University of Groningen

Logical Aspects of Multi-agent Systems

Application of Epistemic Logic and Game Theory to Liar's Dice game

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Abstract

Games are a big field of applications of Multi-Agent systems. Principles of Epistemic Logic allows us to model agent knowledge and beliefs. In this paper we examine an application of Epistemic Logic and Game Theory to a popular gambling game called Liar's Dice. Firstly, the rules of the game will be explained and some examples will be provided. Then, the formalization and application of Epistemic Logic will be proposed. In the further section, results of experiments with our system will be presented. Finally, we will discuss the results and several deductions will be concluded.

1 Introduction: Liar's Dice rules explained

The game is played using a set of dice. Each player receives up-front six fair dice. In each round there is at least one loser, who then has to lay away one of his dice leaving him with one less. The game is over when one person loses his last die.

At the start of the game, the person rolls his remaining dice and is allowed to view them. Note that each player only knows his own dice and not those of his/her opponents. The game is about the beliefs each player has about the distribution of the dice using the dice of all of the players. A player needs to make a claim about this distribution (for instance: "I believe there are **at least six four's** on the table". The next player needs to overbid the previous bidding. This can be done by increasing either the *quantity* (in this case six) or the *value* (in this case four). See example 1.

Example 1: There are four players in the game, each holding three dice. The distribution is as follows: $D = \{(1, 2, 2), (1, 1, 3), (3, 5, 6), (4, 3, 6)\}$. The starting player of the round opens his bidding and says *"Two two's"*. The next player has two choices: raise the *value* or the *quantity*. The player chooses to say *"Three two's"*, which means he has overbid the quantity.

Example 2: Marie has just made the bidding *"Two sixes"*. John is then only allowed to increase the quantity, because no eyes are higher than six. John is then allowed to say *"Three three's*, he does not have to say *"Three Sixes"*. When the quantity is increased, the value may be 'reset'. NOTE: John is also allowed to skip quantities: he could have immediately went from three to seven.

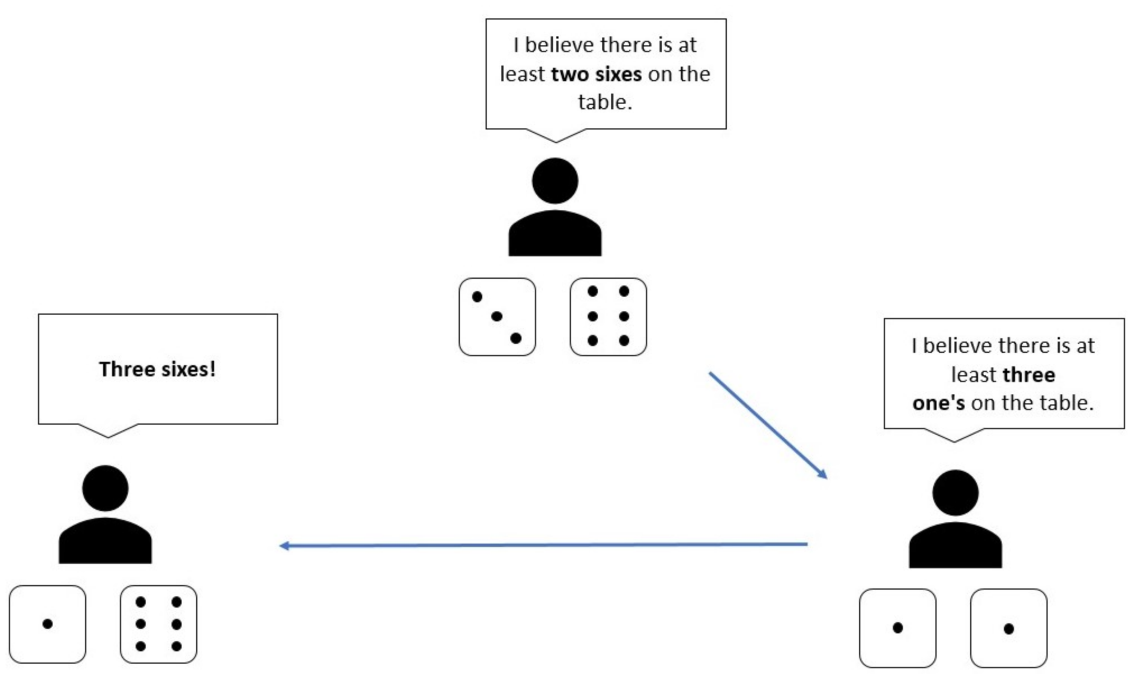

Figure 1: An example round of Liar's Dice.

1.1 End of the game

This continues until a player does not believe a bidding is a proper representation of the distribution of the total dice on the table. He then makes the claim that he does not believe the statement to be true and challenges the last bidding. The remaining players, other than the one who made the final bidding, must also make a claim whether they believe the final bidding to be a proper representation. When everyone has made their stakes, all of the dice will become known and the last bidding will be checked. All of those who made the correct claim will not lose a die, all of those who made the incorrect claim will lose a die.

1.2 The pick die

One final adjustment to the probabilities of this game is the fact that the die with one eye on it will be the **Pick** die. This means that upon evaluation of the game, all of the dice with one eye on them also count towards the number of dice of the value that was chosen during the last bidding. The players in the game need to incorporate the chance of rolling a one into their biddings as well.

1.3 One final rule

The pick die may be used as an alternative to raising the quantity or the value. A player may opt to make an assumption that there are **at least** n many pick die on the table. Notice that the benefit of incorporating the pick die in evaluation is lost when this strategy is chosen. The rule-of-thumb is that the pick die is always worth twice the quantity of that of regular dice + **a little extra**. This means that if the previous bidding had a quantity of four, the player is allowed to make a bidding of *"Two pick"*. The other way around, if two pick dice was the previous bidding, the player needs to have at least a quantity of **five** (because of the little extra).

2 Formalization

After explaining Liar Dice rules we are able to define main principles of the Multi-Agent system we managed to embed in such environment. Our system consist of a fixed number of agents (players). Before the game starts, every player know only the amount of every of his six dice. We can define a Kripke model M with all possible worlds, as well as we can define accesses between those worlds for every agent. For example, in Liar Dice game with 4 players, we will obtain an initial model with $4 \times 6^6 = 186624$ possible worlds. The number of states in the model decreases during the game (as always someone lose one die or is kicked out of the game each round). To show it directly, let's see how the number of possible worlds changes during first couple of rounds of our example game, in which one of the players will lose all of his dice during first six rounds (assuming that bidding are not taken into account):

Number of dice for each player	Number of possible worlds in the Kripke Model
6, 6, 6, 6	$4 \times 6^6 = 186624$
6, 6, 6, 5	$3 \times 6^6 + 6^5 = 147744$
6, 6, 6, 4	$3 \times 6^6 + 6^4 = 141264$
6, 6, 6, 3	$3 \times 6^6 + 6^3 = 140184$
6, 6, 6, 2	$3 \times 6^6 + 6^2 = 140004$
6, 6, 6, 1	$3 \times 6^6 + 6 = 139974$
6, 6, 6, 0	$3 \times 6^6 = 139968$

Table 1: Capacity of Kripke Model depending of dice distribution - 1st example

The situation looks much better, when all of the players lose their dice similarly:

Number of dice for each player	Number of possible worlds in the Kripke Model
6, 6, 6, 6	$4 \times 6^6 = 186624$
6, 6, 6, 5	$3 \times 6^6 + 6^5 = 147744$
6, 6, 5, 5	$2 \times 6^6 + 2 \times 6^5 = 108864$
6, 5, 5, 5	$6^6 + 3 \times 6^5 = 69984$
5, 5, 5, 5	$4 \times 6^5 = 31104$
5, 5, 5, 4	$3 \times 6^5 + 6^4 = 24624$
5, 5, 4, 4	$2 \times 6^5 + 2 \times 6^4 = 10368$

Table 2: Capacity of Kripke Model depending of dice distribution - 2nd example

Processing such a big models is highly inefficient. To decrease the number of states, we decided to perform experiments with less number of players and with smaller amount of dice. Furthermore, to achieve smaller and easier to process models, we change the general approach: except of building one model with all of the agents, we are treating all players separately. This means that every player have his own Kripke model with all possible worlds, that he has access to. Thanks to that, we obtain smaller models, that represent every player's knowledge about the game, and have initially 2592 states.

One of the most important skills in Liar Dice is bluffing and capability to challenge bids in appropriate moment. In our implementation, we decided to model lying and believing by adding two variables, which allow to parameterize those features for every player. Each parameter is a number from 0 to 1, which corresponds to a probability of believing in previous bid or to bluff in the bid of current agent. That brings a big number of possibilities to model different types of players and examine which strategy will provide the highest chance to win the Liar's Dice game.

The number of dice left in the game is common knowledge. It is also common knowledge that every player knows only the value of their own dice. This means that all the players knows these propositions and all players know that all players know of it and so on.

Since a player only has knowledge of the value of each of his dice D, the other agents do not have knowledge of D and therefore worlds with different values of D are indistinguishable by those other agents.
$(M, s) \models K_m D \rightarrow \neg(K_1 D \vee ... \ K_{m-1} D \vee K_{m+1} D \vee ... \ K_n D)$

3 Experiments

As mentioned before, we have created environment, that allows to model various strategies used by agents, and perform a big number of Liar's Dice simulations in relatively short time. To make use of it, we decided to carry out some interesting experiments.

3.1 2 - players game

In case of 2 - players game, we decided to examine different combinations of believing and bluffing parameters against a specific type of opponent. For every set of parameters agents played 100 games of Liar's Dice, as in 2-players simulations the computacional and time cost are easier to handle. To make it clear, all experiments that we have done are shown in the following table:

Experiment	Opponent's believe parameter	Opponent's lying parameter
Exp 1: Truthful opponent	1.0	0.0
Exp 2: Complete liar	Random	1.0
Exp 3: Disbeliever	0.0	Random
Exp 4: Average opponent	0.5	0.5
Exp 5: Random opponent	Random	Random

Table 3: Experiments for 2-players game

3.2 3 and more players game

When we have been performing experiments with 2 players, we have made some assumptions about the opponent, for example, that he is always trying to bluff or that he believe that every bid we make is truthful. In the real world, we can make such an assumptions about our opponents only after several rounds played. It is worth to notice, that in our environment, agents do not have any knowledge about their opponents strategies. Therefore, in further experiments, we have assigned random values for believing and lying parameters for every opponent in 3, 4 and 5 - players game. That brought us a broader view on which parameters are more "universal" - that means which parameters performs better, when the distribution of opponents is differential. To make the simulations in a reasonable time, we reduced the number of dice from 6 to 4 for each agent.

4 Results

4.1 2 - players game

Results of each experiment mentioned in table 3 can be seen below:

Experiment	Believe parameter	Lying parameter	Number of games won
Exp 1: Truthful opponent	1.0	1.0	56/100
Exp 2: Complete liar	1.0	0.3	84/100
Exp 3: Disbeliever	1.0	[0.0, 1.0]	100/100
Exp 4: Average opponent	1.0	0.8	83/100
Exp 5: Random opponent	1.0	0.8	83/100

Table 4: Results for 2-players game

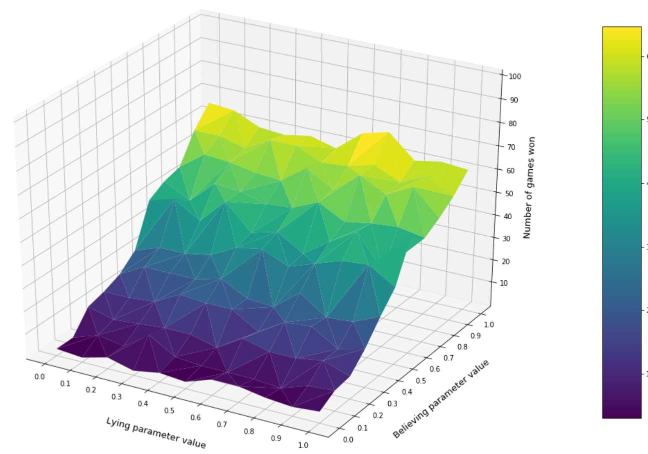

Figure 2: 3-players game simulation

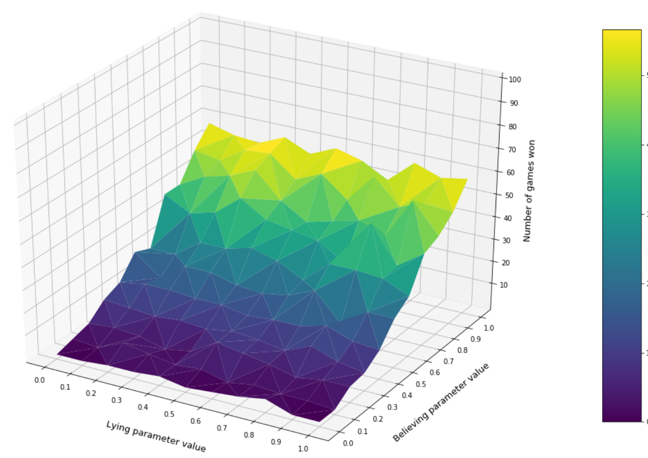

Figure 3: 4-players game simulation

4.2 3 and more players game

The following plots presents results of our agent in number of games won, according to values of lying and believing parameters:

5 Conclusions

According to the results of our experiments, we can conclude that believing parameter played significant role in the simulations. There is almost a linear dependency between its amount and number of games our agent managed to win. According to that correlation, we can easily deduce that believing in others players bids is an optimal strategy, no matter of which strategy the opponent will choose. That is compatible with our intuition. Calling a lie is a bit risky in Liar's Dice, since we have to have quite a lot of knowledge about dice in the game. Therefore it might be better to try overbid the previous player.

Some of results obtained from experiments with 2 contestants are surprising. For example, it is still better to believe in our opponent bids, when we are playing against a liar. Furthermore, it came out that the most difficult opponent is the one who believes in every bid we make and never lie - our agent won 56% of games played against him. In case, when our opponent never believes in our bids, the frequency of our false bets can be any. No matter what we say, our rival will always call a lie, which, as the results indicates, is the worst strategy. In case when parameters of the opponent were randomized, our agent was won 83/100 games with lying parameter set to 0.8.

In case of Liar's Dice with more than 2 players, results are also quite interesting. The strategy

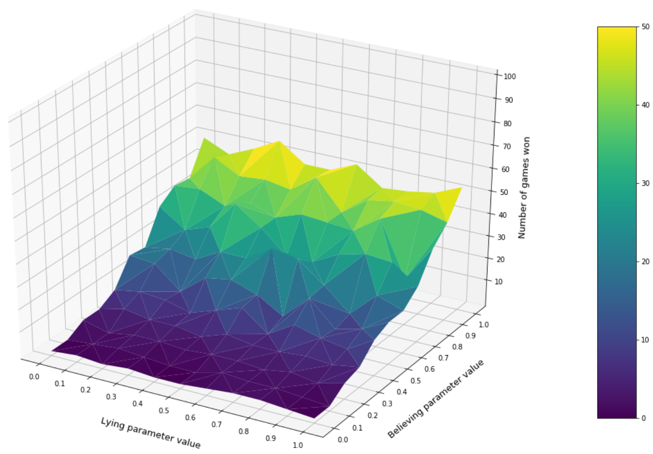

Figure 4: 5-players game simulation

of believing in every bid still guarantee the highest probability of winning the game. With that setting, according to the number of opponents, our agent was able to win:

- 66/100 games with lying parameter value = 0.7, and ≈ 60% of games overall against 2 opponents,
- 56/100 games with lying parameter values = {0.5, 0.3} and ≈ 52% of games overall against 3 opponents,
- 52/100 games with lying parameter value = 0.3, and ≈ 46% of games overall against 4 opponents.

The following histogram shows the results described above, taking into account also the outcome of game with one opponent:

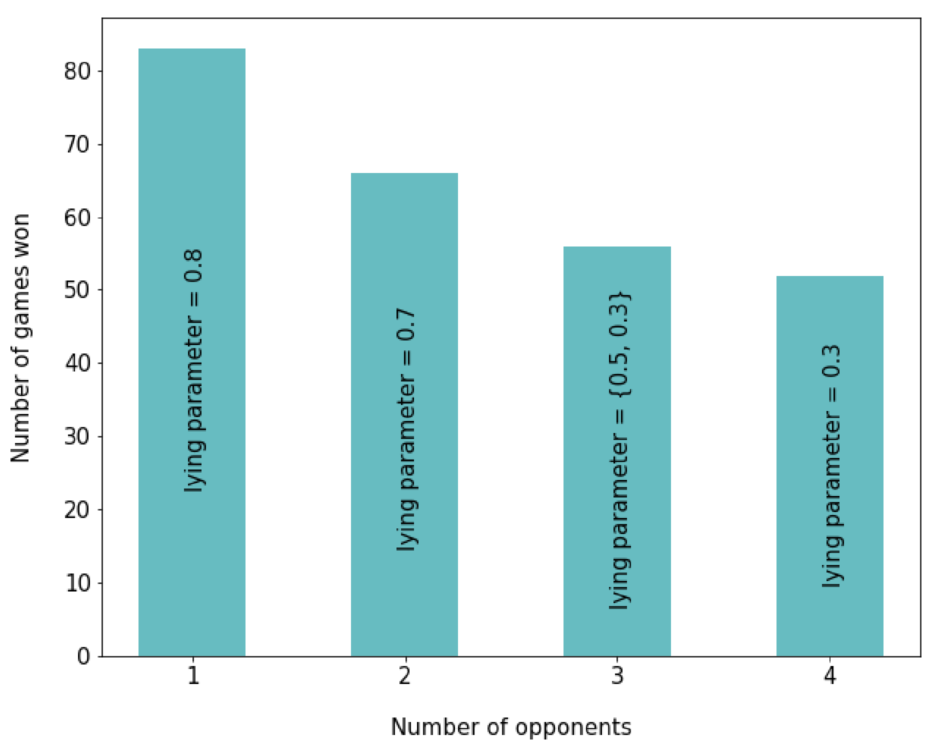

Figure 5: Number of games won according to the number of opponents

That leads to some instructive conclusions. Firstly, when number of players in the game increases, it becomes harder to win. Secondly, it is better to lie less, when we play against more opponents.

To summarize, results from our experiments indicates, that believing in every bid and lying in around half of our turns (depending on the number of competitors) is the optimal strategy in our multi-agent system, when we have not any knowledge about our opponents preferences.

6 Discussion

Liar's Dice is a game which, is very difficult to model. That difficulty follows from the fact, that it is still a big challenge to model humans behaviour, even in games. The biggest advantage of developing simplified environments like we did in this project, is the ability to simulate a big number of games in relatively short time. Analyzing large amount of data gathered from contests played between agents can bring instructive conclusions and give a broader view on the examined problem.

On the other hand, data gathered from games played between humans might be more corresponded with the Liar's Dice reality. Nevertheless, collecting data from human's games would last unimaginably longer.

Regarding our project, the research might be continued by evaluating strategies concluded from experiments in multi-agent system in a number of games played between humans. That would give us a valuable information about quality and authenticity of results we obtained from the developed environment.