MMD Problem Set 10

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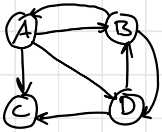
Exercise 3



$k=2$

	A	B	C	D
A	0	1/2	0	0
B	1/3	0	0	1/2
C	1/3	0	0	1/2
D	1/3	1/2	0	0

M_{11}, M_{21}, M_{12}, M_{22}

M_{11}

source	degree	Dest.
A	1	B
B	1	A

M_{21}

source	degree	Dest.
A	2	C, D
B	1	D

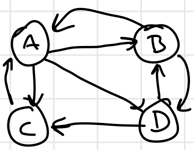
M_{12}

source	degree	Dest.
D	1	B

M_{22}

source	degree	Dest.
D	1	C

Exercise 4



M

	A	B	C	D
A	0	1/2	1	0
B	1/3	0	0	1/2
C	1/3	0	0	1/2
D	1/3	1/2	0	0

with $\beta = 0.8$, $\beta \cdot M = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix}$

a) $S = \{A\}$

To be iterated: $v' = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \cdot v + \begin{pmatrix} 1/5 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$r_0 = \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix}$ $\quad r_1 = r_0 \cdot \beta M + \begin{pmatrix} 1/5 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/4 \cdot 0 + 1/4 \cdot 2/5 + 1/4 \cdot 4/5 + 1/4 \cdot 0 + 1/5 \\ 1/4 \cdot 4/15 + 1/4 \cdot 0 + 1/4 \cdot 0 + 1/4 \cdot 2/5 + 0 \\ 1/4 \cdot 4/15 + 1/4 \cdot 0 + 1/4 \cdot 0 + 1/4 \cdot 2/5 + 0 \\ 1/4 \cdot 4/15 + 1/4 \cdot 2/5 + 1/4 \cdot 0 + 1/4 \cdot 0 + 0 \end{pmatrix}$

$= \begin{pmatrix} 1/10 + 2/10 + 2/10 \\ 1/15 + 1/10 + 0 \\ 1/15 + 1/10 + 0 \\ 1/15 + 1/10 + 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/6 \\ 1/6 \\ 1/6 \end{pmatrix}$

$r_2 =$ too lazy to calculate any further, but we get the gist! :)

b) $S = \{A, C\}$

To be iterated: $v' = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \cdot v + \begin{pmatrix} 1/10 \\ 0 \\ 1/10 \\ 0 \end{pmatrix}$

$r_0 = \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix}$ $\quad r_1 = r_0 \cdot \beta M + \begin{pmatrix} 1/10 \\ 0 \\ 1/10 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/4 \cdot 0 + 1/4 \cdot 2/5 + 1/4 \cdot 4/5 + 1/4 \cdot 0 + 1/10 \\ 1/4 \cdot 4/15 + 1/4 \cdot 0 + 1/4 \cdot 0 + 1/4 \cdot 2/5 + 0 \\ 1/4 \cdot 4/15 + 1/4 \cdot 0 + 1/4 \cdot 0 + 1/4 \cdot 2/5 + 1/10 \\ 1/4 \cdot 4/15 + 1/4 \cdot 2/5 + 1/4 \cdot 0 + 1/4 \cdot 0 + 0 \end{pmatrix}$

$\begin{pmatrix} 1/10 + 2/10 + 1/10 \\ 1/15 + 1/10 + 0 \\ 1/15 + 1/10 + 1/10 \\ 1/15 + 1/10 + 0 \end{pmatrix} = \begin{pmatrix} 12/30 \\ 5/30 \\ 8/30 \\ 5/30 \end{pmatrix}$

$r_2 =$ too lazy to calculate any further, but we get the gist! :)

Exercise 5

$y = a \cdot x + b \cdot \frac{?}{m} + c \cdot \frac{k}{n}$

Contribution from outside to t: $\underline{x}$

Contribution from supporting pages: $\frac{\beta \cdot y}{m}$, with m supporting pages $\underline{\beta \cdot y}$

Contribution from second-tier pages: Each page links to $\frac{m}{k}$ supporting pages

PageRank: $\frac{\beta y}{k} + \frac{(1-\beta)}{n}$

$\Rightarrow$ contribution to t: $\beta \cdot \left(\frac{\beta y}{k} + \frac{(1-\beta)}{n}\right)$,

with k second tier pages

$$k \cdot \beta \cdot \left(\frac{\beta y}{k} + \frac{(1-\beta)}{n}\right) = \underline{\beta^2 y + \beta(1-\beta)\frac{k}{n}}$$

Total PageRank of t:

$$y = x + \beta y + \beta^2 y + \beta(1-\beta) \cdot \frac{k}{n}$$

$$y - \beta y - \beta^2 y = x + \beta(1-\beta)\frac{k}{n}$$

$$y \cdot (1-\beta-\beta^2) = x + \beta(1-\beta)\frac{k}{n}$$

$$\underline{y = \frac{x}{1-\beta-\beta^2} + \frac{\beta(1-\beta)}{1-\beta-\beta^2}\frac{k}{n}}$$

$b \cdot \frac{m}{n}$ comes from the contribution of supporting pages, $\beta \cdot y$. Since y already includes this, we drop the $\frac{m}{n}$ term for simplicity

$\Rightarrow$ Now set $a = \frac{1}{1-\beta-\beta^2}$ and $c = \frac{\beta \cdot (1-\beta)}{1-\beta-\beta^2}$, $b = 0$

$\Downarrow$

$$y = a \cdot x + c \cdot \frac{k}{n}$$

For $\beta = 0{,}85$:

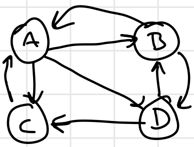
$$a = \frac{1}{1-0{,}85-0{,}85^2} \approx \underline{-1{,}746}$$

$b = \underline{0}$

$$c = \frac{0{,}85 \cdot (1-0{,}85)}{1-0{,}85-0{,}85^2} \approx \underline{-0{,}223}$$

$$\Rightarrow \underline{\underline{y = -1{,}746\,x - 0{,}223 \cdot \frac{k}{n}}}$$

Exercise 6



M

	A	B	C	D
A	0	1/2	1	0
B	1/3	0	0	1/2
C	1/3	0	0	1/2
D	1/3	1/2	0	0

with $\beta = 0{,}8$, $\beta \cdot M = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix}$

a) $S = \{B\}$

To be iterated: $v' = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \cdot v + \begin{pmatrix} 0 \\ 1/5 \\ 0 \\ 0 \end{pmatrix}$

$r_0 = \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix}$ $\quad r_1 = r_0 \cdot \beta M + \begin{pmatrix} 0 \\ 1/5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/4 \cdot 0 + 1/4 \cdot 2/5 + 1/4 \cdot 4/5 + 1/4 \cdot 0 + 0 \\ 1/4 \cdot 4/15 + 1/4 \cdot 0 + 1/4 \cdot 0 + 1/4 \cdot 2/5 + 1/5 \\ 1/4 \cdot 4/15 + 1/4 \cdot 0 + 1/4 \cdot 0 + 1/4 \cdot 2/5 + 0 \\ 1/4 \cdot 4/15 + 1/4 \cdot 2/5 + 1/4 \cdot 0 + 1/4 \cdot 0 + 0 \end{pmatrix}$ $\frac{38}{60}$

$$\text{Trustrank} = \begin{pmatrix} 1/10 + 2/10 \\ 1/15 + 1/10 + 2/10 \\ 1/15 + 1/10 + 0 \\ 1/15 + 1/10 + 0 \end{pmatrix} = \begin{pmatrix} 9/30 \\ 11/30 \\ 5/30 \\ 5/30 \end{pmatrix} \begin{matrix} 18/60 \\ 22/60 \\ 10/60 \\ 10/60 \end{matrix}$$

r_2 = too lazy to calculate a further, but we get the gist! :)

$$\text{Pagerank } r_1 = \begin{pmatrix} 1/4 \cdot 0 + 1/4 \cdot 2/5 + 1/4 \cdot 4/5 + 1/4 \cdot 0 \\ 1/4 \cdot 4/15 + 1/4 \cdot 0 + 1/4 \cdot 0 + 1/4 \cdot 2/5 \\ 1/4 \cdot 4/15 + 1/4 \cdot 0 + 1/4 \cdot 0 + 1/4 \cdot 2/5 \\ 1/4 \cdot 4/15 + 1/4 \cdot 2/5 + 1/4 \cdot 0 + 1/4 \cdot 0 + \end{pmatrix}$$

$$= \begin{pmatrix} 1/10 + 2/10 \\ 1/15 + 1/10 \\ 1/15 + 1/10 \\ 1/15 + 1/10 \end{pmatrix} = \begin{pmatrix} 3/10 \\ 1/6 \\ 1/6 \\ 1/6 \end{pmatrix}$$

Spam mass:

node A: $r_p^- = r_p - r_p^+ = \frac{3}{10} - \frac{3}{10} = 0$

spam mass(A) $= \frac{r_p^-}{r_p} = \frac{0}{3/10} = 0$

node B: $r_p^- = r_p - r_p^+ = \frac{10}{60} - \frac{22}{60} = -\frac{12}{60}$

Spam mass(B) $= \frac{-12/60}{10/60} = -1{,}2$

node C: $r_p^- = r_p - r_p^+ = \frac{10}{60} - \frac{10}{60} = 0$

Spam mass(C) $= \frac{0}{10/60} = 0$

node D: $r_p^- = r_p - r_p^+ = \frac{10}{60} - \frac{10}{60} = 0$

Spam mass(D) $= \frac{0}{10/60} = 0$

Final spam mass = [0, -1.2, 0, 0]