VTOL Design Point

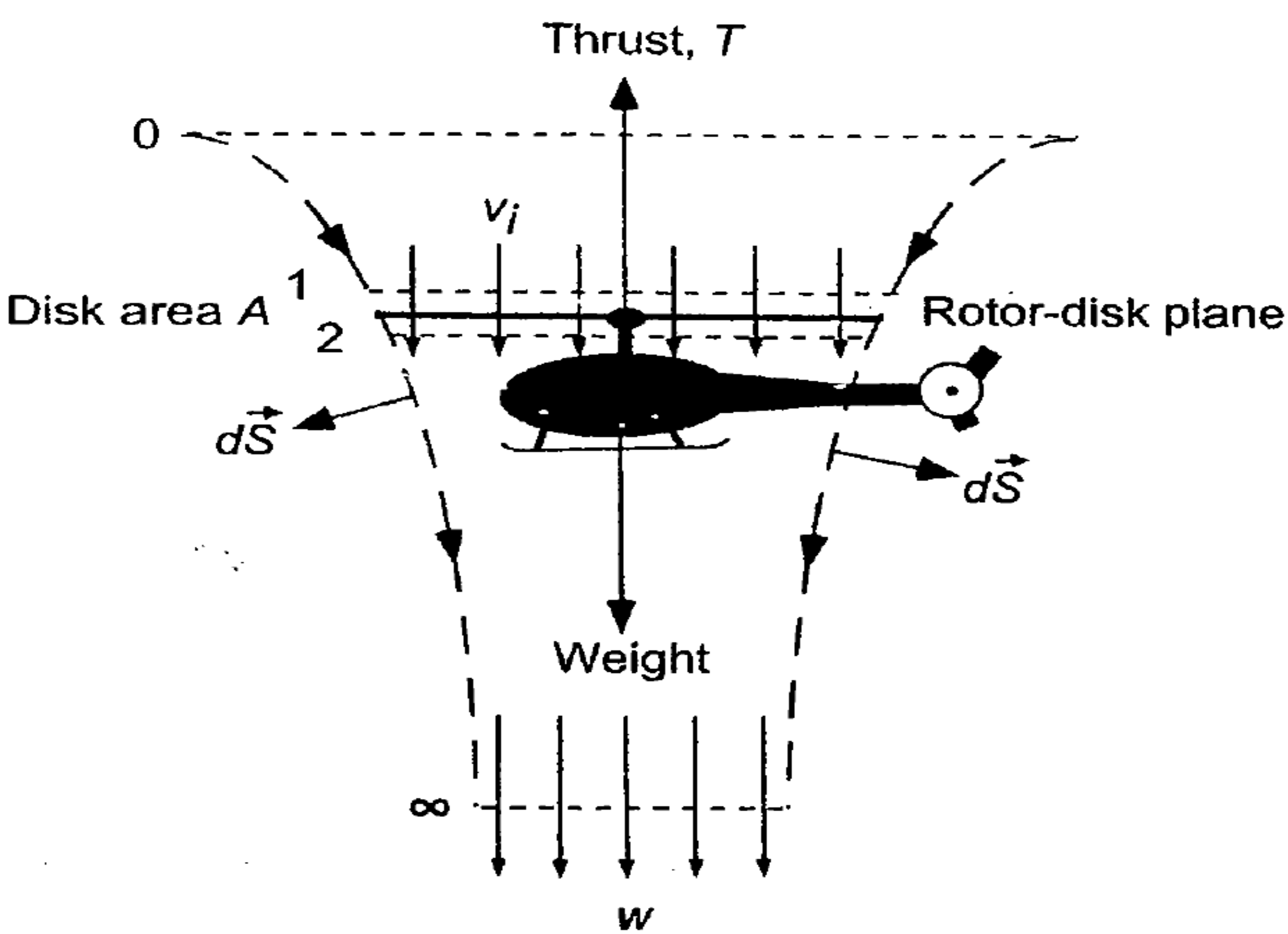
Projecto Aeroespacial| Aircraft Design

TÉCNICO LISBOA

Introduction

- The design of the **fixed-wing** and **rotors** is performed separately.
- Disc loading (DL) is relevant for rotor sizing, whereas wing loading (W/S) is relevant for wing sizing.
- Rotorcraft theory is developed based on the **linear momentum theory**: conservation of mass and linear momentum must be verified.
- This theory was mostly applied for **helicopter sizing**, and can be replicated to other VTOL configurations.

Hovering flight



Hovering flight

Assuming **no wings are located below the rotors** (i.e. no downward force is generated in the wings by the air flow passing through the rotors), the total power required for hovering is:

$$P = P_{i,R} + P_o$$

where $P_{i,R}$ is the real induced power, and P_o is the rotor's profile power.

Ideal Induced Power (w/o viscosity effects): $P_i = T v_{i,h} = T\sqrt{\frac{T}{2\rho A}}$,where

- $v_{i,h}$ - induced velocity at the rotor in hover;
- T - rotor thrust;
- ρ - air density at hover altitude;
- A - rotor disk area.

Hovering flight

In vertical flight, $T = W$; Moreover, the ideal power required calculated considers incompressible, stationary, inviscid and irrotational fluid. The real power required must account for sources of drag and inefficiencies (e.g. rotor tip losses, non-uniform flow, wake swirl, non-ideal wake contraction, effect of finite blades' length). A multiplicative factor, k_i is included to account for those effects (≈ 1.15).

Real Induced Power: $P_{i,R} = Tv_{i,R} = Tk_i v_{i,h} = k_i W\sqrt{\frac{W}{2\rho A}} = k_i \frac{W^{3/2}}{\sqrt{2\rho A}}$

Rotor's profile power: Considers rectangular blades and C_d independent of Mach and Reynold's numbers.

$$P_o = \rho A V_{tip}^3 \left(\frac{\sigma C_d}{8}\right)$$

- C_d - average blade drag coefficient;
- $\sigma = \Sigma_i \left(A_{Blade,i}\right)/A_{rotor}$ - main rotor solidity ratio (between 0.07-0.12);
- V_{tip} - rotor's tip speed.

Hovering flight

The total power equation can be rearranged as:

$$P = P_{i,R} + P_o = k_i \frac{W^{3/2}}{\sqrt{2\rho A}} + \rho A V_{tip}^3 \left(\frac{\sigma C_d}{8}\right)$$

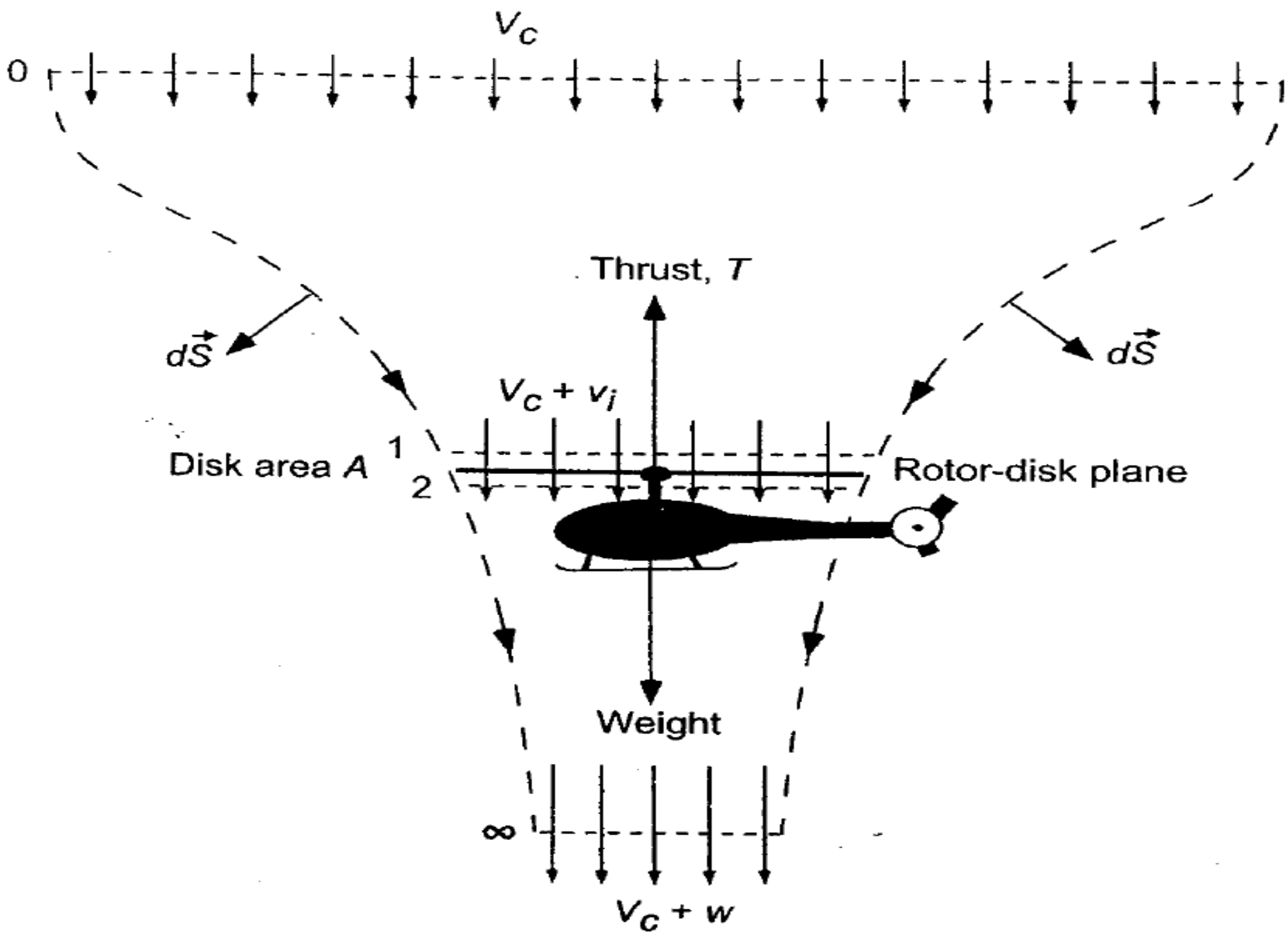
Yet, at this stage, no precise data exists to calculate P_o. Therefore, the **Figure of Merit** (FoM) is introduced as the ratio between the ideal induced power and the total consumed power. An $FoM \approx 0.8$ is the optimal case, whereas $FoM \approx 0.6$ is optimized for forward flight capability:

$$FoM = \frac{P_i}{P} \Leftrightarrow P = \frac{W}{FoM}\sqrt{\frac{W}{2\rho A}}$$

For design point purposes, we shall rewrite the above equation as a function of **power loading**, $PL = W/P$ and **disc loading**, $DL = W/A$:

$$FoM = \frac{P_i}{P} \Leftrightarrow PL = FoM\sqrt{\frac{2\rho}{DL}}$$

Vertical climb flight



Vertical climb flight

Assuming again the absence of a wing positioned underneath the rotors, the total power required for vertical climb is given by:

$$P = P_o + P_c + P_i = T(V_y + k_i v_{i,c}) + \rho A V_{tip}^3 \left(\frac{\sigma C_d}{8}\right)$$

where

- P_c - Power to climb;
- V_y - Rate of climb (at low subsonic speeds, at service ceiling, $\approx 0.5 m/s$);
- $v_{i,c}$ - Induced velocity in climb, $v_{i,c} = -\frac{1}{2}V_y + \frac{1}{2}\sqrt{V_y^2 + \frac{2T}{\rho A}}$

Rearranging the initial equation by assuming that the drag generated during climb is negligible with respect to the aircraft's weight (i.e. $W >> D \Rightarrow T = W$):

$$P = W\left(V_y - \frac{k_i}{2}V_y + \frac{k_i}{2}\sqrt{V_y^2 + \frac{2W}{\rho A}}\right) + \rho A V_{tip}^3 \left(\frac{\sigma C_d}{8}\right)$$

Vertical climb flight

Dividing the above equation by W and inverting the fractions, one can rearrange the equation for vertical climb flight as a function of PL and DL:

$$\frac{P}{W} = \left(V_y - \frac{k_i}{2}V_y + \frac{k_i}{2}\sqrt{V_y^2 + \frac{2W}{\rho A}}\right) + \frac{\rho A V_{tip}^3}{W}\left(\frac{\sigma C_d}{8}\right) \Leftrightarrow$$

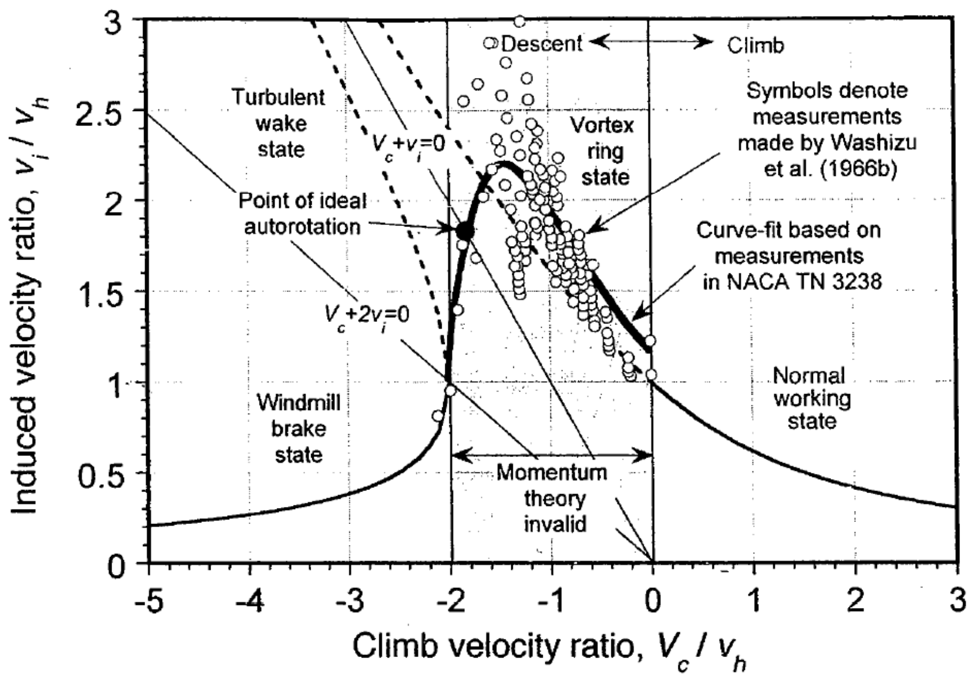
$$PL = \frac{1}{\left(V_y - \frac{k_i}{2}V_y + \frac{k_i}{2}\sqrt{V_y^2 + \frac{2DL}{\rho}}\right) + \frac{\rho V_{tip}^3}{DL}\left(\frac{\sigma C_d}{8}\right)}$$

This equation must be assessed for $\rho = \rho_0$ (ISL/SEA level conditions) and $\rho = \rho_c$ (ceiling altitude).

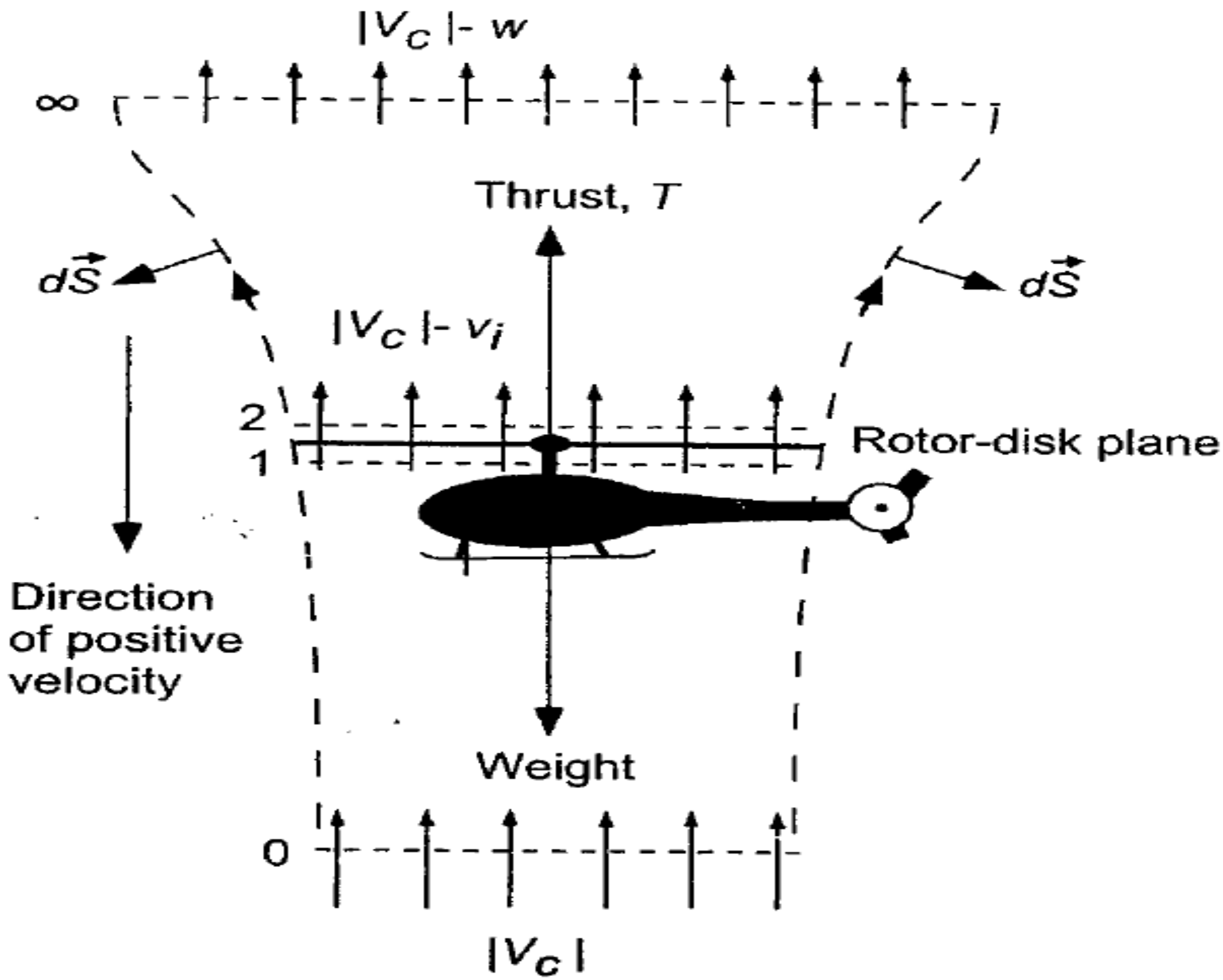
Vertical descent flight

- When the descent velocity is $-2v_{i,h} \leq V_y \leq 0$, a more complicated recirculating flow pattern can exist at the rotor, therefore invalidating the momentum theory - **empirical relations will be used**.

- To proceed with the momentum theory, the assumption must be made that $V_y/v_{i,h} \leq -2$ so that a well-defined slipstream will always exist above the rotor and encompassing the rotor disk.



Vertical descent flight



Vertical descent flight: $V_y/v_{i,h} \leq -2$

- The induced velocity for the descent condition, $v_{i,d}$ can be written as:

$$v_{i,d} = -\frac{V_y}{2} - \frac{1}{2}\sqrt{V_y^2 - \frac{2DL}{\rho}}$$

where $V_y < 0$ and $T \approx W$.

- The total power required can be calculated with the same method implemented for the vertical climb:

$$P = P_o + P_c + P_i = T(V_y + k_i v_{i,d}) + \rho A V_{tip}^3 \left(\frac{\sigma C_d}{8}\right)$$

$$= W\left[V_y - \frac{k_i}{2}\left(V_y + \sqrt{V_y^2 - \frac{2DL}{\rho}}\right)\right] + \rho A V_{tip}^3 \left(\frac{\sigma C_d}{8}\right)$$

Vertical descent flight: $V_y/v_{i,h} \leq -2$

- Dividing both sides of the equation by W, one can obtain a similar expression to the one attained for vertical climb as a function of PL and DL:

$$\frac{P}{W} = \left[V_y - \frac{k_i}{2}\left(V_y + \sqrt{V_y^2 - \frac{2DL}{\rho}}\right)\right] + \frac{\rho A V_{tip}^3}{W}\left(\frac{\sigma C_d}{8}\right) \Leftrightarrow$$

$$PL = \frac{1}{\left[V_y - \frac{k_i}{2}\left(V_y + \sqrt{V_y^2 - \frac{2DL}{\rho}}\right)\right] + \frac{\rho V_{tip}^3}{DL}\left(\frac{\sigma C_d}{8}\right)}$$

Vertical descent flight: $0 \leq V_y/v_{i,h} \leq -2$

- Since the nature of the induced velocity is not analitically determined, experiments can be used to describe 'best-fit' approximations valid for any rate of descent.

- A linear approximation was proposed by some authors:

$$\frac{v_{i,d}}{v_{i,h}} = \begin{cases} k_i - \frac{V_y}{v_{i,h}}, & \text{for } -1.5 \leq \frac{V_y}{v_{i,h}} \leq 0 \\ k_i \left[7 + 3\frac{V_y}{v_{i,h}}\right], & \text{for } -2 \leq \frac{V_y}{v_{i,h}} \leq -1.5 \end{cases}$$

- A better approximation to the measured curve is:

$$\frac{v_{i,d}}{v_{i,h}} = \begin{cases} k_i - \frac{3}{4}\frac{V_y}{v_{i,h}}, & \text{for } \frac{-8k_i}{4k_i+1} \leq \frac{V_y}{v_{i,h}} \leq 0 \\ k_i \left[7 + 3\frac{V_y}{v_{i,h}}\right], & \text{for } -2 \leq \frac{V_y}{v_{i,h}} \leq \frac{-8k_i}{4k_i+1} \end{cases}$$

Vertical descent flight: $0 \leq V_y/v_{i,h} \leq -2$

- A continuous approximation to the measured induced velocity curve is the quartic:

$$\frac{v_{i,d}}{v_{i,h}} = k_i + K_1\left(\frac{V_y}{v_{i,h}}\right) + K_2\left(\frac{V_y}{v_{i,h}}\right)^2 + K_3\left(\frac{V_y}{v_{i,h}}\right)^3 + K_4\left(\frac{V_y}{v_{i,h}}\right)^4,$$

where $K_1 = -1.125, K_2 = -1.372, K_3 = -1.718,$ and $K_4 = -0.655.$

Design Point – Transition flight

The total power required is defined as:

$$P = P_{i,R} + P_o + P_p$$

where

- P_p - Parasite power.

Assumptions:

- Transition flight is performed at **constant altitude**;
- Angle of attack is **small** $\Rightarrow V_\infty >> V_\perp$, where V_∞ is the horizontal component of the forward flight velocity that leads to the wing-borne phenomenon, and $V_\perp$ is the perpendicular one.

Real induced power

$$P_{i,R} = k_i T v_i = k_i T \sqrt{\frac{-V_\infty^2}{2} + \sqrt{\left(\frac{V_\infty^2}{2}\right)^2 + \left(\frac{T}{2\rho A}\right)^2}}$$

where $k_i \approx 1.2$.

Design Point – Transition flight

Rotor's profile power

$$P_o = \rho A V_{tip}^3 \left(\frac{\sigma C_d}{8} (1 + 4.6\mu^2) \right)$$

where μ is the advance ratio given by:

$$\mu = \frac{V_\infty \cos(\alpha)}{V_{tip}}$$

Parasite power

$$P_p = D V_\infty = \frac{1}{2} \rho V_\infty^3 C_D S$$

where C_D is the wing's drag coefficient considering the wing-borne condition:

$$C_D = C_{D0} + K C_L^2 = C_{D0} + \frac{4(W/S)^2}{(\pi.e.AR)\rho^2 V_\infty^4}$$

where e is the Oswald factor, and AR is the wing's aspect ratio.

Parasite power

Assuming again that $D << W$ along the vertical direction, the thrust can be obtained as:

$$T = \frac{W}{\sin(\theta_{tilt})}$$

where θ_{tilt} is the rotor tilt angle measured counter clock-wise from the horizontal axis.

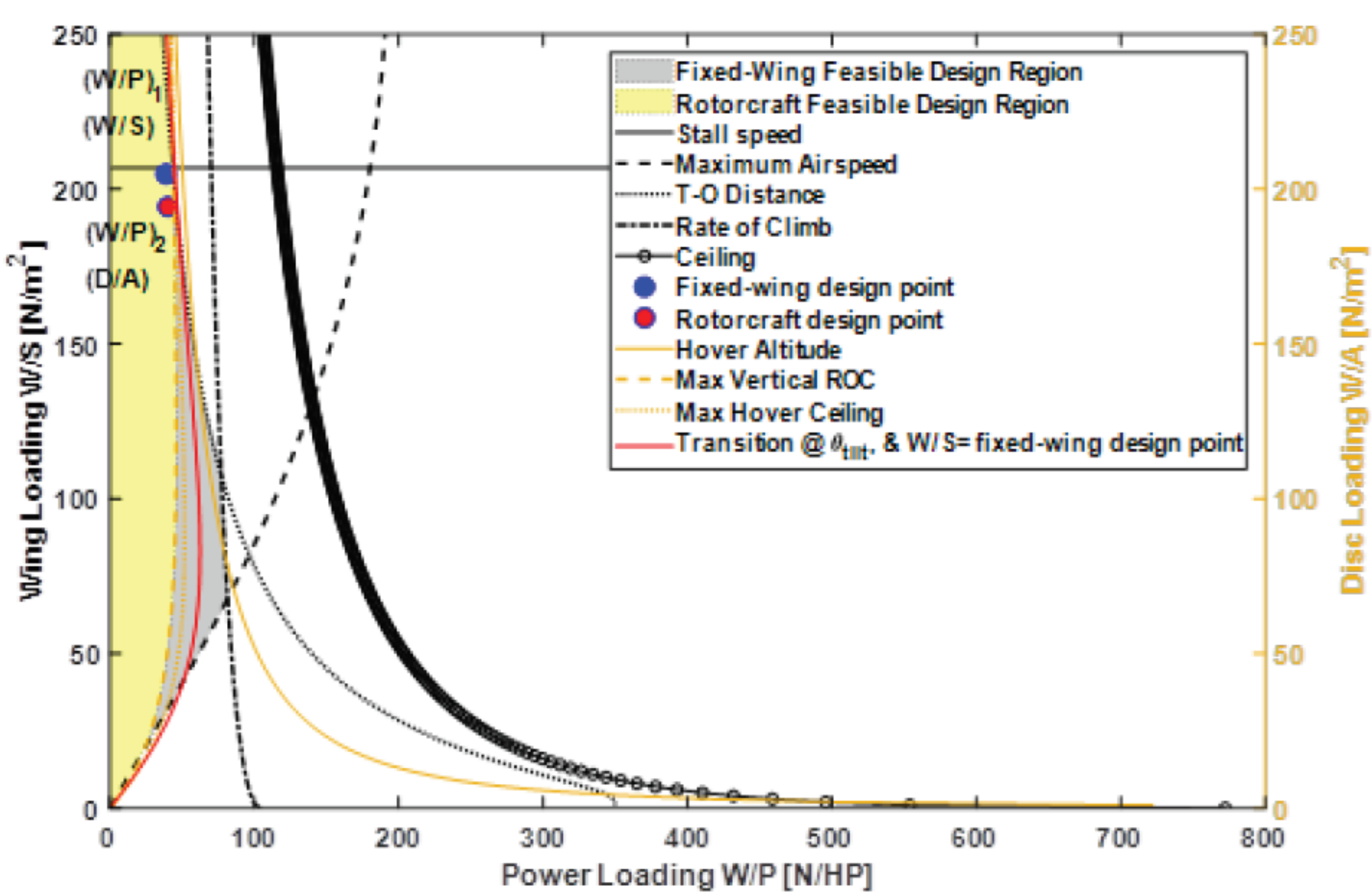
Summing the three aforementioned power components, and rewritting the final formulation as a function of DL and PL, the design constrain for transition flight yields:

$$PL = \frac{1}{\frac{k_i}{\sin(\theta_{tilt})}\sqrt{\frac{-V_\infty^2}{2} + \sqrt{\left(\frac{V_\infty^2}{2}\right)^2 + \left(\frac{DL}{2\rho\sin(\theta_{tilt})}\right)^2}} + \frac{\rho V_{tip}^3}{DL}\left(\frac{\sigma C_d}{8}(1 + 4.6\mu^2)\right) + X}$$

where $X = \frac{1}{2}\rho V_\infty^3 C_{D0}\frac{1}{(W/S)} + \frac{2(W/S)}{(\pi.e.AR)\rho V_\infty}$

The approach is the same as the one adopted for fixed-wing aircrafts. The constraint equations can be resumed by the following:

- Stall speed;
- Maximum forward speed;
- Maximum Rate of Climb (RoC);
- Maximum ceiling.



Two design points must be selected:

- **Fixed-wing**: combination of maximum PL and W/S;
- **Rotorcraft**: combination of maximum PL and DL