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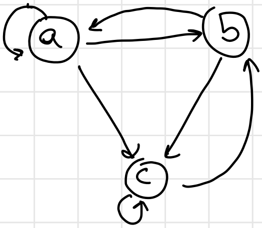
MMD Problem Set 9 Exercise 1

a)



	a	b	c
a	1/3	1/2	0
b	1/3	0	1/2
c	1/3	1/2	1/2

$$r^0 = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} \qquad M = \begin{pmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{pmatrix}$$

$$r^1 = r^0 \cdot M$$

$$= \begin{pmatrix} 1/9 + 1/6 + 0 \\ 1/9 + 0 + 1/6 \\ 1/9 + 1/6 + 1/6 \end{pmatrix} = \begin{pmatrix} 5/18 \\ 5/18 \\ 4/9 \end{pmatrix}$$

$$r^2 = r^1 \cdot M$$

$$= \begin{pmatrix} 5/18 \cdot 1/3 + 5/18 \cdot 1/2 + 4/9 \cdot 0 \\ \vdots \end{pmatrix} = \underline{\underline{\begin{pmatrix} 25/108 \\ 17/54 \\ 49/108 \end{pmatrix}}}$$ stop

$$r^3 = r^2 \cdot M = \begin{pmatrix} 19/81 \\ 197/648 \\ 299/648 \end{pmatrix}$$

b) We want to find Eigenvector v, such that

$M \cdot v = v$

$(M - I) \cdot v = 0$ with I being the identity matrix

$$M - I = \begin{bmatrix} -2/3 & 1/2 & 0 \\ 1/3 & -1 & 1/2 \\ 1/3 & 1/2 & -1/2 \end{bmatrix}$$

Solve the resulting linear equation

$\vdots$

The eigenvector corresponding to eigenvalue 1 and thus our stationary distribution is

$$\underline{\underline{v = \begin{pmatrix} 3/13 \\ 4/13 \\ 6/13 \end{pmatrix}}}$$

c) $A = \beta \cdot M + (1-\beta) \cdot [1/N]$ $\quad \beta = 0.8$

$$A = 0.8 \cdot \begin{pmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{pmatrix} + 0.2 \cdot \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix} 1/3 & 7/15 & 1/15 \\ 1/3 & 1/15 & 7/15 \\ 1/3 & 7/15 & 7/15 \end{pmatrix}}}$$

$$r^0 = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}, \quad r^1 = r^0 A = \underline{\underline{\begin{pmatrix} 13/45 \\ 13/45 \\ 19/45 \end{pmatrix}}},$$ stop

$$r^2 = r^1 \cdot A = \begin{pmatrix} 7/27 \\ 211/675 \\ 289/675 \end{pmatrix}$$

d) same process as in b),

$\vdots$

$$\underline{\underline{\vec{v} = \begin{pmatrix} 7/27 \\ 25/81 \\ 35/81 \end{pmatrix}}}$$

Exercise 2

c) We utilize the properties of the clique.

Each node has a total of n+1 outgoing links

The pagerank x of a node in the clique is given by:

$$x = \beta \cdot \left(\frac{x}{n+1} \cdot (n-1) + \frac{y}{n+1} \right) + \frac{1-\beta}{N}$$

$$= \beta \cdot \frac{x \cdot (n-1)}{n+1} + \beta \cdot \frac{y}{n+1} + \frac{1-\beta}{N}$$

Now, pagerank y for node outside the clique is given by

$$y = \beta \cdot \frac{n \cdot x}{n+1} + \frac{1-\beta}{N}$$

Now substituting y in equation for x, we get:

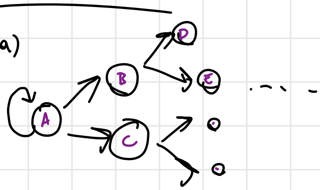
$$x = \beta \cdot \frac{x \cdot (n-1)}{n+1} + \beta \cdot \left[\beta \cdot \frac{n \cdot x}{n+1} + \frac{1-\beta}{N} \right] + \frac{1-\beta}{N}$$

simplify $\Rightarrow$ $x = \frac{\beta - 1}{\beta n - n - 1}$, $y = \frac{\beta - 1}{\beta n - n - 1}$

same value!!!

Exercise 5

a)



After deleting all dead-end nodes, the resulting graph has to look like this:

Thus the matrix is not really a matrix but more a number, $M = [1]$ the pagerank of A is therefore $\underline{\underline{\text{also } 1}}$

b) The page-rank of a dead-end node r_i at level i is proportional to pagerank r_p of its parent node

If a parent node has k children, the pagerank assigned to each child is

$r_i = \frac{r_p}{k}$, in our case $k=2$.

So at level i, the pagerank is derived from parent node $i-1$: $r_i = \frac{r_{i-1}}{2}$

↳ recursion back until root node $r_{root} = 1$ leads to: $\underline{\underline{r_i = \frac{1}{2^i}}}$

At i_{th} level, there are 2^i nodes in the tree, the total pagerank for all nodes at this level is

sum of pageranks at level i:

$2^i \cdot \frac{1}{2^i} = 1$ (constant at each level i)

summing over all levels of the tree

$\sum_{i=1}^{n} 1 = \underline{\underline{n}}$

The sum of pageranks exceeds 1 in this model. This happens because the dropping method we use doesn't enforce normalization at the end. In this recursive approach the assigned pagerank grows as we move deeper into the binary tree.
In real-world scenarios, the pagerank model normalizes the pagerank across all nodes (e.g. by taxation or teleportation).