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Banerjee's Test:

	$i=0$	$i=n-1$
$j=0$	$f(0,0)$	$f(n-1,0)$
$j=n-1$	$f(0,n-1)$	$f(n-1,n-1)$

if $b_0 - a_0 < L$ OR $b_0 - a_0 > U$

$\Rightarrow$ then dependency is broken

if $L > b_0 - a_0$ OR $U < b_0 - a_0$ $\Rightarrow$ it can be parallelized

Complete Banerjee's Test

$f(i,j) = a_1 \times i - b_1 \times j$ $\quad 0 \leq j < i < n$

Anti-dependence: $f(i,j) = a_1 \times i - b_1 \times j$ $\quad 0 \leq j < i < n$

We evaluate the 3 "boundary" values for (i,j): $(1,0)$, $(n-1,0)$, $(n-1,n-2)$

	$i=1$	$i=n-1$
$j=0$	$(1,0)$	$(n-1,0)$
$j=n-2$	N/A	$(n-1,n-2)$

$\Rightarrow$ U (upper bound) = max result

L (Lower bound) = min result

if $b_0 - a_0 < L$ OR $b_0 - a_0 > U$ then the anti-dependence is broken.

True-dependence: $f(i,j) = a_1 \times i - b_1 \times j$ $\quad 0 < i \leq j < n$

We evaluate 3 "boundary" values: $(0,0)$, $(0,n-1)$, $(n-1,n-1)$

	$i=0$	$i=n-1$
$j=0$	$f(0,0)$	N/A
$j=n-1$	$f(0,0)$	$f(n-1,n-1)$

$\Rightarrow \left\{ \begin{array}{l} U = \text{max result} \\ L = \text{min result} \end{array} \right.$

Assignment 1: Test Data Dependencies
Comp30250 : Parallel Computing

$N = 3150 \quad k_1 = 395 \quad k_2 = 395 \quad k_3 = 36 \quad k_4 = 85 \quad k_5 = 135 \quad k_6 = 315.$

Loop 1:
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for (i=0, i<3150, i++)
    a[(3151)*i + 395] = b[i]*c[i]
    d[i]              = a[(3150+1)*i + 395*3150] + e
}
```

GCD Test: $a_1 \times i - b_1 \times j = b_0 - a_0$ $\quad GCD(a_1, b_1) = G \quad \frac{b_0 - a_0}{G} \overset{?}{=} \mathbb{Z}$

$f(i) = (3151)i + 395 \qquad g(j) = (3151)i + (395)(3150)$

$a_0 = 395 \quad a_1 = 3151 \qquad b_0 = (395)(3150) = 1244250 \quad b_1 = 3151$

$GCD(3151, 1244250) = G \qquad b_0 - a_0 = 3151 - 395$

$G = 1 \qquad = 2756$

$\Rightarrow \left(\frac{a_1}{G}\right) * i - \left(\frac{b_1}{G}\right) j = \frac{(b_0 - a_0)}{G} \qquad \Rightarrow \frac{2756}{1} = 2756$ valid integer ∴ divides $b_0 - a_0$ and there could be a dependency.

Banerjee's Test: $a_1 \times i - b_1 \times j \quad 0 \leq i < n \;\&\; 0 \leq j < n$

	$i=0$	$i=n-1$
$j=0$	$(0,0)$	$(0,n-1)$
$j=n-1$	$(n-1,0)$	$(n-1,n-1)$

$a_1 * i - b_1 * j = 3151i - 3151j$

$\Rightarrow$

$3151(0) - 3151(0)$	$3151(3149) - 0$
$0 - 3151(3149)$	$3151(3149) - 3151(3149)$

$\Rightarrow$

0	9925650
$0 - 9925650$	0

$\Rightarrow$

0	9925650
-9925650	0

Upper bound $(U) = (0, n-1) = 9925650$

Lower bound $(L) = (n-1, 0) = -992560$

Dependency Check: if $b_0 - a_0 < L$ OR $b_0 - a_0 > U$ then the dependency is broken.

$b_0 - a_0 = 2756 \qquad 2756 \not< L = -992560 \qquad 2756 < U\,?$

$b_0 - a_0$ is not $< L \qquad 2756 \not> 9925650$

$\Rightarrow$ dependency is not broken

Loop 1:

Complete Banerjee's Test: $b_0 - a_0 = 2756$ $\quad a_1 x i - b_1 x j = 3151i - 1244250j$

Anti-dependence:

3 points: $(1,0)\ (n-1,0), (n-1,n-2)$

	$i=0$	$i=n-1$					
$j=0$	$f(1,0)$	$f(n-1,0)$	$\Rightarrow$	3151	9922499	$\Rightarrow$	$U = 992499$
$j=n-2$	N/A	$f(n-1,n-2)$		N/A	-3906976501		$L = -3906976501$

if $b_0 - a_0 < L$ or $b_0 - a_0 > U$ then anti-dependence is broken.

is $2756 \overset{?}{<} L$ OR is $2756 \overset{?}{>} U$

$2756 < -3906976501$ $\quad 2756 \overset{?}{>} 992499$

FALSE $\quad$ FALSE

$\therefore$ Niether $b_0 - a_0 < L$ or $b_0 - a_0 > U$ is true

$\Rightarrow$ s1 & s2 are dependend & cannot be parallelized.

True-dependence: $\quad a_1 x i - b x j = 3151i - 1244250j$

3 points: $(0,0), (0,n-1), (n-1,n-1)$

	$i=0$	$i=n-1$				
$j=0$	$f(0,0)$	N/A	$\Rightarrow$	0	N/A	$U = -3908220751$
$j=n-1$	$f(0,n-1)$	$f(n-1,n-1)$		-3918143250	-3908220751	$L = -3918143250$

if $b_0 - a_0 < L$ OR $b_0 - a_0 > U$: then true-dependency is broken.

$2756 \overset{?}{<} L$ OR $2756 \overset{?}{>} U$

FALSE $\quad 2756 > -3908\ldots$

TRUE

The true-dependency is broken.

Loop 2: for (i=0, i < 3150 , i++)

$a[(36)(3150)+i] = \ldots$

$\ldots \quad = a[(37)(3150)+i]+e$

}

GCD Test: $a_1 i - b_1 j = b_0 - a_0$

$f(i) = (36)(3150)+i$ $\qquad g(j) = 116550 + j$

$a_0 = 113400 \quad a_1 = 1 \qquad b_0 = 116550 \quad b_1 = 1$

$GCD(a_1, b_1) = G = GCD(1,1) = 1$

and 1 divides $b_0 - a_0$ $\therefore$ There could be a dependency.

Banerjee's Test: $a_1 * i - b_1 * j = i - j$

	i=0	i=n-1
j=0	(0,0)	(0,n-1)
j=n-1	(n-1,0)	(n-1,n-1)

$\Rightarrow$

0	3149
-3149	0

$\Rightarrow \begin{cases} U = 3149 \\ L = -3149 \end{cases}$

Dependency check: $b_0 - a_0 < L$ or $b_0 - a_0 > U$

$1-1 < -3149 \qquad 1-1 > 3149$

$0 \not< -3149 \qquad 0 \not> 3149$

$\therefore$ dependency is not broken.

Complete banerjee's Test: $a_1 x i - b_1 x j = i - j \qquad b_0 - a_0 = 3150$

Anti-dependence

3 points: $(i,j) := (1,0), (n-1,0), (n-1,n-2)$

	i=1	i=n-1
j=0	f(1,0)	f(n-1,0)
j=n-2	N/A	f(n-1,n-2)

$\Downarrow$

0	3149
N/A	3149-3148 = 1

$U = 3149 \quad L = 1$

True-dependency:

3 points: $(i,j) := (0,0), (0,n-1), (n-1,n-2)$

	i=0	i=n-1
j=0	0	N/A
j=n-2	-3148	1

$\Downarrow$

$U = 1$

$L = -3148$

If $b_0 - a_0 < L$ OR $b_0 - a_0 > U$: dependency is broken.

$3150 \overset{?}{<} L$ OR $3150 \overset{?}{<} U$

FALSE FALSE

$3150 < L$ or $3150 \overset{?}{>} U$

$3150 \overset{?}{<} -3148 \qquad 3150 \overset{?}{>} 1$

FALSE TRUE

$\therefore$ True dependency b/w S1 & S2 are broken

S1 & S2 can be parallelized

Loop 3: for (i=0, i<3150, i++)

a[3150*i + 85] = ...

--- = a[(3151)i + 135] + e

}

Gcd Test: $a_1 \times i - b_1 \times j = b_0 - a_0$

$f(i) = 3150i + 85$ $g(j) = 3151i + 135$

$a_0 = 85$ $a_1 = 3150$ $b_0 = 135$ $b_1 = 3151$

$GCD(a_1, b_1) = G = GCD(3150, 3151) = 1$

and $\frac{135 - 85}{1} \Rightarrow 50 \in \mathbb{Z}$ ∴ GCD fails and there could exist a dependency btw S1 & S2

Banerjee's Test: $3150i - 3151j$

	i=0	i=n-1			
j=0	f(0,0)	f(0,n-1)	⇒ 0	9919350	⇒ U = 9919350
j=n-1	f(n-1,0)	f(n-1,n-1)	-9922499	-3149	L = -9922499

Dependency Checks: $b_0 - a_0 < L$ or $b_0 - a_0 > U$

$b_0 - a_0 = 135 - 85 = 50$

$50 \not< L$ or $50 \not> U$

$b_0 - a_0$ is not less than the lower limit NOR is it greater than the upper limit

Therefore S1 & S2 can not be parallelized.

Complete banerjee's Test:

Anti-dependence:

3 points $(i,j) := (1,0), (n-1,0), (n-1,n-2)$

	i=1	i=n-1
j=0	3150	9919350
j=n-2	N/A	3150

⇓

$U = 9919350$

$L = 3150$

True-dependency:

3 points $(i,j): (0,0), (0,n-1), (n-1,n-1)$

	i=0	i=n-1
j=0	0	N/A
j=n-1	-9922499	-3149

⇓

$U = -3149$

$L = -9922499$

if $b_0 - a_0 < L$ or $b_0 - a_0 > U$: then dependency is broken.

is $50 < L$ OR $50 > U$

$50 \overset{?}{<} 3150$ $50 \overset{?}{>} 99...$

TRUE FALSE

is $b_0 - a_0 < L$ OR $b_0 - a_0 > U$

$50 \overset{?}{<} L$ $50 > U$

$50 < -992...$ $50 \overset{?}{>} -3149$

FALSE TRUE

∴ S1 & S2 Anti & True dependencies are broken and can be parallelized.

Loop 4: for (i=0, i<3150, i++)

$a[(3151)*i] = \ldots$

$\ldots = a[(315+1)*(3150)+i]+e$

}

GCD Test:

$f(i) = (3151)*i \qquad g(j) = (316)*(3150)+j$

$a_0 = 0 \qquad a_1 = 3151 \qquad b_0 = 995400 \qquad b_1 = 1$

$a_1 x i - b_1 x j = 1(i) - 1(j) \qquad b_0 - a_0 = 995400$

$GCD(a_1, b_1) = GCD(3151, 995400) = G$

$G = 1$ and G divides $b_0 - a_0$ ∴ GCD Test fails.

there could be dependencies

Banerjee's Test: $a_1 x i - b_1 x j = i - j \qquad 0 \le i < n, 0 \le j < n$

	i=0	i=n-1			
j=0	(0,0)	(0,n-1)	⇒	0	3149
j=n-1	(n-1,0)	(n-1,n-1)		-3149	0

⟹ Upper bound = 3149, Lower bound = -3149

Dependency checks: if $b_0 - a_0 < L$ or $b_0 - a_0 > U$ then dependency is broken.

$995400 \overset{?}{<} -3149$ or $995400 \overset{?}{>} 3149$

$b_0 - a_0$ is NOT less than Lower bound

$b_0 - a_0$ is NOT greater than upper bound.

Niether is true therefore s1 & s2 can not be parallelized.

Complete Banerjee's Test: $a_1 x i - b_1 x j = i - j \qquad b_0 - a_0 = 995400$

Anti-dependence

3 points (i,j): (1,0), (n-1,0), (n-1,n-2)

	i=1	i=n-1
j=0	1	3149
j=n-2	N/A	1

⇓

$U = 3149$

$L = 1$

True-dependence:

3 points (i,j): (0,0), (0,n-1), (n-1,n-1)

	i=0	i=n-1
j=0	0	N/A
j=n-1	-3149	0

⇓

$U = 0$

$L = -3149$

if $b_0 - a_0 < L$ or $b_0 - a_0 > U$: ⇒ dependency is broken.

is $b_0 - a_0 \overset{?}{<} L$ or $b_0 - a_0 \overset{?}{>} U$

$995400 \overset{?}{<} 1 \qquad 995400 \overset{?}{>} 3149$

FALSE TRUE

is $b_0 - a_0 < L$ or is $b_0 - a_0 > U$

$995400 \overset{?}{<} -3149 \qquad 995400 > 0$

FALSE TRUE

S1 & S2 can be parallized