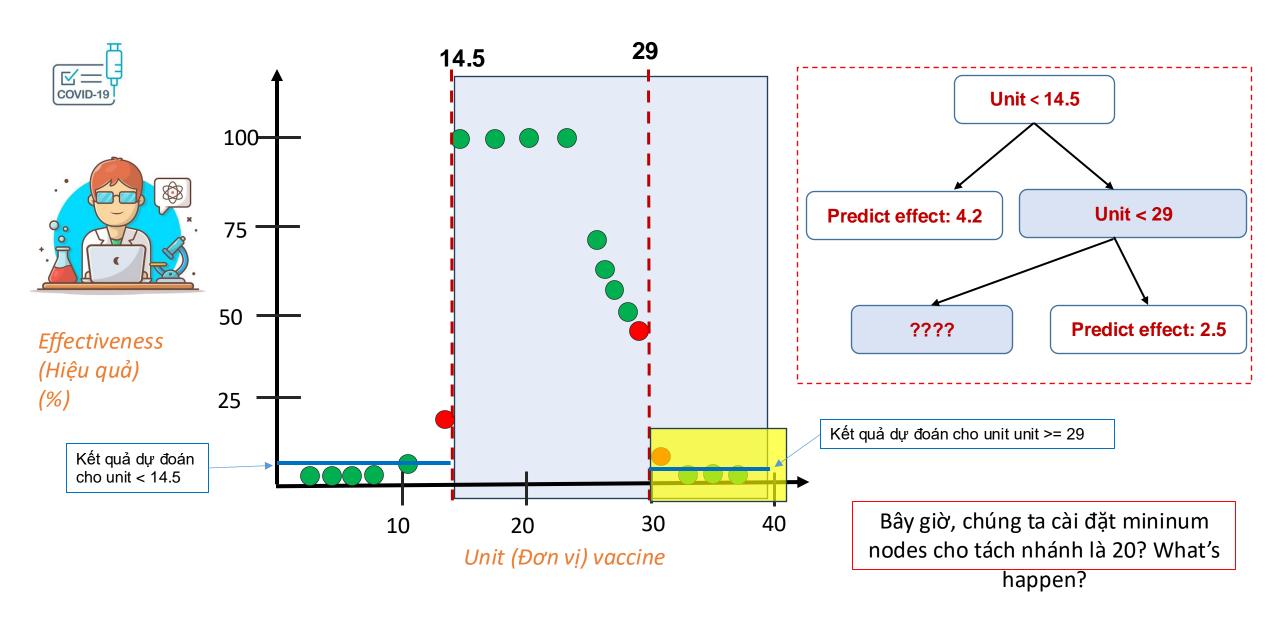




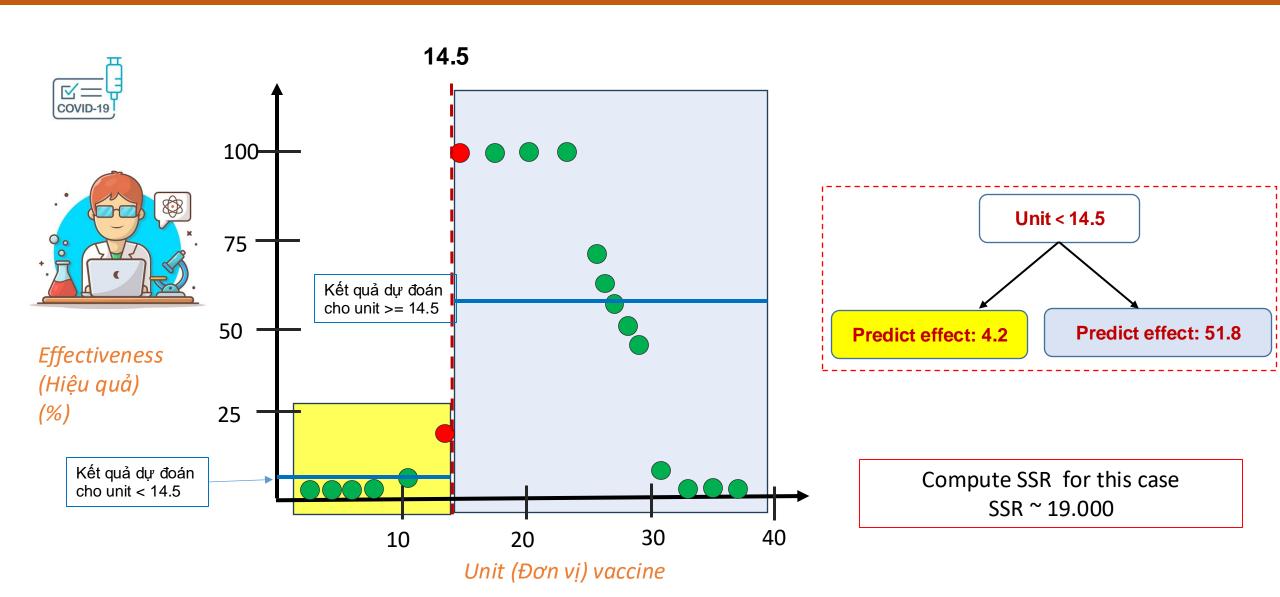


















Unit	Age	Sex	Effect (%)
10	25	Female	98
20	73	Male	0
35	54	Female	100
5	12	Male	44
•••	•••		

Tiêm 5 đơn vị vaccine, 12 tuổi, giới tính nam

quả

vaccine:

Hiệu

44%

Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với liều lượng dùng cố định, tuổi và giới tính của bệnh nhân.



Age note is a root?





Age	Effect (hiệu quả) (%)
25	98
73	0
54	100
12	44
	•••

Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với tuổi (age) của bệnh nhân.

12 tuổi

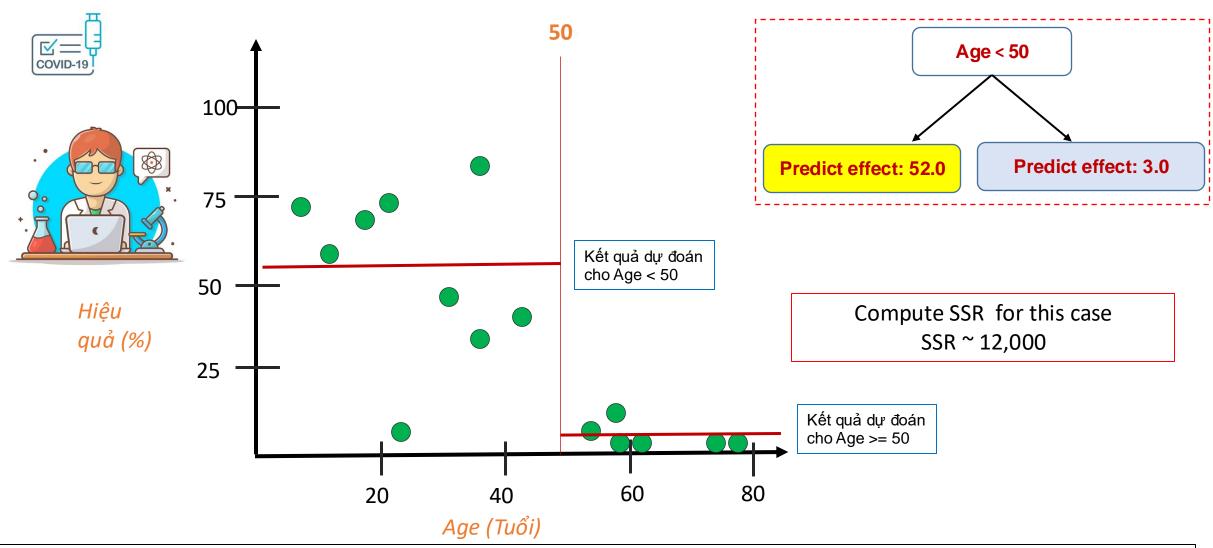




Hiệu quả vaccine:



Age note is a root?



Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với từng liều lượng dùng trên bênh nhân.





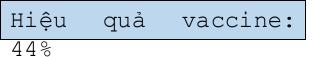


Unit	Age	Sex	Effect (%)
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•••	•••	•••	•••

Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với liều lượng dùng cố định, tuổi và giới tính của bệnh nhân.

Giới tính nam







Sex note is a root?





Sex	Effect (hiệu quả) (%)
Female	98
Male	0
Female	100
Male	44
	•••

Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với giới tính (sex) của bệnh nhân.

Giới tính Male

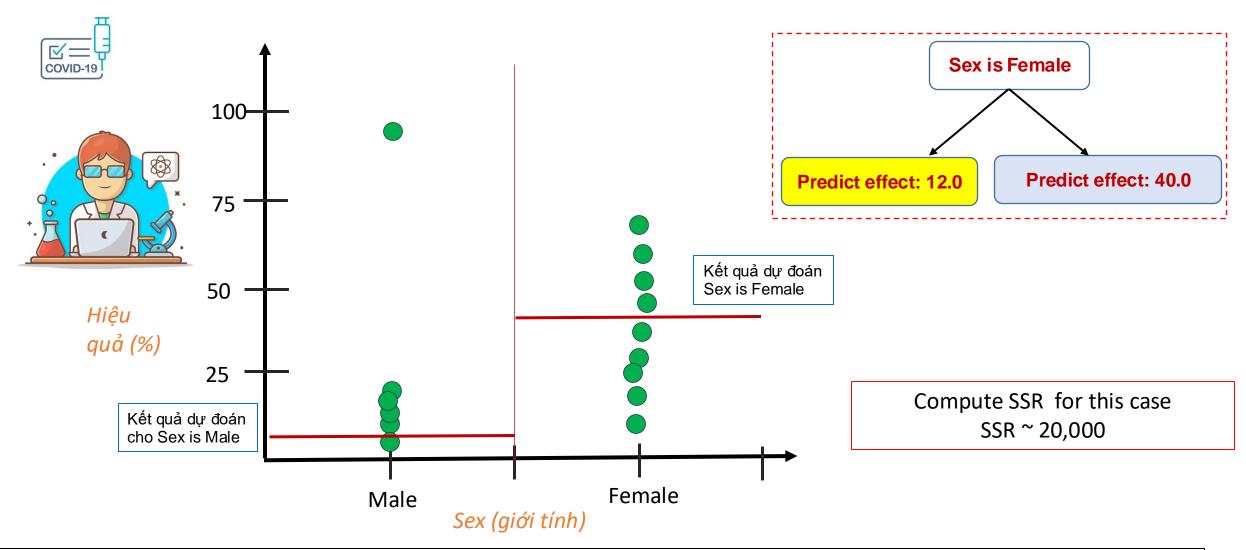




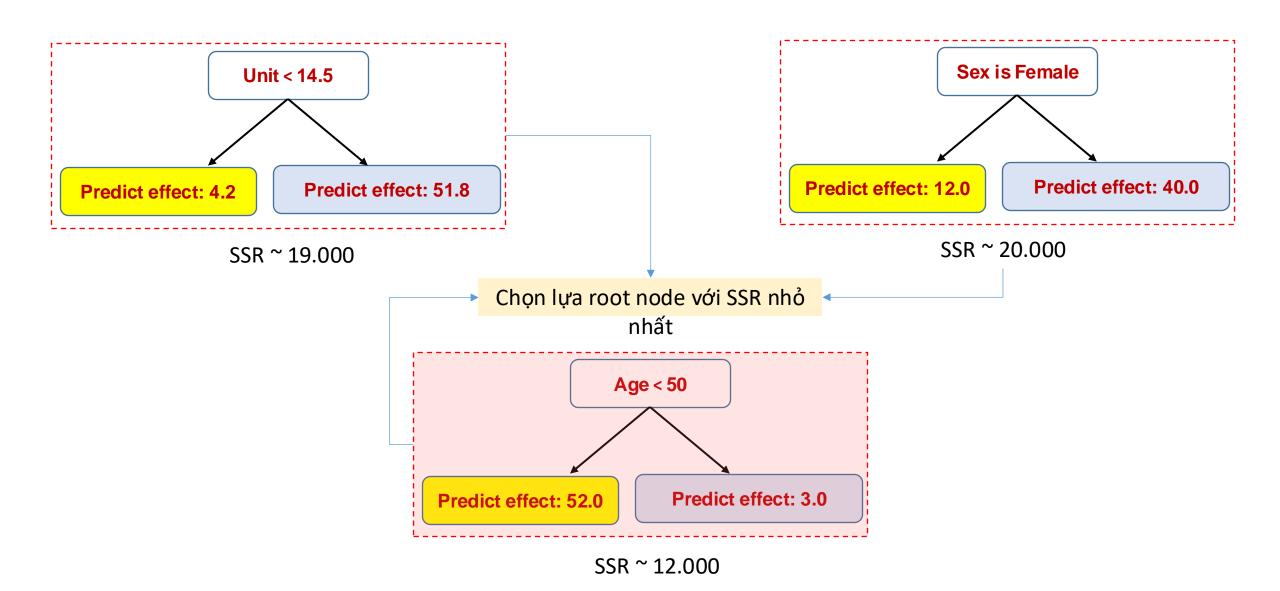
Hiệu quả vaccine:



Sex note is a root?



Khi có 1 vaccine ra đời, chúng ta muốn dự đoán xem nó hiệu quả bao nhiêu % ứng với từng liều lượng dùng trên bênh nhân.

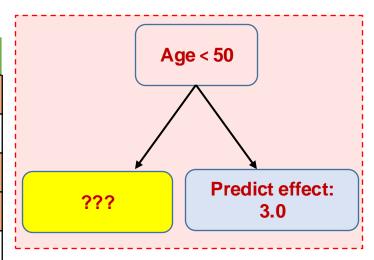








Unit	Age	Sex	Effect (%)
10	25	Female	98
20	73	Male	0
35	54	Female	100
5	12	Male	44
7	80	Male	5



Tiếp tục mở rộng cho trường hợp Age < 50

Unit hoặc Sex là node kế tiếp????



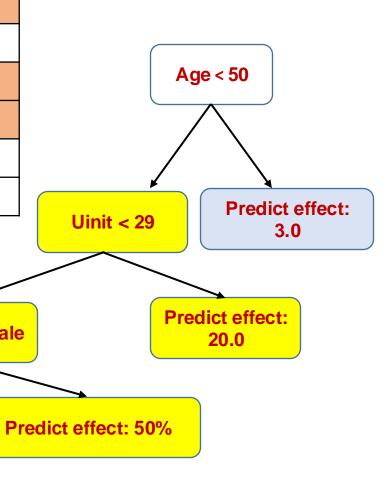
Sex is Female



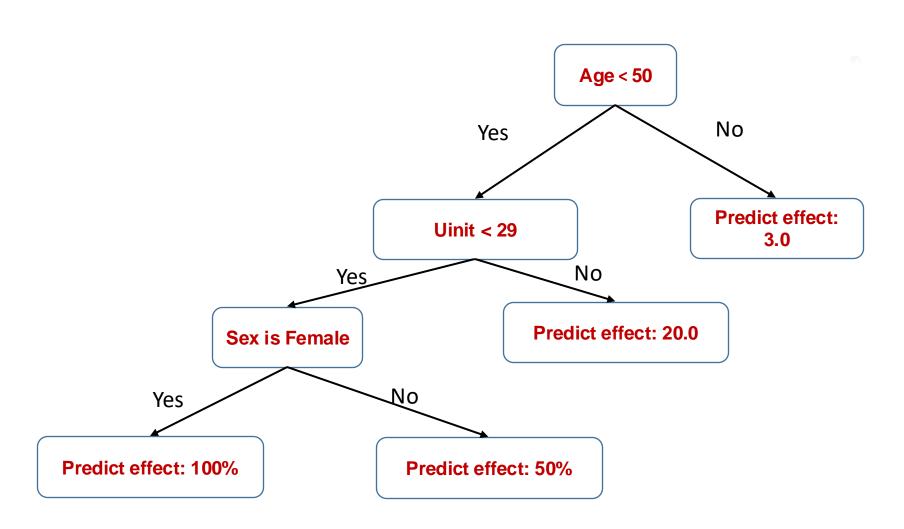


Unit	Age	Sex	Effect (%)
10	25	Female	98
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5	12	Male	44
7	80	Male	5
	•••	•••	•••

Predict effect: 100%







Outline

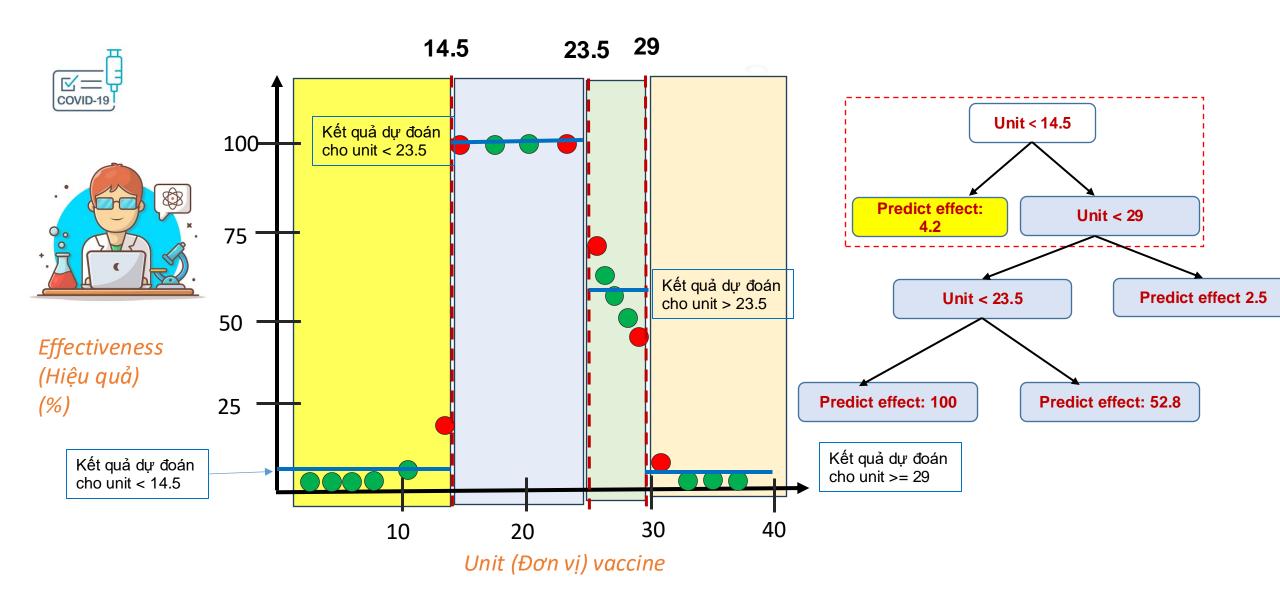
- **Classification Tree: Review**
- **Regression Tree: Motivation**
- **Regression Tree: Clearly Explain**



- **Regression Tree: Overfitting Problem**
- **Examples**

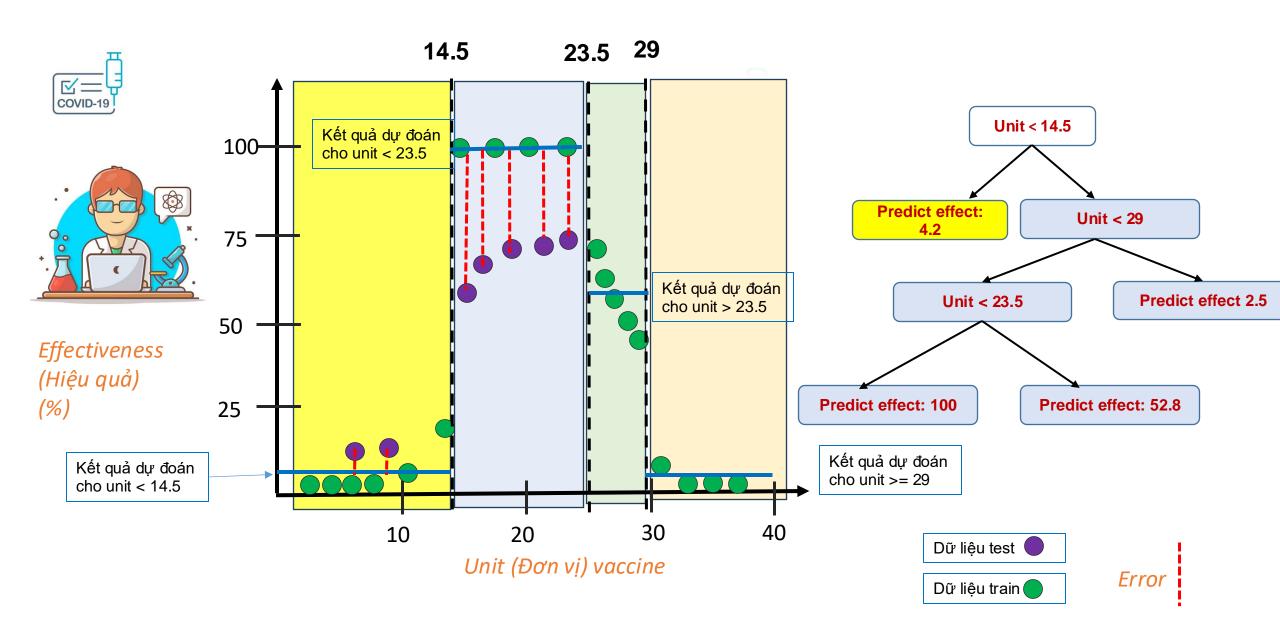


Overfitting Problem



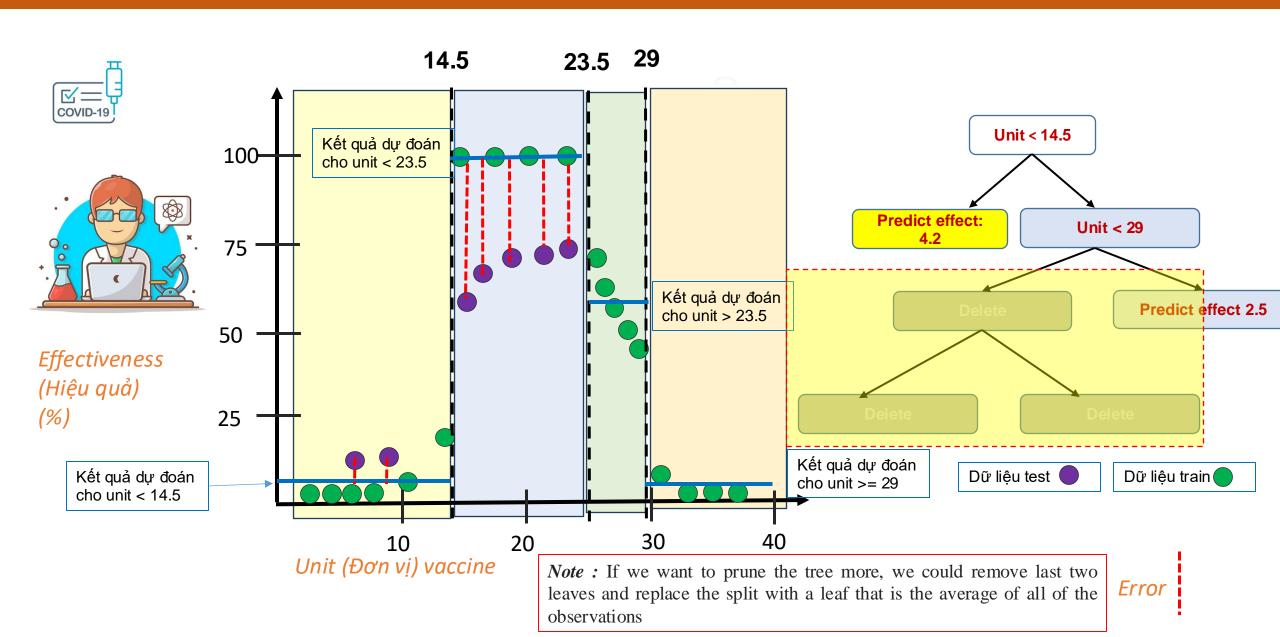


Overfitting Problem



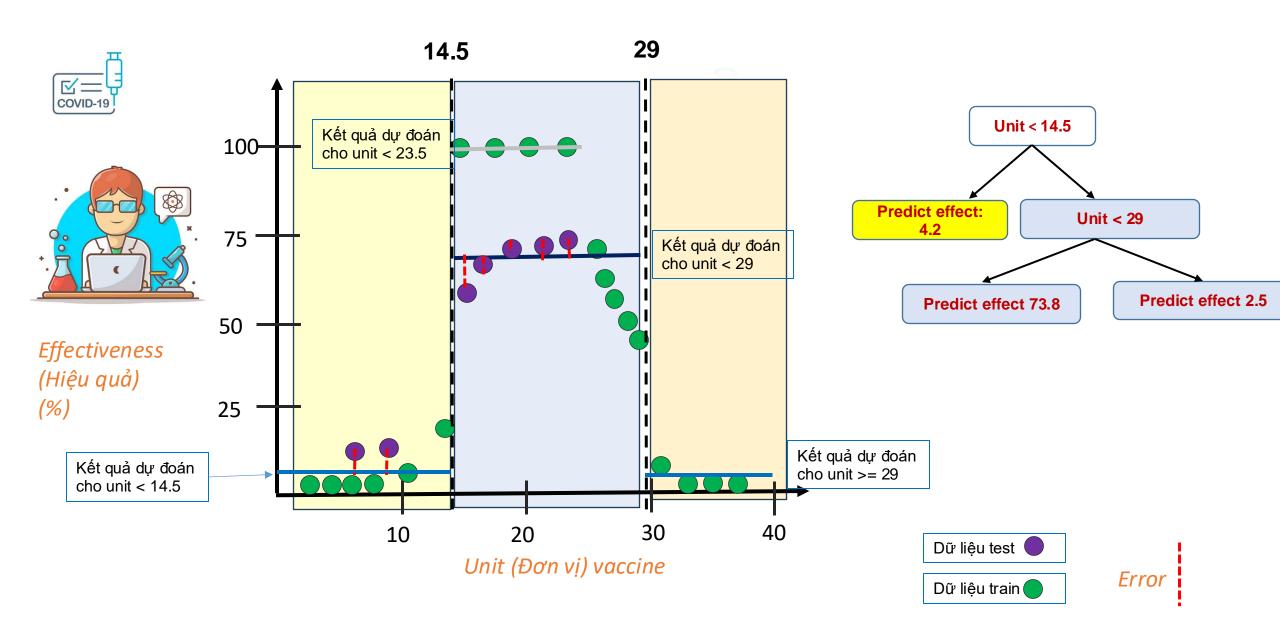


Pruning Solution

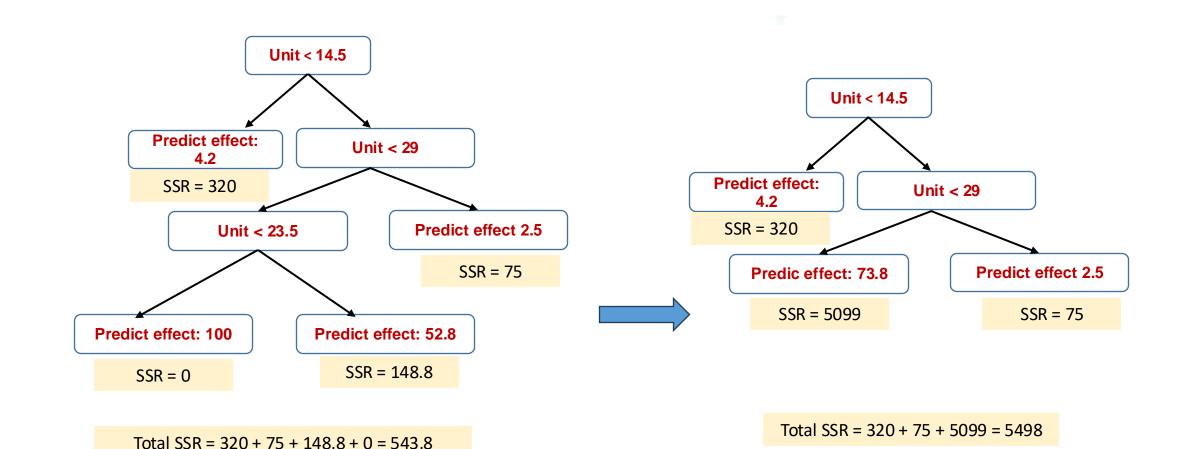




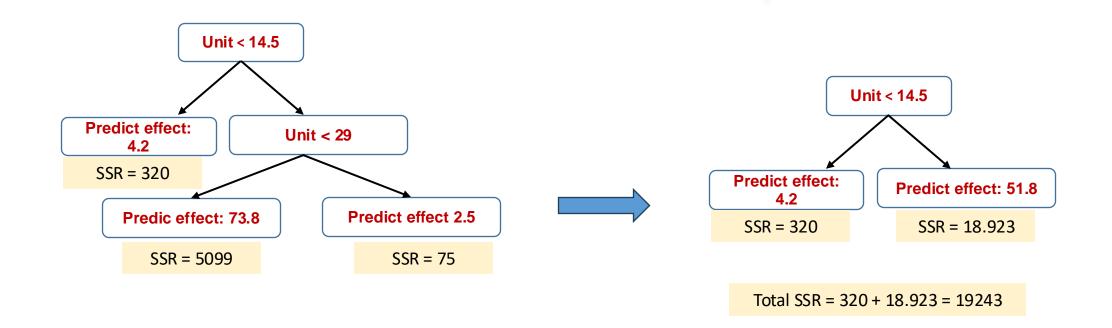
Pruning Solution





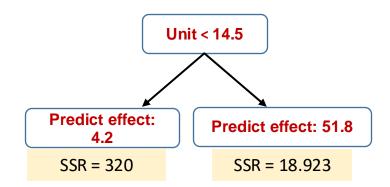






Total SSR = 320 + 75 + 5099 = 5498



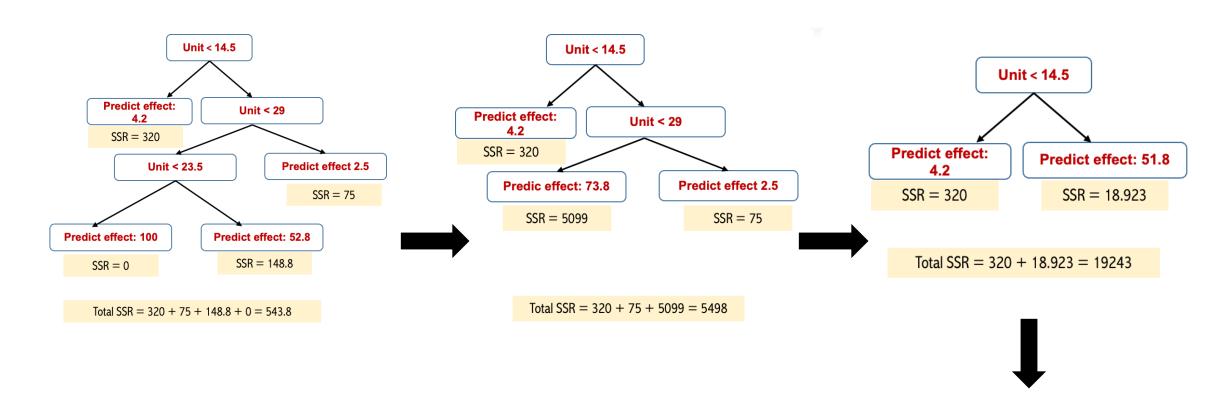


Total SSR = 320 + 18.923 = 19243

Predict effect: 50.5

SSR = 28.897



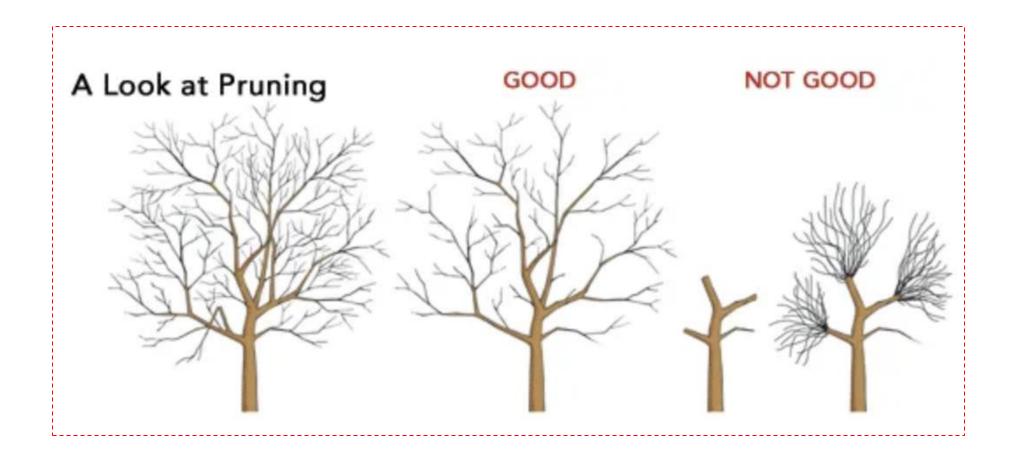


The sum of squared residual is relatively small for the original, full sized tree. But each time we remove a leaf, the sum of squared residual gets larger.

Predict effect: 50.5



Decision Tree-Pruning-Cost Complexity Method



Tree Complexity Penalty

The tree complexity penalty compensates for the difference in the number of leaves.

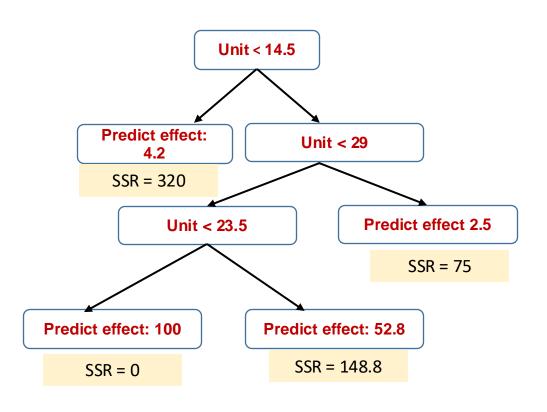
Tree Score = sum of squared residual + α T

 α (alpha) is a tuning parameter that we finding using cross validation. T is the total number of terminal nodes/the total number of leaves

For now, let's let $\alpha = 10,000$ and calculate tree score for each tree.



Tree Score

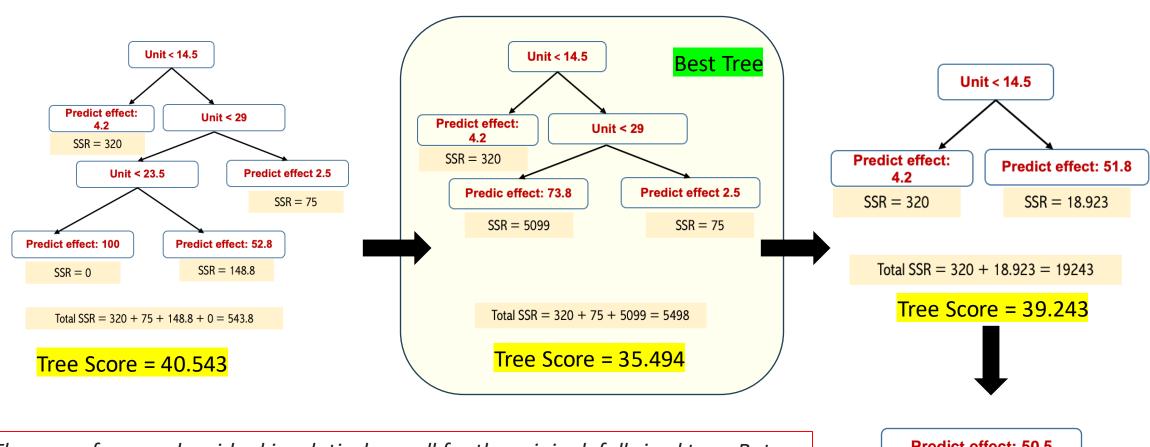


Total SSR =
$$320 + 75 + 148.8 + 0 = 543.8$$

$$\alpha$$
 = 10000, T = 4
Tree Score = Total SSR+ α T
Tree Score = 543 + α T = 40.543

Tree Score

 $\alpha = 10.000$



The sum of squared residual is relatively small for the original, full sized tree. But each time we remove a leaf, the sum of squared residual gets larger.

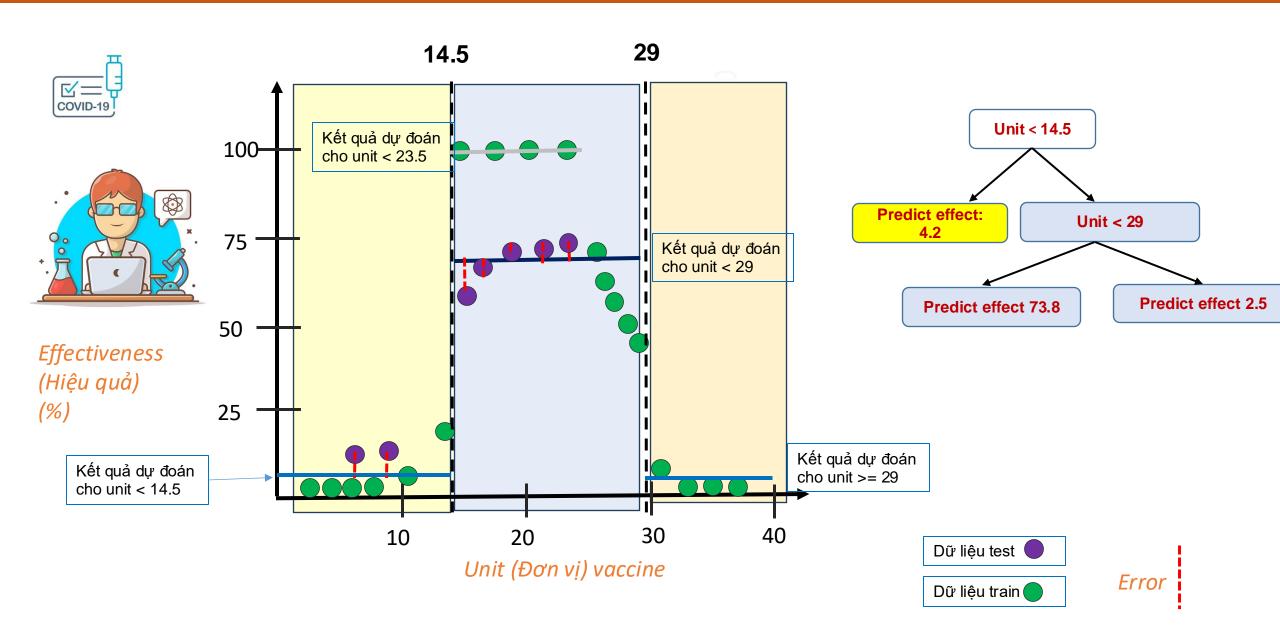
Predict effect: 50.5

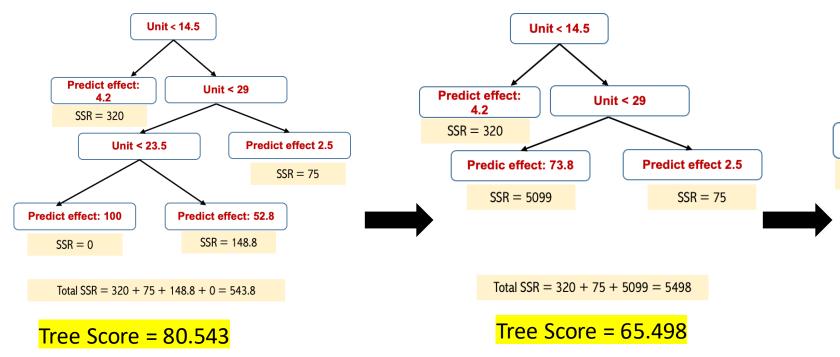
Tree Score = 38.897

SSR = 28.897

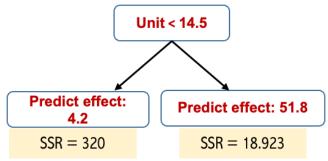


Pruning Solution





The sum of squared residual is relatively small for the original, full sized tree. But each time we remove a leaf, the sum of squared residual gets larger.



Total SSR = 320 + 18.923 = 19243

Tree Score = 59.243

Best Tree

Predict effect: 50.5

Tree Score = 48.897

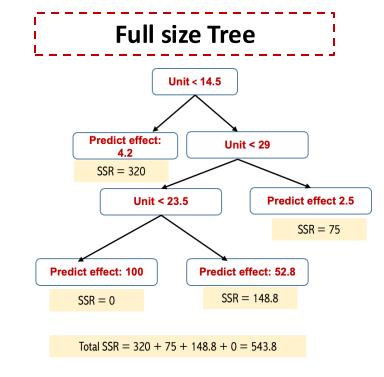
SSR = 28.897



1

Entire dataset							
Unit Age Sex Effect (%							
10	25	Female	98				
20	73	Male	0				
35	54	Female	100				
5	12	Male	44				
7	80	Male	5				

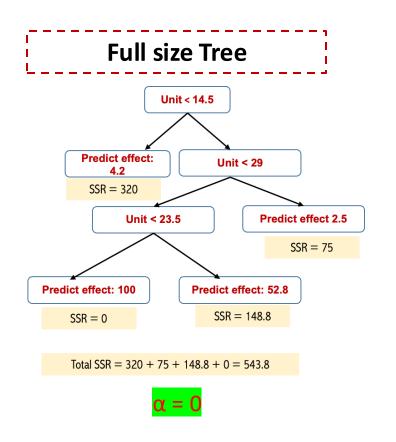
Tree Score = sum of squared residual + αT



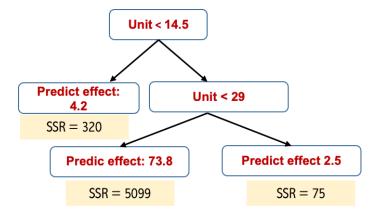
This full size tree has lowest Tree Score when $\alpha = ??$



2



Prunning Tree



Total SSR = 320 + 75 + 5099 = 5498

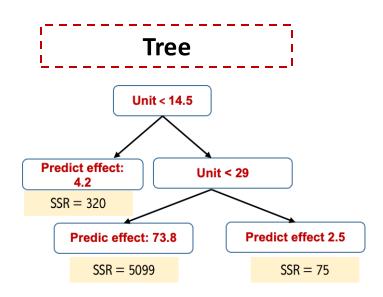
 $\alpha = 10.000$

Tree Score = sum of squared residual + αT

Increase untill pruning leaves will give us a lower Tree Score



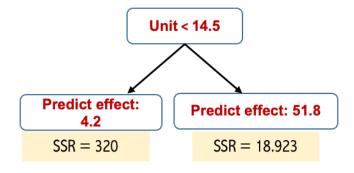
3



Total SSR = 320 + 75 + 5099 = 5498

 $\alpha = 10.000$

Prunning Tree



Total SSR = 320 + 18.923 = 19243

 $\alpha = 15.000$

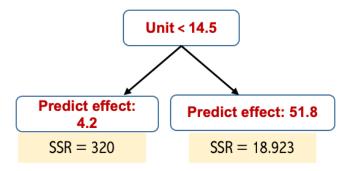
Tree Score = sum of squared residual + α T

Increase untill pruning leaves will give us a lower Tree Score



3





Total SSR = 320 + 18.923 = 19243

 $\alpha = 15.000$

Tree Score = sum of squared residual + αT

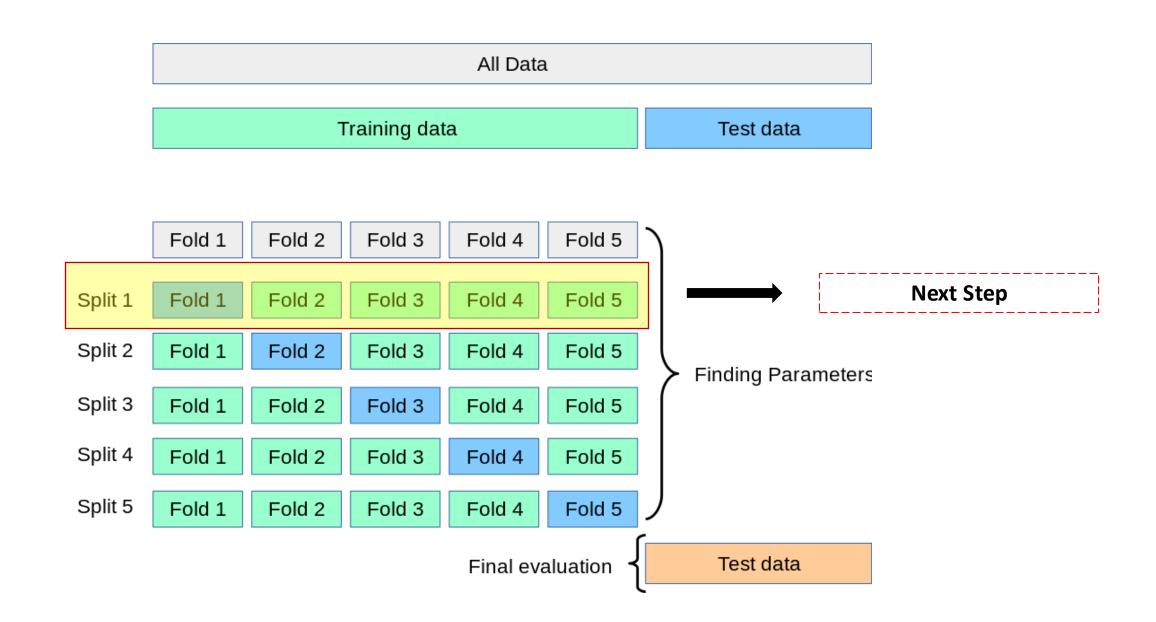
Leaf

Predict effect: 50.5

 $\alpha = 20.000$

Increase untill pruning leaves will give us a lower Tree Score







For each Split							
	Entire	dataset					
Unit	Unit Age Sex Effect (%)						
10	25	Female	98				
20	73	Male	0				
35	54	Female	100				
5	12	Male	44				
7	80	Male	5				



Training dataset							
Unit Age Sex Effect (%							
10	25	Female	98				
20	73	Male	0				



Build Tree with α = 0 , α = 10000, α = 15000, α = 20,000



Testing dataset						
Unit Age Sex Effect (%)						
5	12	Male	44			
7	80	Male	5			



Tree Score with $\alpha = 0$, $\alpha = 10000$, $\alpha = 15000$, $\alpha = 20,000$



	α = 0	α = 10,000	α = 15000	α =20,000
Split 1				
Split 2		•••		•••
Split 3				•••
Split 4		•••		•••
Split 5		•••		•••
Average	50,000	5000	11,000	30,000

In this case, the optimal trees built with α = 10,000 had, on average, the lowest sum of square residuals. So α = 10,000 is our final value.

Outline

- **Classification Tree: Review**
- **Regression Tree: Motivation**
- Regression Tree: Clearly Explain
- Regression Tree: Overfitting Problem



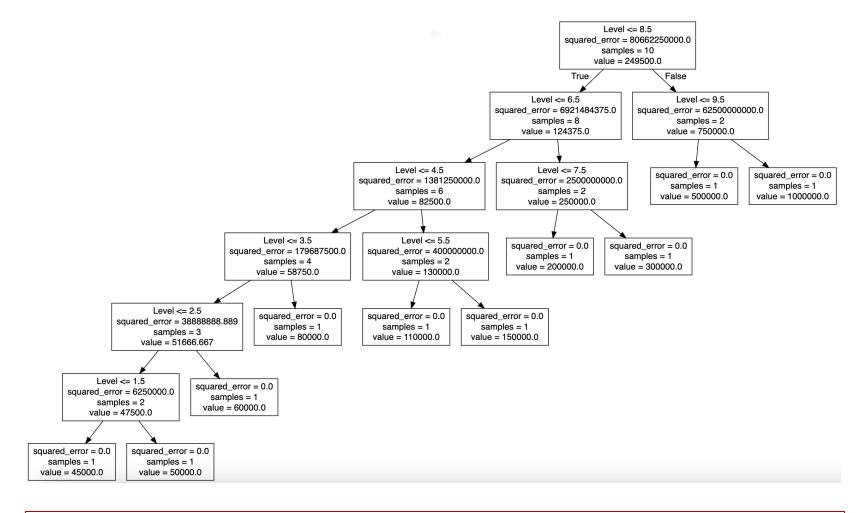
Examples



Position Salaries

Position	Level	Salary
Business Analyst	1	45000
Junior Consultant	2	50000
Senior Consultant	3	60000
Manager	4	80000
Country Manager	5	110000
Region Manager	6	150000
Partner	7	200000
Senior Partner	8	300000
C-level	9	500000
CEO	10	1000000

http://www.webgraphviz.com/

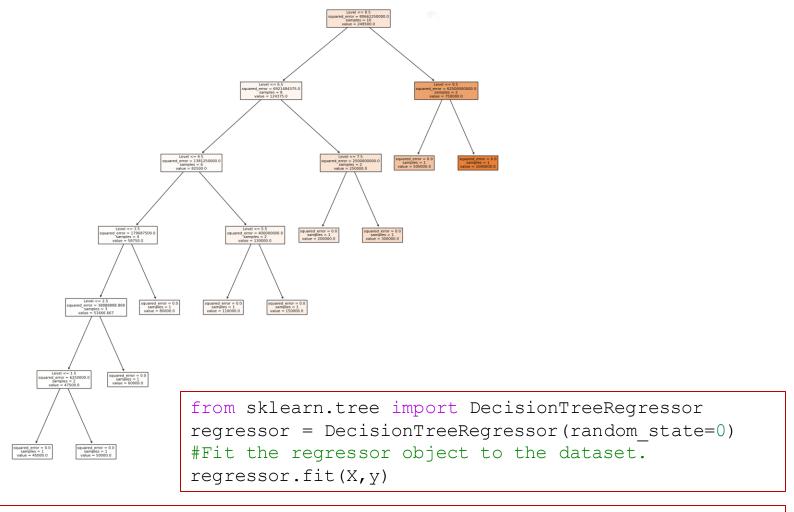


```
export_graphviz(regressor, out_file ='tree.dot',
feature names =["Level"])
```



Position_Salaries

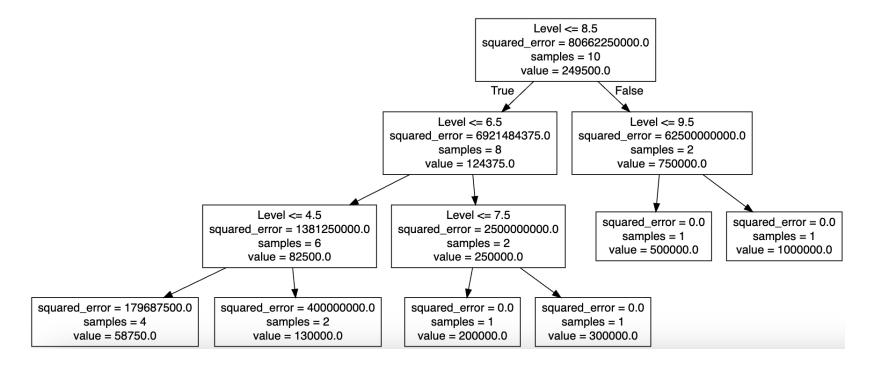
Position	Level	Salary
Business Analyst	1	45000
Junior Consultant	2	50000
Senior Consultant	3	60000
Manager	4	80000
Country Manager	5	110000
Region Manager	6	150000
Partner	7	200000
Senior Partner	8	300000
C-level	9	500000
CEO	10	1000000





Position_Salaries

Position	Level	Salary	
Business Analyst	1	45000	
Junior Consultant	2	50000	
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Manager	4	80000	
Country Manager	5	110000	
Region Manager	6	150000	
Partner	7	200000	
Senior Partner	8	300000	
C-level	9	500000	
CEO	10	1000000	

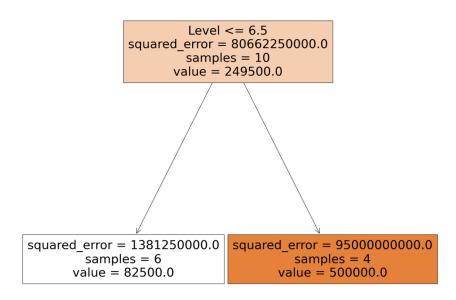


regressor = DecisionTreeRegressor(random_state=0,
max_depth=3)



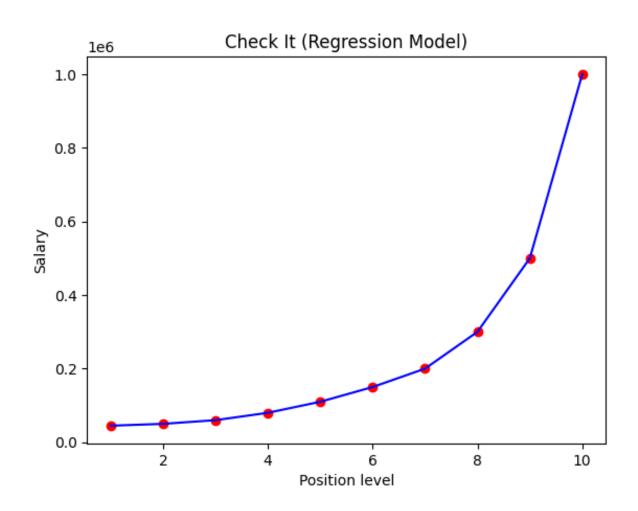
Position_Salaries

Position	Level	Salary
Business Analyst	1	45000
Junior Consultant	2	50000
Senior Consultant	3	60000
Manager	4	80000
Country Manager	5	110000
Region Manager	6	150000
Partner	7	200000
Senior Partner	8	300000
C-level	9	500000
CEO	10	1000000

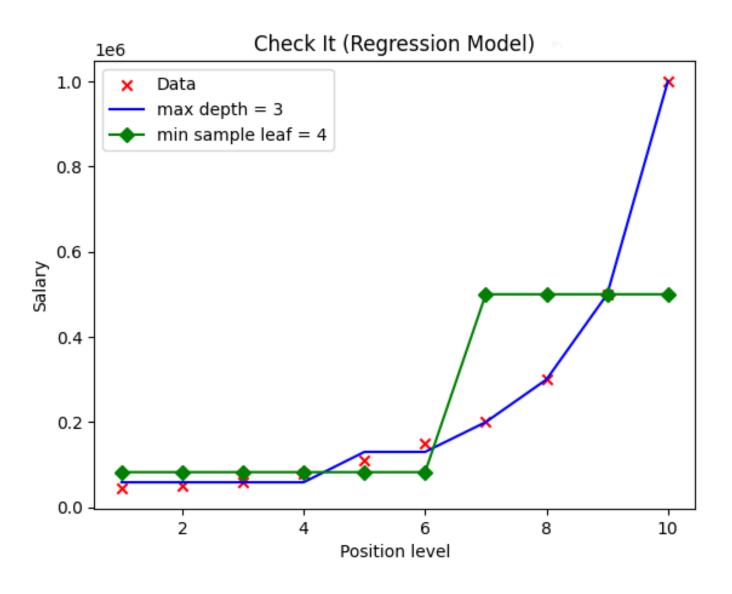


regressor = DecisionTreeRegressor(random_state=0,
min_samples_leaf=4)





```
Visualising the Decision Tree Regression
results
plt.scatter(X, y, color = 'red')
plt.plot(X, regressor.predict(X), color
= 'blue')
plt.title('Check It (Regression Model)')
plt.xlabel('Position level')
plt.ylabel('Salary')
plt.show()
```





Microsoft Stock Price Prediction

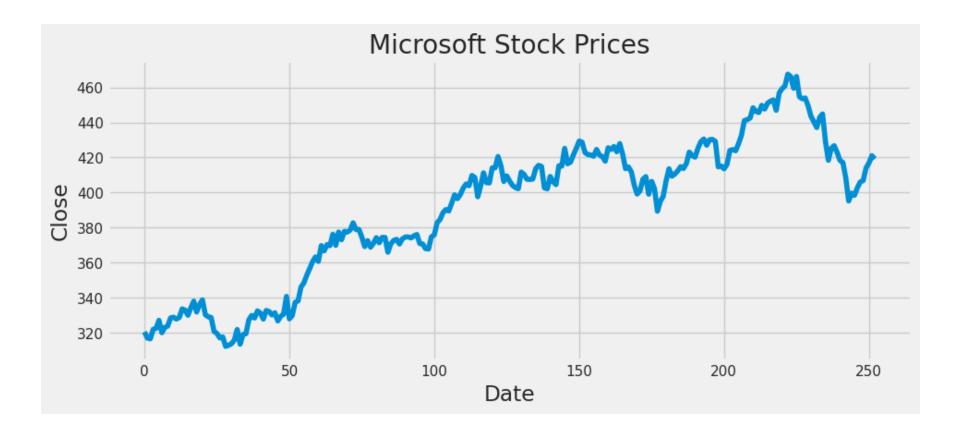
- 1. Visit **Yahoo Finance**
- 2. Search for "MSFT"
- 3. Click on "Historical Data"
- 4. Click on "Download"

		MSFT
Open	High	Low

Date	Open	High	Low	Close	Adj Close	Volume
2023-08-16	320.799988	324.420013	319.799988	320.399994	318.013000	20698900
2023-08-17	320.540009	321.869995	316.209991	316.880005	314.519196	21257200
2023-08-18	314.489990	318.380005	311.549988	316.480011	314.122192	24744800
2023-08-21	317.929993	322.769989	317.040009	321.880005	319.481964	24040000
2023-08-22	325.500000	326.079987	321.459991	322.459991	320.057648	16102000
2023-08-23	323.820007	329.200012	323.459991	327.000000	324.563812	21166400
2023-08-24	332.850006	332.980011	319.959991	319.970001	317.586182	23281400
2023-08-25	321.470001	325.359985	318.799988	322.980011	320.573761	21684100
2023-08-28	325.660004	326.149994	321.720001	323.700012	321.288422	14808500
2023-08-29	321.880005	328.980011	321.880005	328.410004	325.963287	19284600
2023-08-30	328.670013	329.809998	326.450012	328.790009	326.340485	15222100
2023-08-31	329.200012	330.910004	326.779999	327.760010	325.318146	26411000

Microsoft Stock Price Prediction

```
plt.figure(figsize=(10, 4))
plt.title("Microsoft Stock Prices")
plt.xlabel("Date")
plt.ylabel("Close")
plt.plot(data["Close"])
plt.show()
```





Microsoft Stock Price Prediction

```
[5] x = data[["Open", "High", "Low"]]
    y = data["Close"]
    x = x_{to}_{numpy}()
    y = y.to_numpy()
    y = y.reshape(-1, 1)
    from sklearn.model_selection import train_test_split
    xtrain, xtest, ytrain, ytest = train_test_split(x, y, test_size=0.2, random_state=42)
    from sklearn.tree import DecisionTreeRegressor
    model = DecisionTreeRegressor()
    model.fit(xtrain, ytrain)
    ypred = model.predict(xtest)
    data = pd.DataFrame(data={"Predicted Rate": ypred})
    print(data.head())
\overline{\Rightarrow}
       Predicted Rate
           442.570007
           326.670013
           371.299988
           424.589996
           407.570007
```





