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Logical Aspects of Multi-agent Systems

Application of Epistemic Logic and Game Theory to Liar's Dice game

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Abstract

Games are a big field of applications of Multi-Agent systems. Principles of Epistemic Logic allows to model agent knowledge and beliefs. In this paper we examine an application of Epistemic Logic and Game Theory to a popular gambling game called Liar's Dice. Firstly, the rules of the game will be explained and some of examples will be provided. Then, the formalization and application of Epistemic Logic will be proposed. In the further section, results of experiments with our system will be presented. Finally, we will discuss the results and several deductions will be concluded.

1 Introduction: Liar's Dice rules explained

The game is played using a set of dice. Each player receives up-front six fair dice. Each round there is at least one loser, who then has to lay away one of his dice leaving him with one less. The game is over when one person loses his last die.

At the start of the game, the person rolls his remaining dice and is allowed to view them. Note that each player only knows his own dice and not those of his/her opponents. The game is about the beliefs each player has about the distribution of the dice using the dice of all of the players. A player needs to make a claim about this distribution (for instance: "I believe there are **at least six four's** on the table". The next player needs to overbid the previous bidding. This can be done by increasing either the *quantity* (in this case six) or the *value* (in this case four). See example 1.

Example 1: There are four players in the game, each holding three dice. The distribution is as follows: $D = \{(1,2,2),(1,1,3),(3,5,6),(4,3,6)\}$. The starting player of the round opens his bidding and says *"Two two's"*. The next player has two choices: raise the *value* or the *quantity*. The player chooses to say *"Three two's"*, which means he has overbid the quantity.

Example 2: Marie has just made the bidding *"Two sixes"*. John is then only allowed to increase the quantity, because no eyes are higher than six. John is then allowed to say *"Three three's*, he does not have to say *"Three Sixes"*. When the quantity is increased, the value may be 'reset'. NOTE: John is also allowed to skip quantities: he could have immediately went from three to seven.

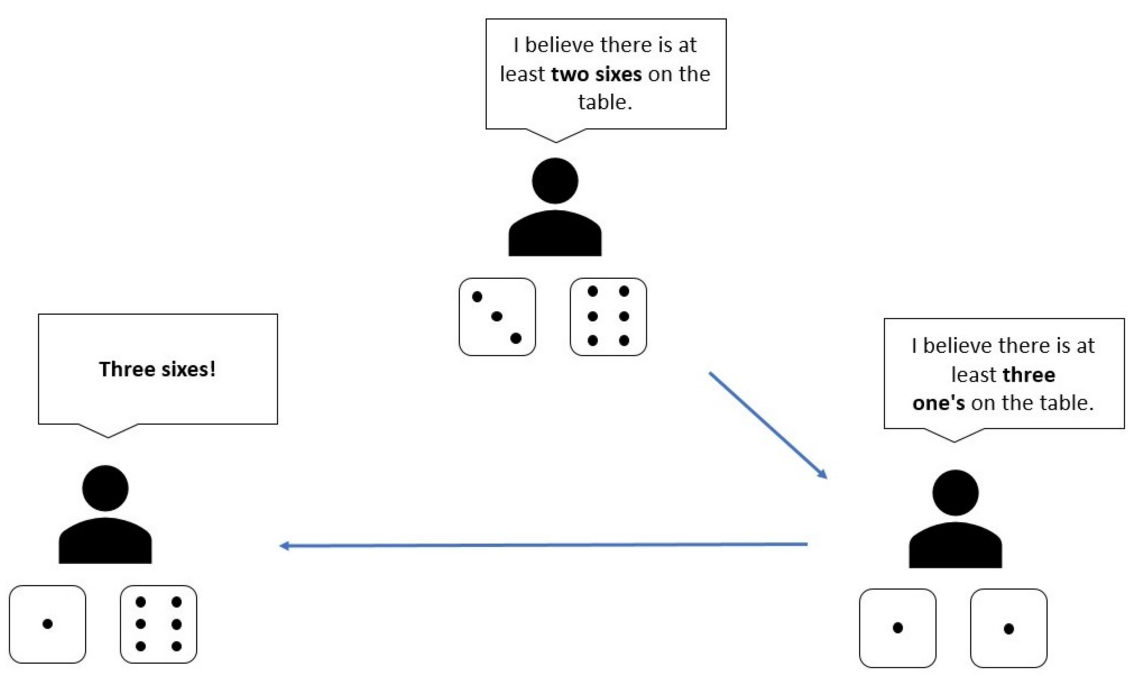

Figure 1: An example round of Liar's Dice.

1.1 End of the game

This continues until a player does not believe a bidding is a proper representation of the distribution of the total dice on the table. He then makes the claim that he does not believe the statement to be true and challenges the last bidding. The remaining players, other than the one who made the final bidding, must also make a claim whether they believe the final bidding to be a proper representation. When everyone has made their stakes, all of the dice will become known and the last bidding will be checked. All of those who made the correct claim will not lose a die, all of those who made the incorrect claim will lose a die.

1.2 The pick die

One final adjustment to the probabilities of this game is the fact that the die with one eye on it will be the **Pick** die. This means that upon evaluation of the game, all of the dice with one eye on them also count towards the number of dice of the value that was chosen during the last bidding. The players in the game need to incorporate the chance of rolling a one into their biddings as well.

1.3 One final rule

The pick die may be used as an alternative to raising the quantity or the value. A player may opt to make an assumption that there are **at least** n many pick die on the table. Notice that the benefit of incorporating the pick die in evaluation is lost when this strategy is chosen. The rule-of-thumb is that the pick die is always worth twice the quantity of that of regular dice + **a little extra**. This means that if the previous bidding had a quantity of four, the player is allowed to make a bidding of *"Two pick"*. The other way around, if two pick dice was the previous bidding, the player needs to have at least a quantity of **five** (because of the little extra).

2 Possible Strategies

3 Formalization

After explaining Liar Dice rules we are able to define main principles of the Multi-Agent system we managed to embed in such environment. Our system consist of a fixed number of agents (players). Before the game starts, every player know only the amount of every of his six dices. We can define a Kripke model M with all possible worlds, as well as we can define accesses between those worlds for every agent. For example, in Liar Dice game with 4 players, we will obtain an initial model with $4 \times 6^6 = 186624$ possible worlds. The number of states in the model decreases during the game (as always someone lose one die or is kicked out of the game each round). To show it directly, let's see how the number of possible worlds changes during first couple of rounds of our example game, in which one of the players will lose all of his dices during first six rounds (assuming that bidding are not taken into account):

Number of dices for each player	Number of possible worlds in the Kripke Model
6, 6, 6, 6	$4 \times 6^6 = 186624$
6, 6, 6, 5	$3 \times 6^6 + 6^5 = 147744$
6, 6, 6, 4	$3 \times 6^6 + 6^4 = 141264$
6, 6, 6, 3	$3 \times 6^6 + 6^3 = 140184$
6, 6, 6, 2	$3 \times 6^6 + 6^2 = 140004$
6, 6, 6, 1	$3 \times 6^6 + 6 = 139974$
6, 6, 6, 0	$3 \times 6^6 = 139968$

Table 1: Capacity of Kripke Model depending of dice distribution - 1st example

The situation looks much better, when all of the players looses their dices similarly:

Number of dices for each player	Number of possible worlds in the Kripke Model
6, 6, 6, 6	$4 \times 6^6 = 186624$
6, 6, 6, 5	$3 \times 6^6 + 6^5 = 147744$
6, 6, 5, 5	$2 \times 6^6 + 2 \times 6^5 = 108864$
6, 5, 5, 5	$6^6 + 3 \times 6^5 = 69984$
5, 5, 5, 5	$4 \times 6^5 = 31104$
5, 5, 5, 4	$3 \times 6^5 + 6^4 = 24624$
5, 5, 4, 4	$2 \times 6^5 + 2 \times 6^4 = 10368$

Table 2: Capacity of Kripke Model depending of dice distribution - 2nd example

Processing such a big models is highly unefficient. To decrease the number of states we decided to perform experiments with less number of players with smaller ammount of dices. Furthermore, to achieve smaller and easier to process models, we change the general approach: except of building one model with all of the agents, we are treating all players separately. That means, that every player have his own Kripke model with all possible worlds, that he have access to. Thanks to that, we obtain smaller models, that represents every player's knowledge about the game, and have initially 2592 states.

One of the most important skills in Liar Dice is bluffing and capability to challenge biddings in appropriate moment. In our implementation, we decided to model lying and believing by adding two variables, which allow to parameterize those features for every player. Each parameter is a number from 0 to 1, which corresponds to a probability of believing in previous bid or to bluff in the bid of current agent. That brings a big number of possibilities to model different types of players and examine which strategy will provide the highest chance to win the Liar's Dice game.

4 Experiments

As mentioned before, we have created environment, that allows to model various strategies used by agents, and perform a big number of Liar's Dice simulations in relatively short time. To make use of it, we decided to carry out some interesting experiments.

4.1 2 - players game

In case of 2 - players game, we decided to examine different combinations of believing and bluffing parameters against a specific type of opponent. For every set of parameters agents played 100 games of Liar's Dice, as in 2-players simulations the computacional and time cost are easier to handle. To make it clear, all experiments that we have done are shown in the following table:

Experiment	Opponent's believe parameter	Opponent's lying parameter
Exp 1: Truthful opponent	1.0	0.0
Exp 2: Complete liar	Random	1.0
Exp 3: Disbeliever	0.0	Random
Exp 4: Average opponent	0.5	0.5
Exp 5: Random opponent	Random	Random

Table 3: Experiments for 2-players game

5 Plan of implementation

We decided to make progress in this project by dividing it into significant steps that we will accomplish one by one.

5.1 Step 1: Develop a working game model.

The first task of the plan was to build an environment for agents, which is able to run a simulation of Liar's Dice game respectful to its rules. We managed to develop such a model, which can simulate a game with a fixed number of players, until the winner will not be known. For tests of the simulator a "dummy" strategy was implemented, but agents have access to the common knowledge and their beliefs. The working code can be found there: *https://github.com/Ruben103/epistemiclogic*

5.2 Step 2: Start working on the Web Application

In the end of the project we want to have a simulator on our website, which will be able to run simulated games of Liar's Dice and allow the user to track the decision process of every agent. To achieve this we have started building an app which will be displaying the output of the simulator.

5.3 Step 3: Apply Game Theory and Epistemic Logic for playing agents

This is the crucial step and the biggest challenge of our project. We need to figure out how to implement the decision process, which is respectful to accessible common knowledge, agent knowledge and its beliefs.

5.4 Step 4: Apply bluffing

In Liar's Dice, like in some of hazard games, the player has the ability to bluff. Public announcement logic, which we are using to developing agents assumes, that the announcement are truthful - it means that every agent are making his bids about what they **really believe**, what the configuration of dices is on the table. The next step is to model bluffing, which is also a very challenging task. To accomplish it, some literature where studied.

5.5 Step 5: Perform experiments

To examine if the Epistemic Logic is helpful in Liar's Dice, we will need to perform some experiments and compare our model with different strategies.