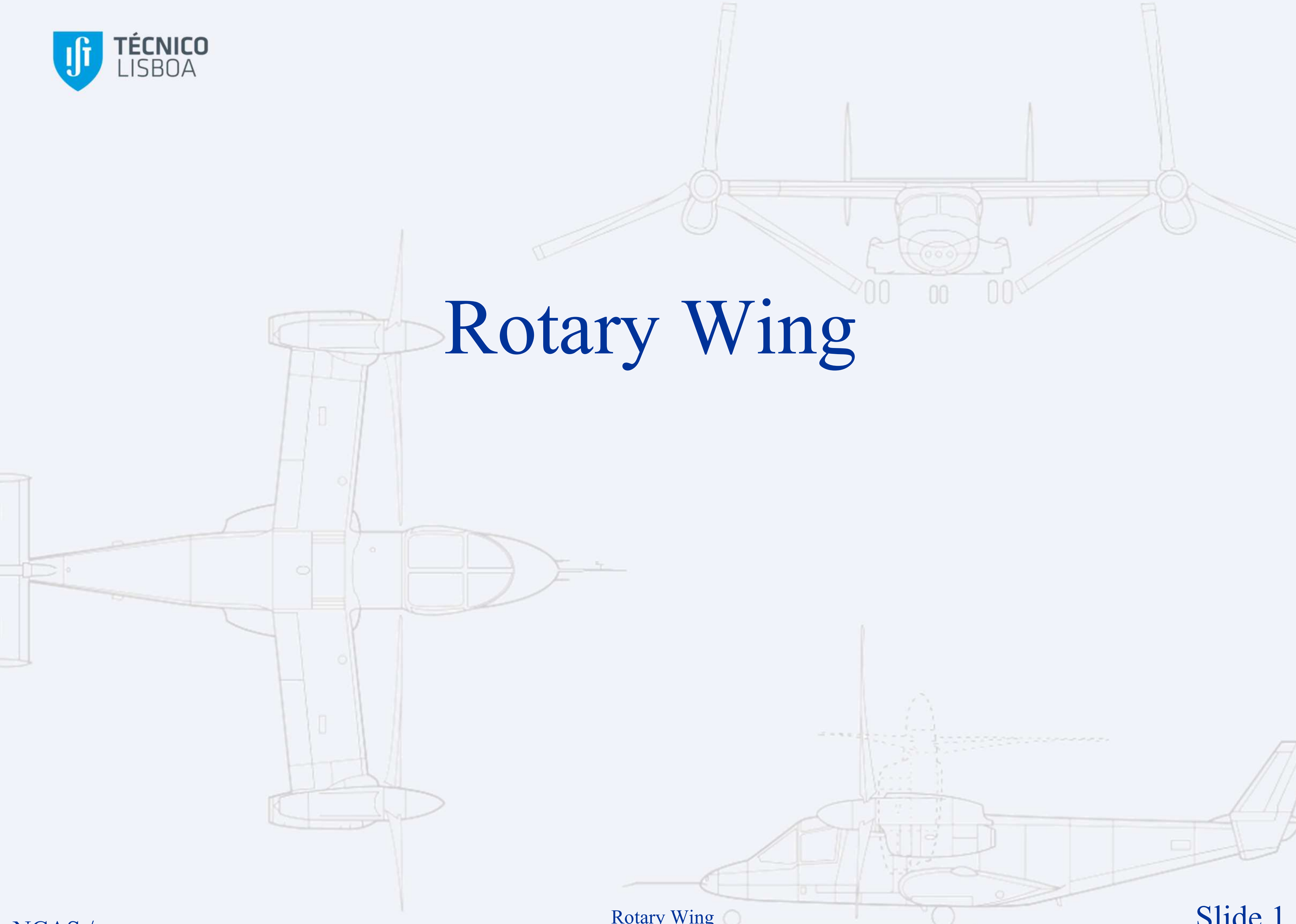
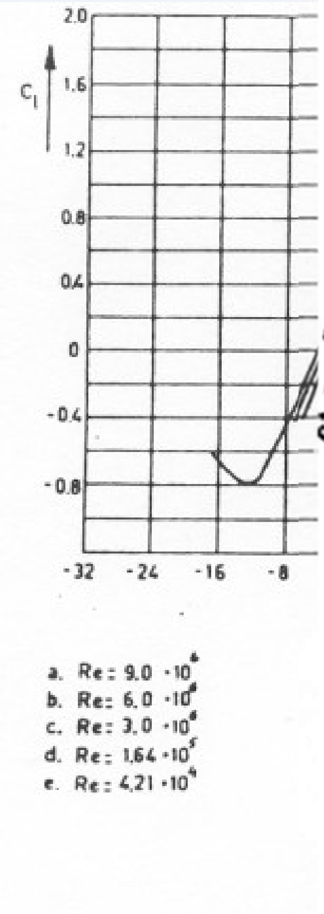
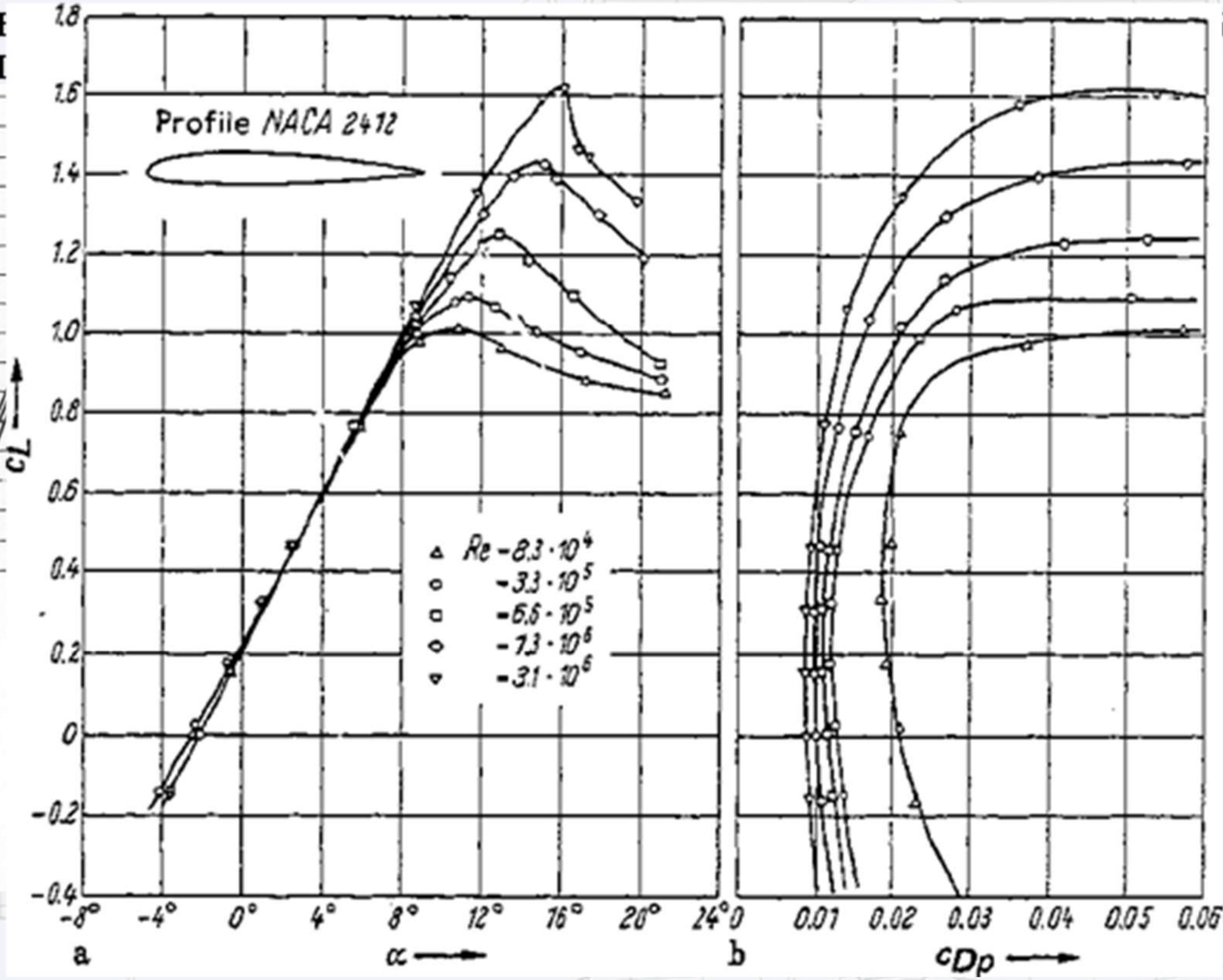
Rotary Wing



Steady flow around an airfoil

Reynolds influence





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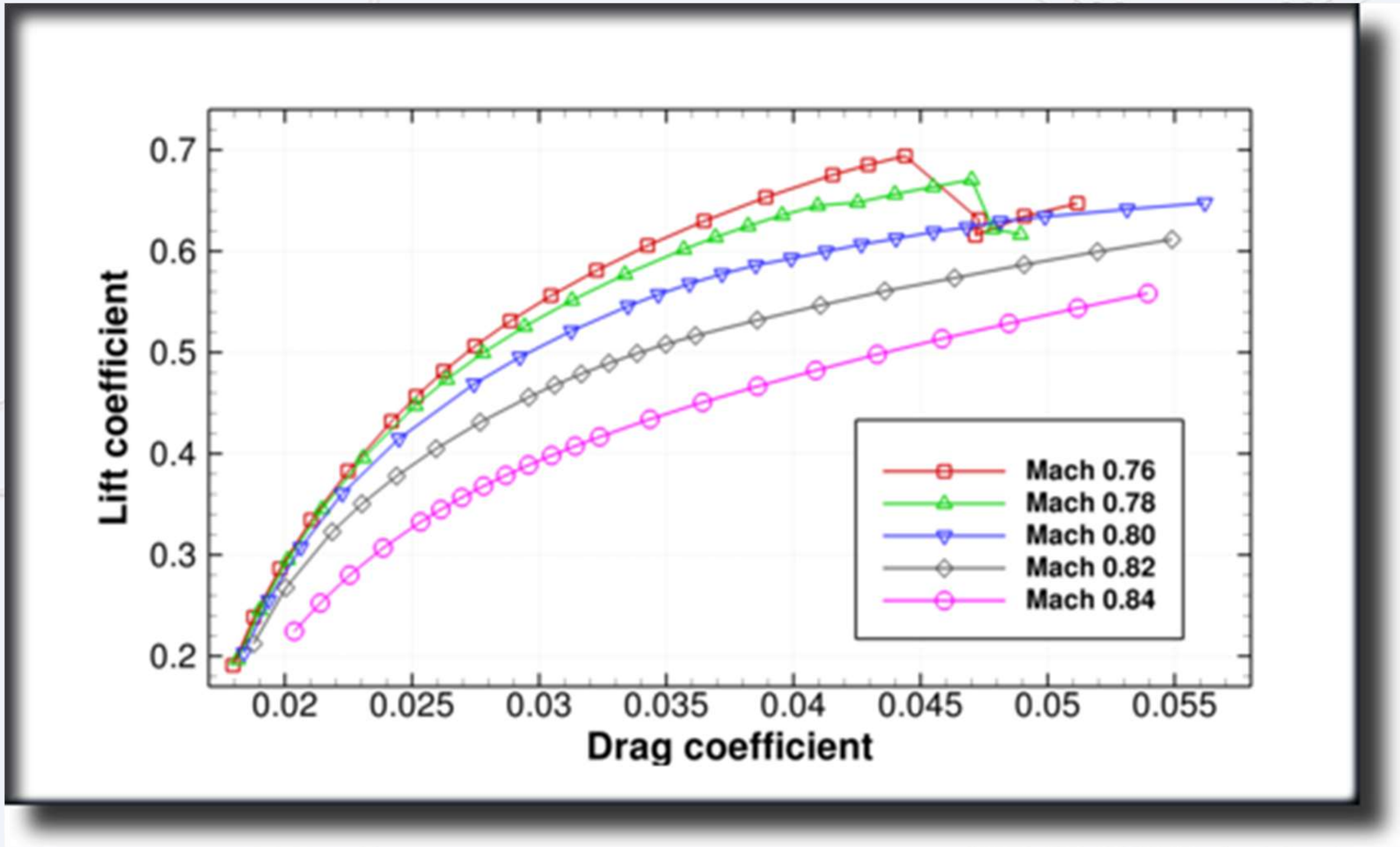
MACH influence



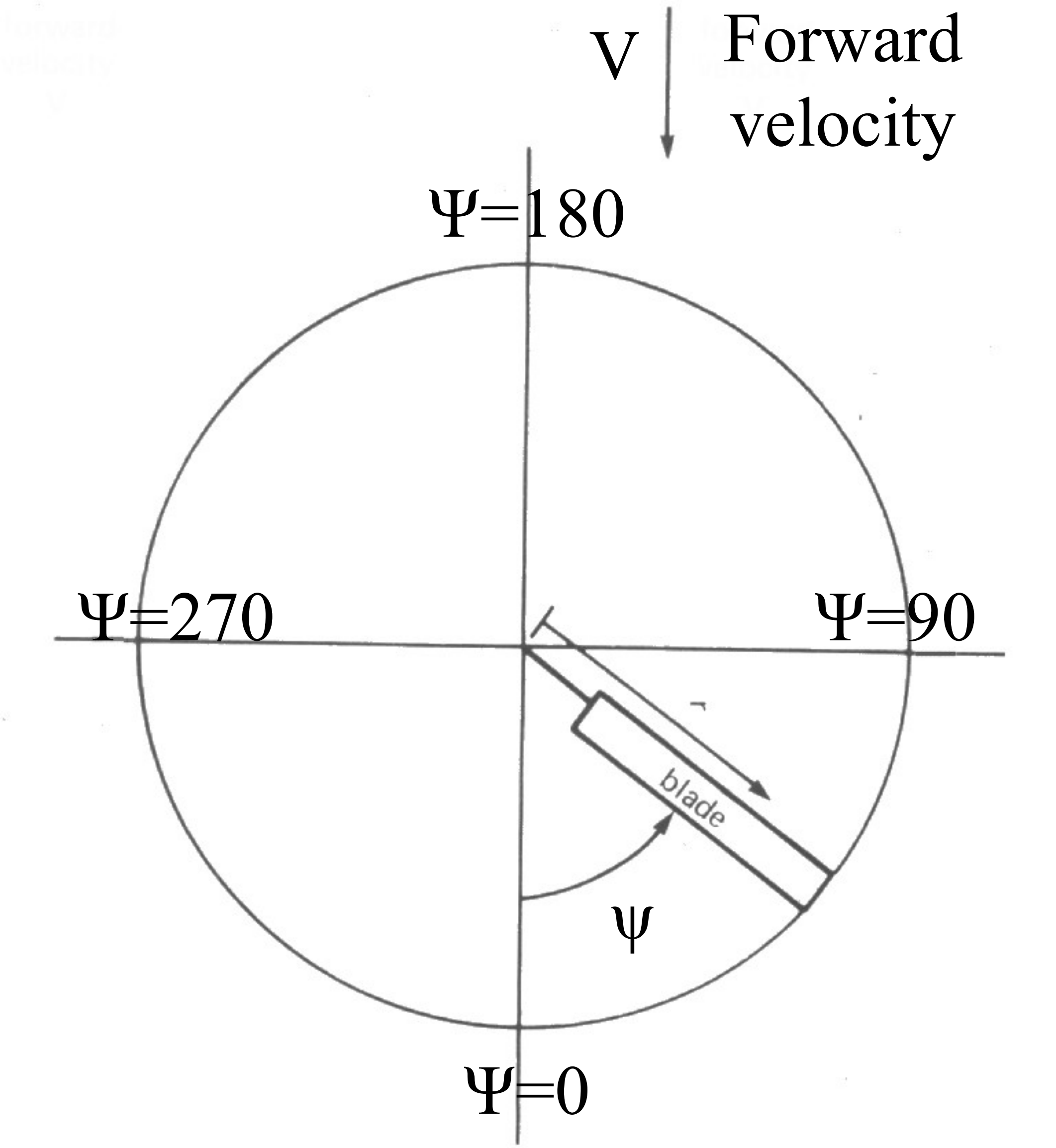
Rotary wing

- The rotor provides 3 basic functions
  - Generation of lift
  - Generation of trust
  - Control the attitude and position

Velocities on the rotor plane

Azimuth Angle

Retreating Side

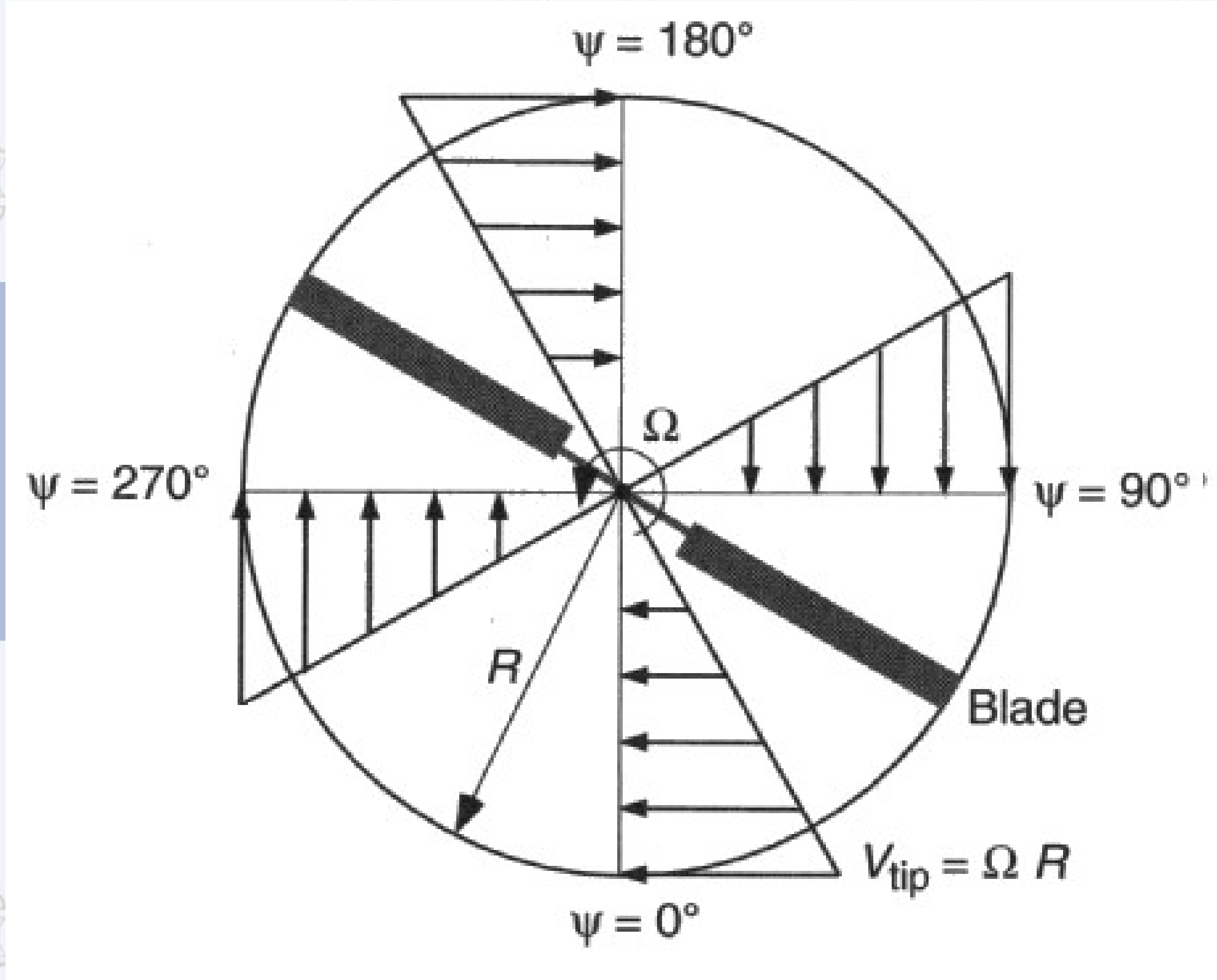


Advancing Side

Velocities on the rotor plane

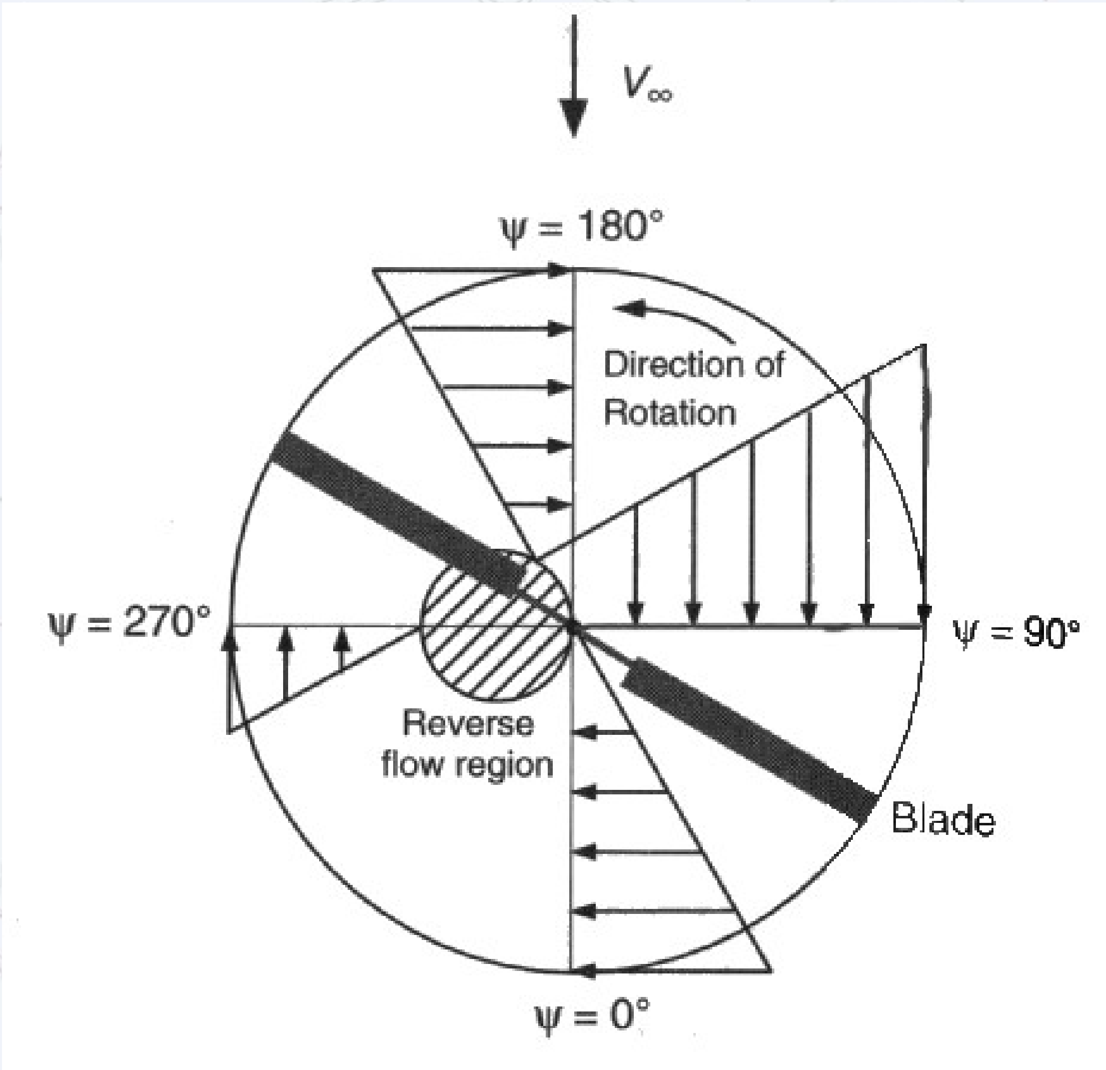
Velocity in hover

Velocity Perpendicular to the blade



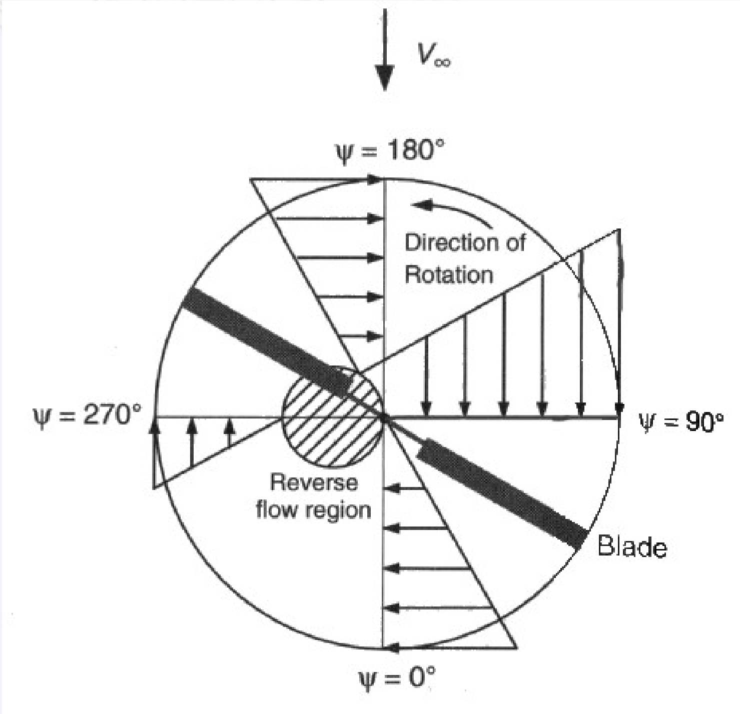
Velocities on the rotor plane

Velocity in Forward Flight



Velocities on the rotor plane

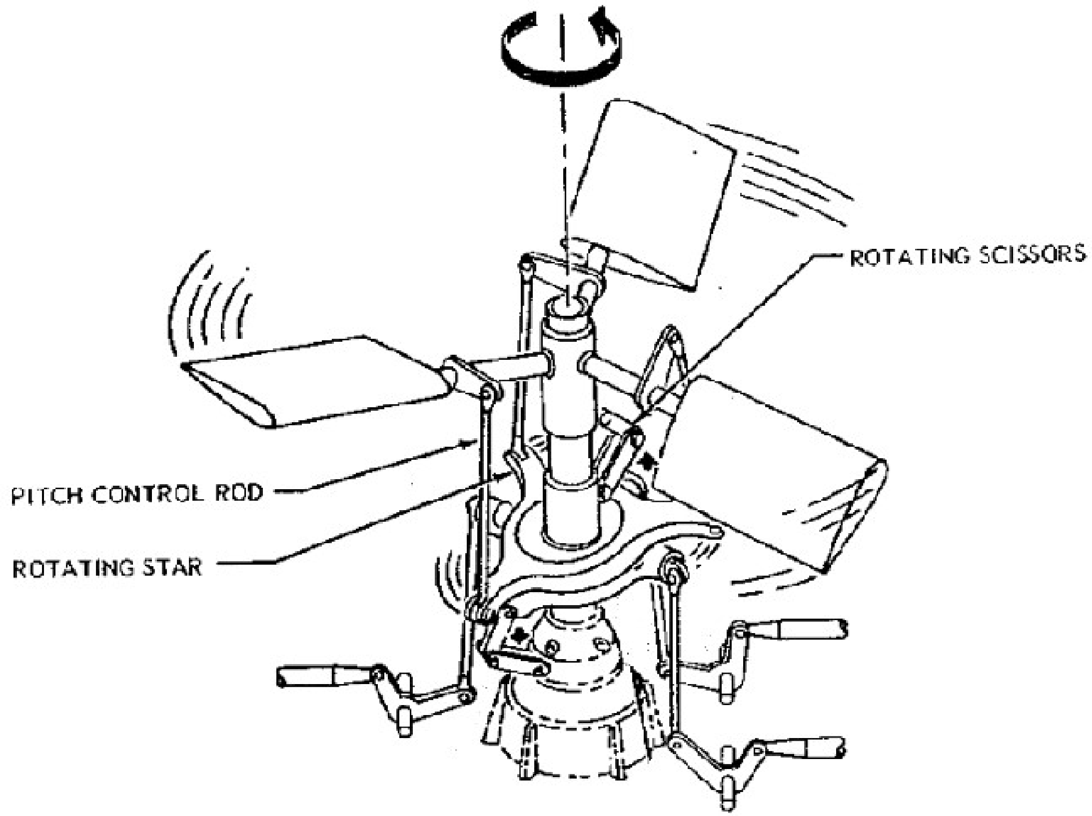
- Velocity distribution is not uniform
- Generation of lift is asymmetric



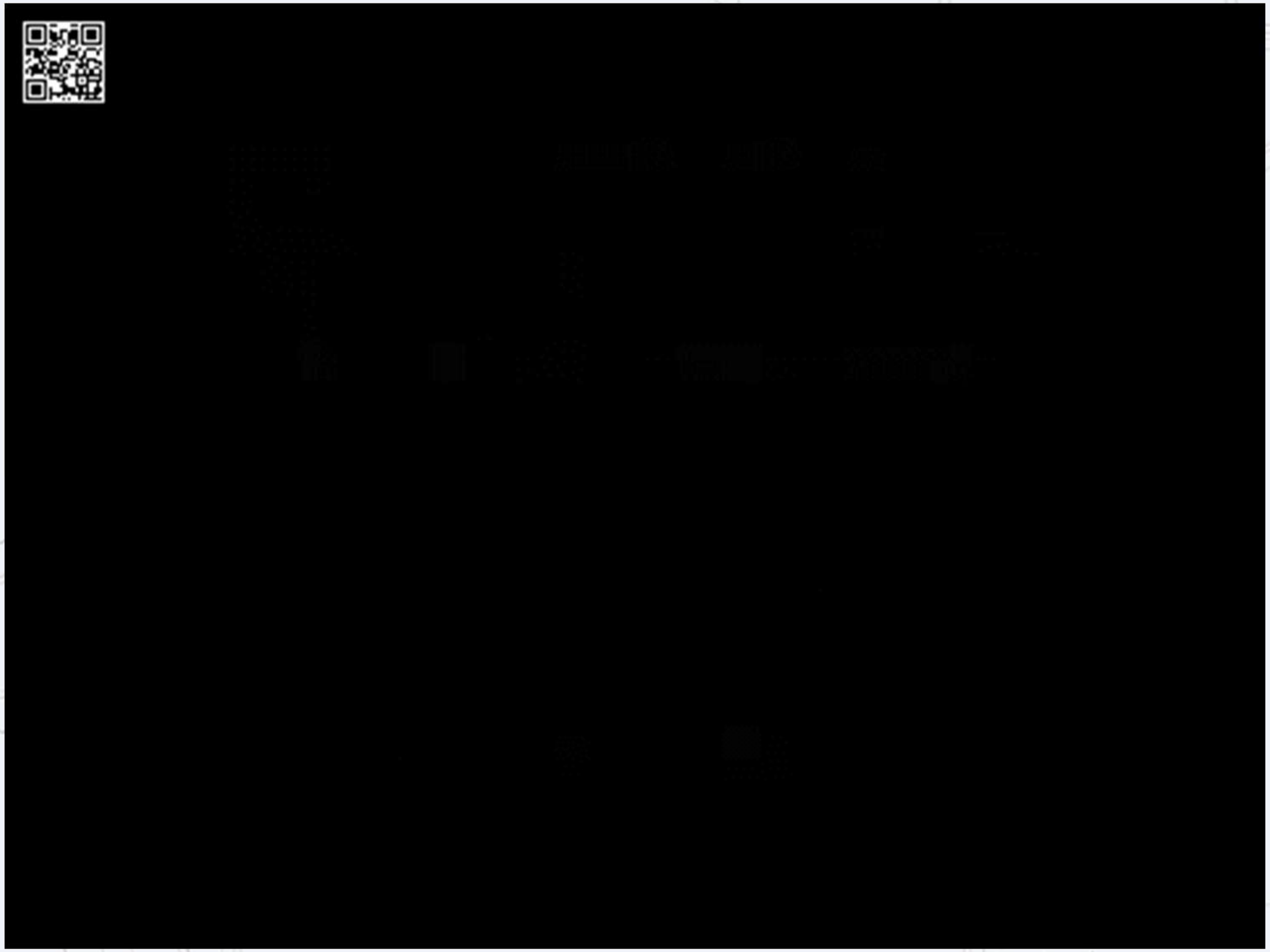
Blade Movement

To compensate de asymmetry

- It's necessary to change the blade AOA (lower on the advancing side, higher on the retreating side)

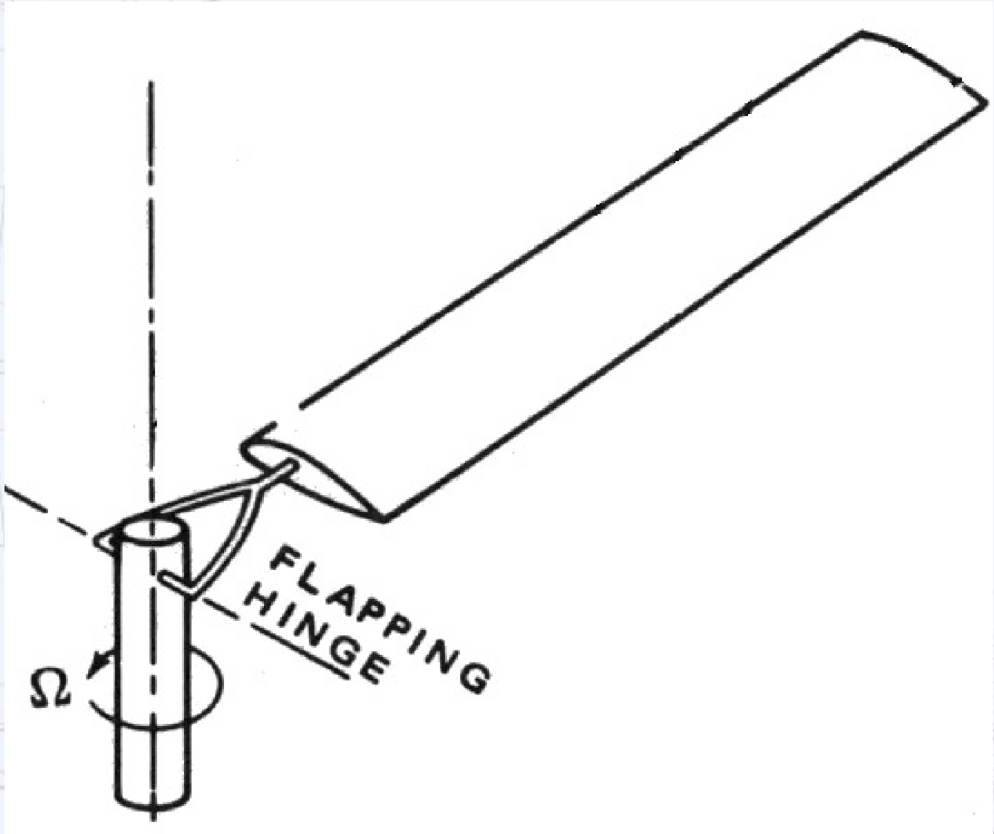


Blade Movement



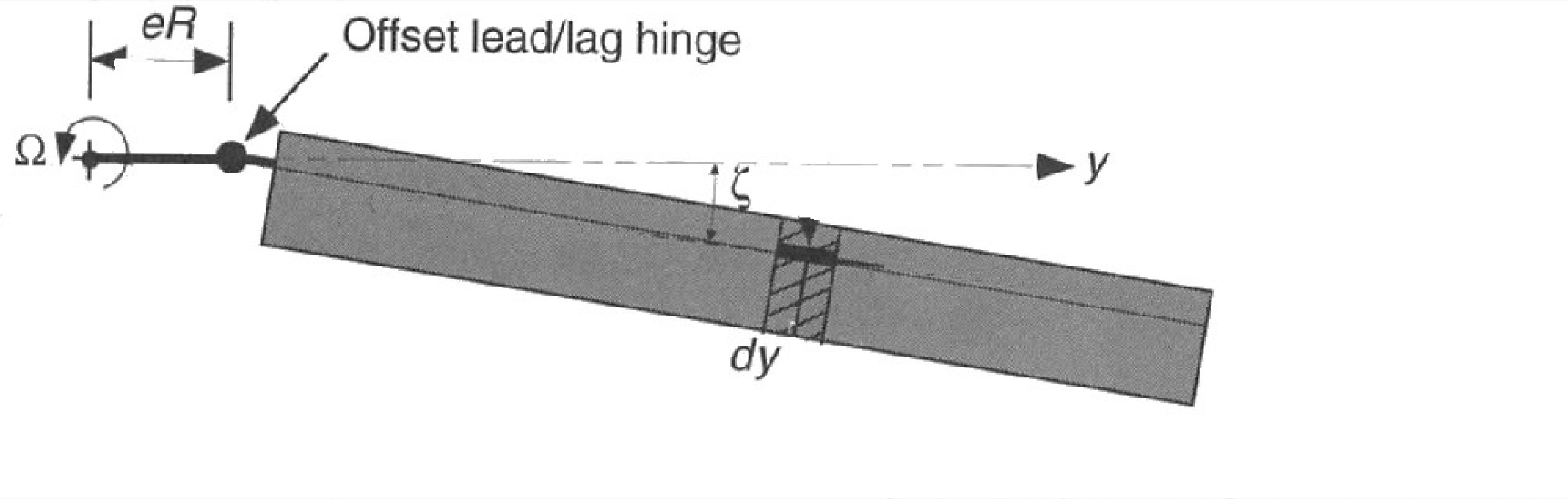
Blade Movement

- To avoid high moments due to the blade on the rotor hub the blade is allowed to flap



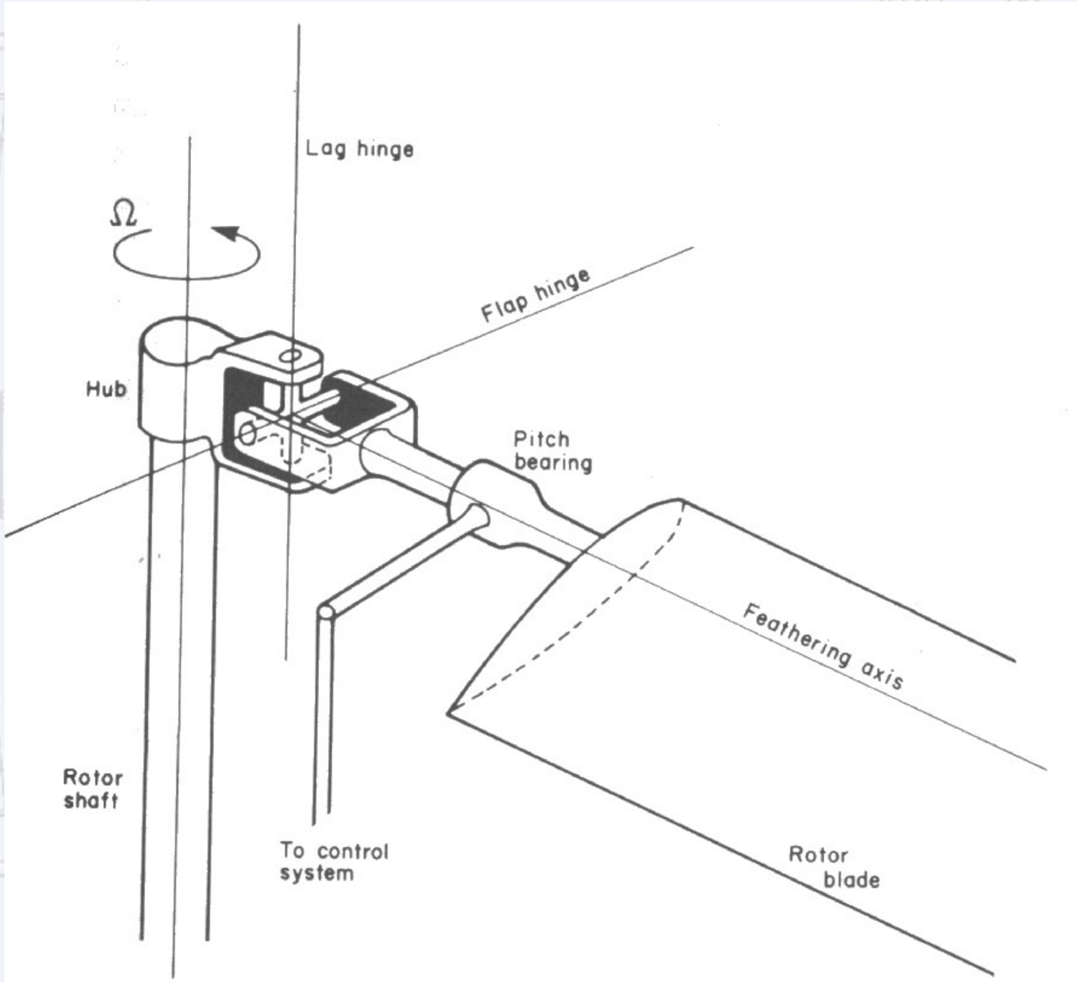
Blade Movement

- To allow the blades movement at constant velocity the blade is allowed to lag.



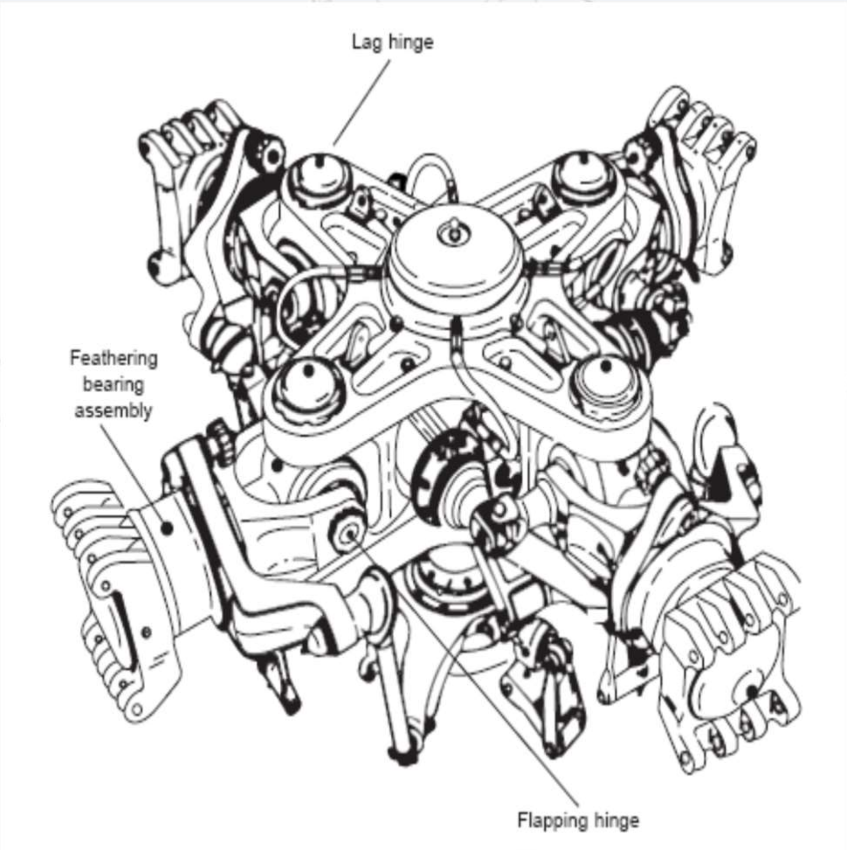
Blade Movement

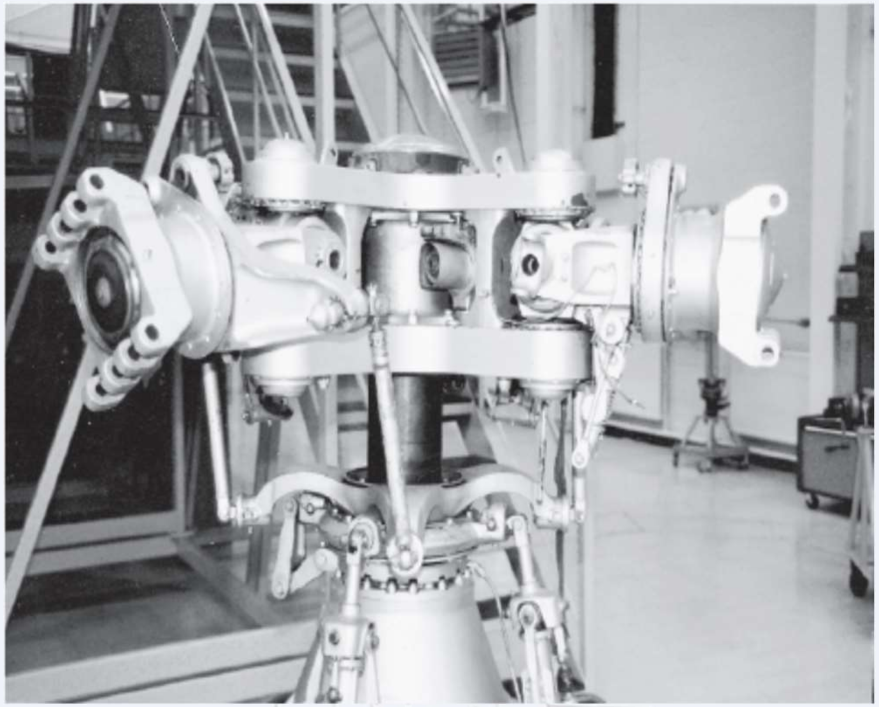
Rotor



Blade Movement

The rotor Hub



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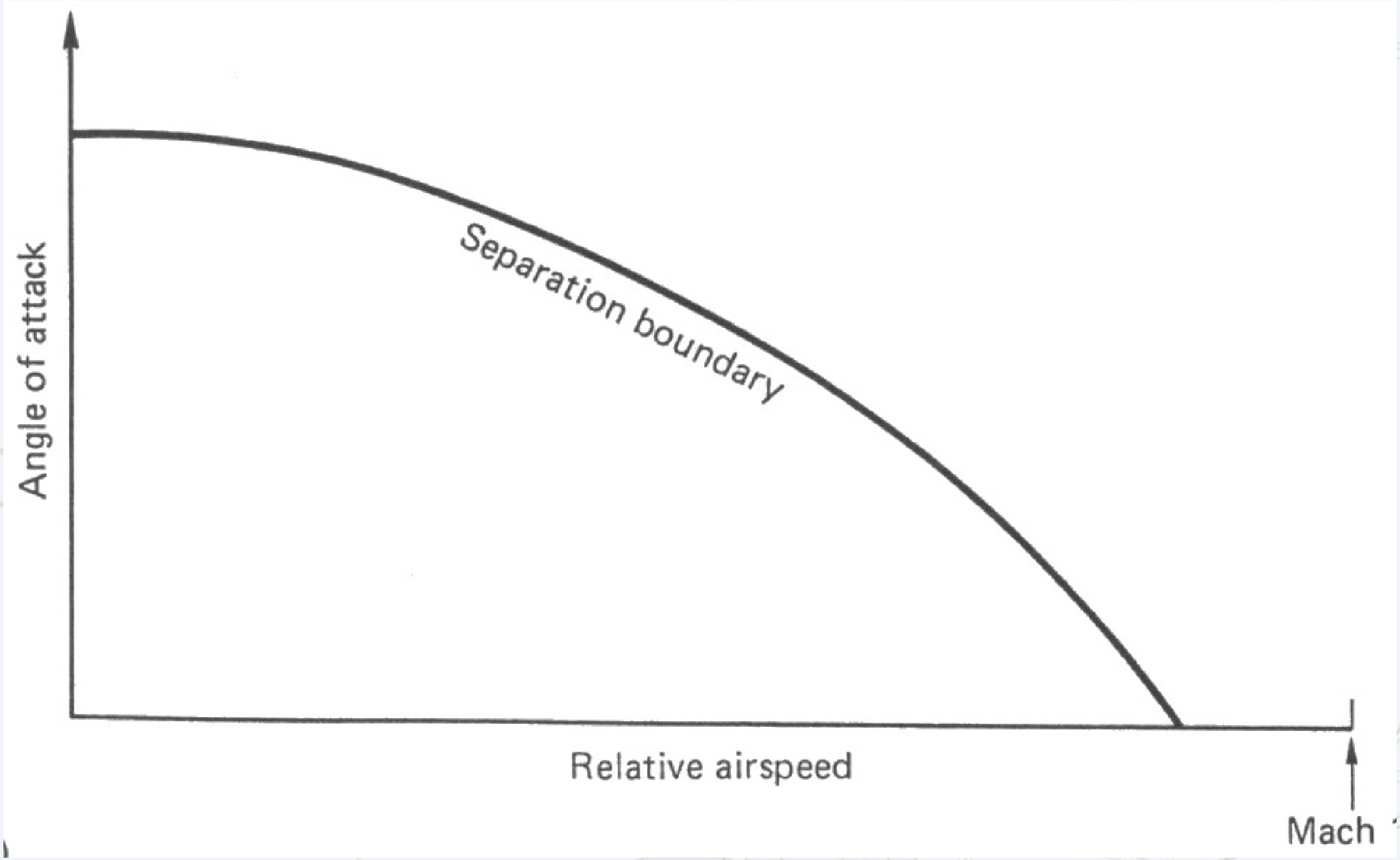
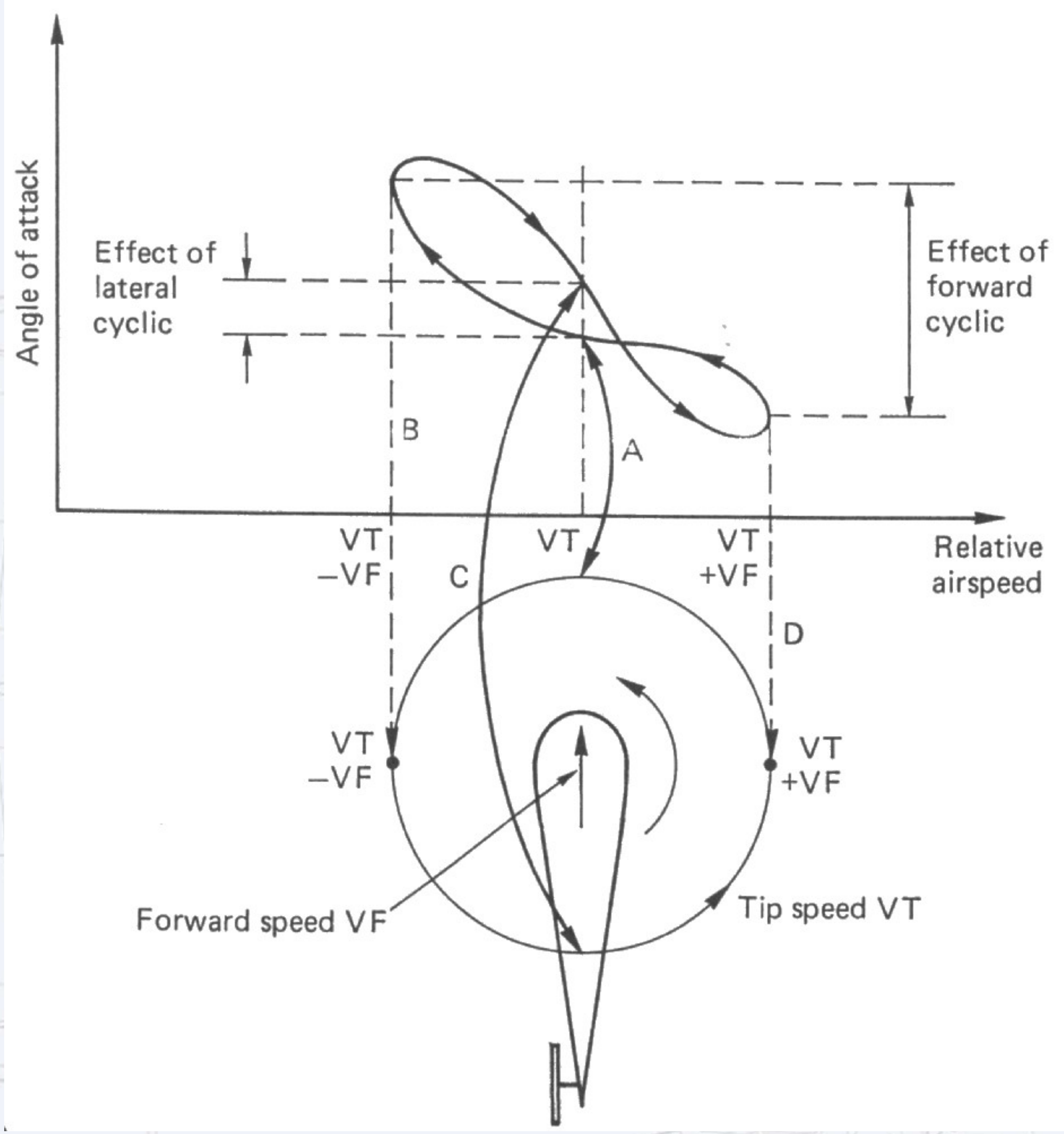
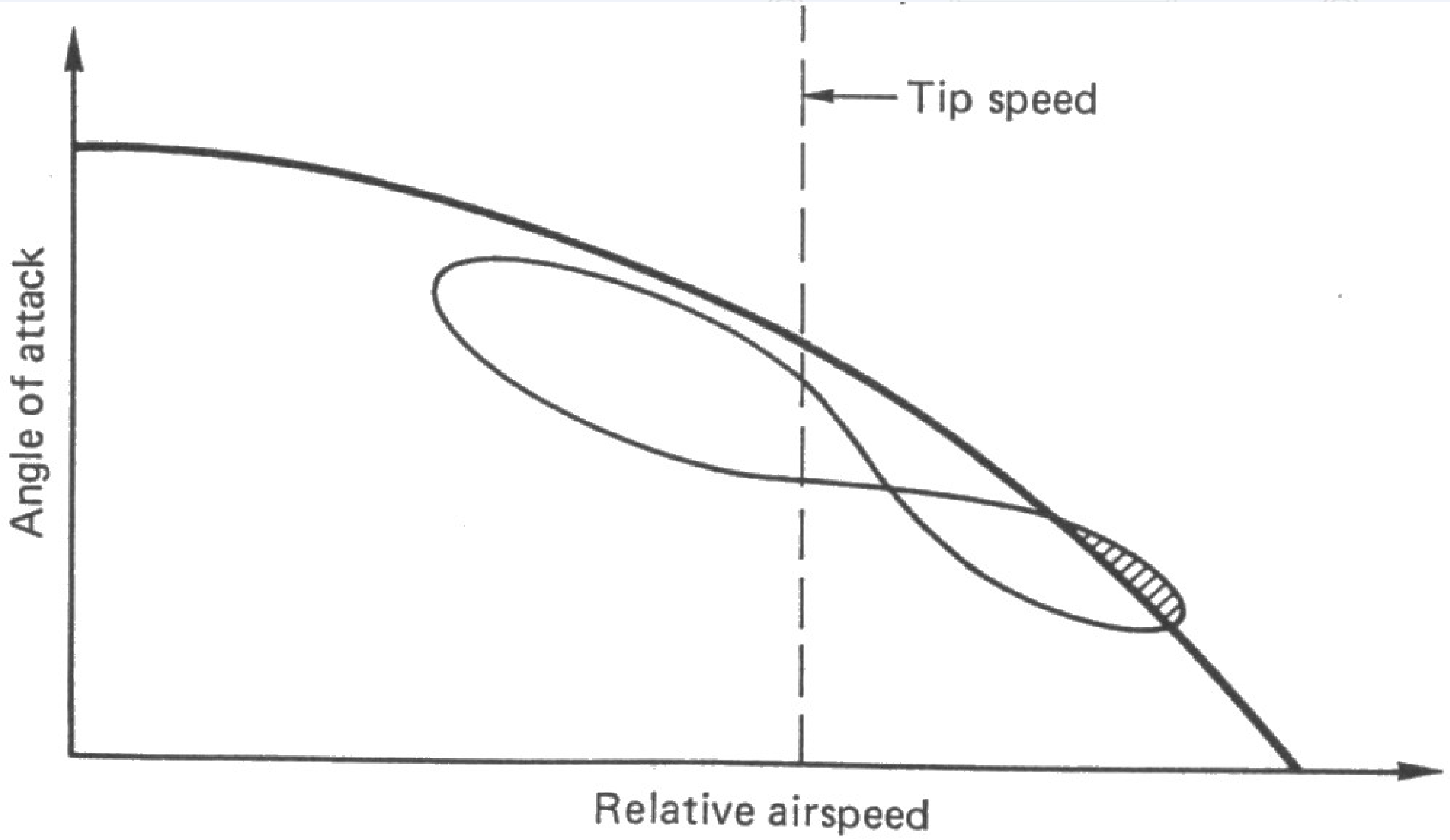
Airfoils Sections



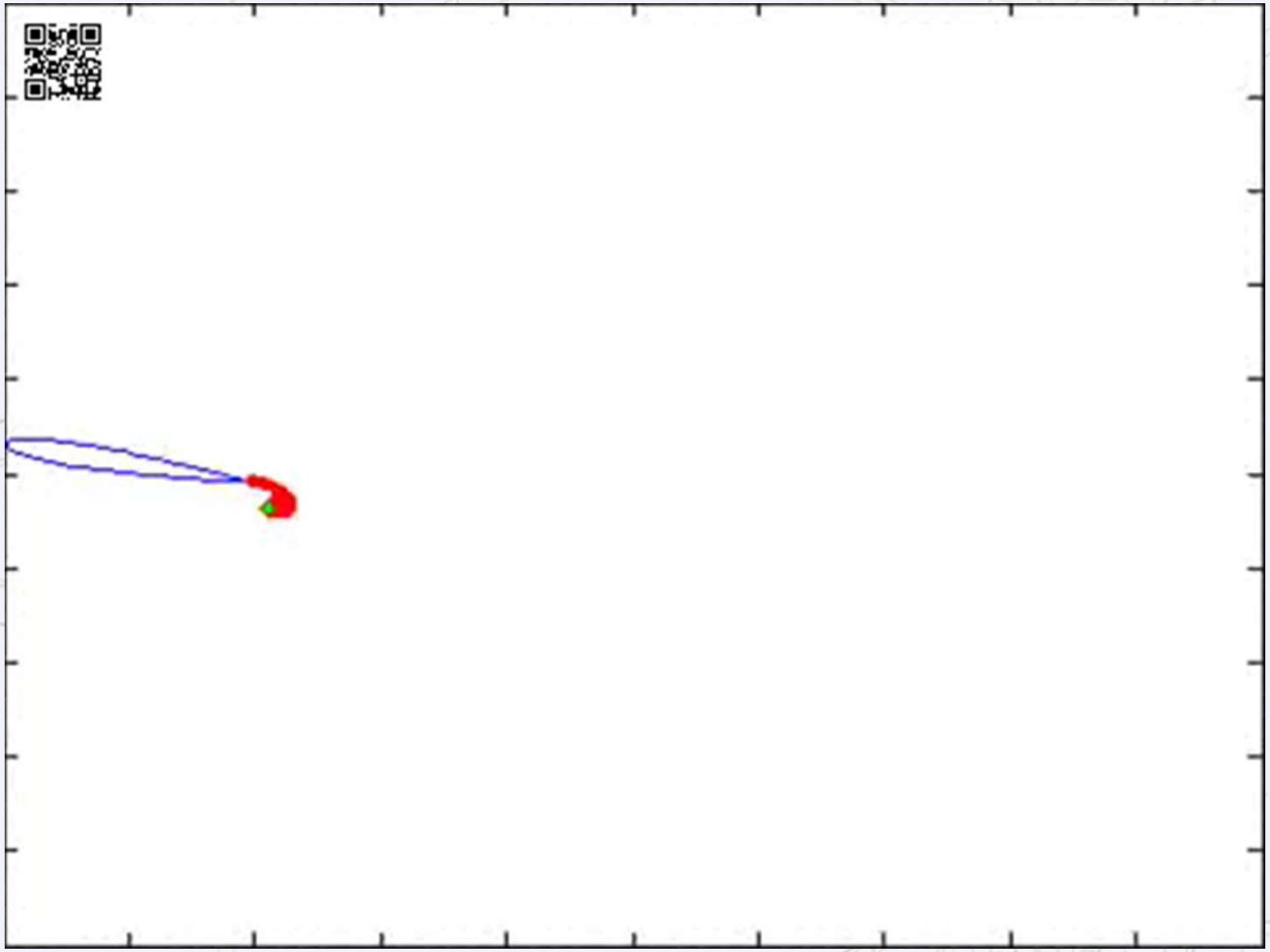
Airfoils Sections



Airfoils Sections



Airfoil Behaviour



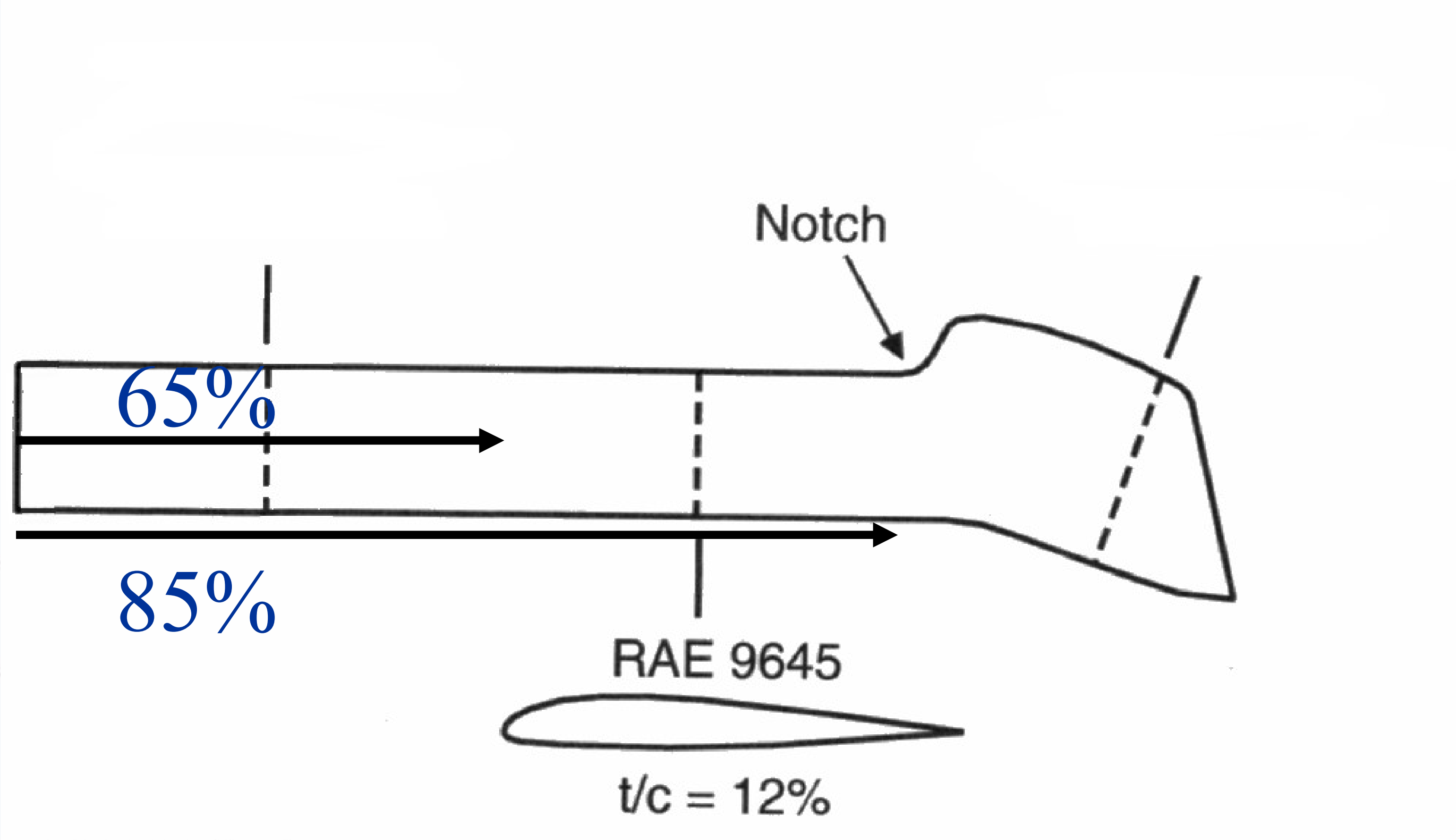
Airfoils Sections

- During each revolution the blade encounters a wide variety of operating conditions.
  - Transonic flow on the advancing region
    - Blade must be thin enough to maximize the M_{dd}.
  - High AOA on the retreating side
    - Minimum thickness and incorporate some camber to give a relatively high $C_{l_{max}}$.
- No single airfoil will meet all requirements. The balance is between:
  - High M_{dd}.
  - High $C_{l_{max}}$ at low M.

BERP rotor

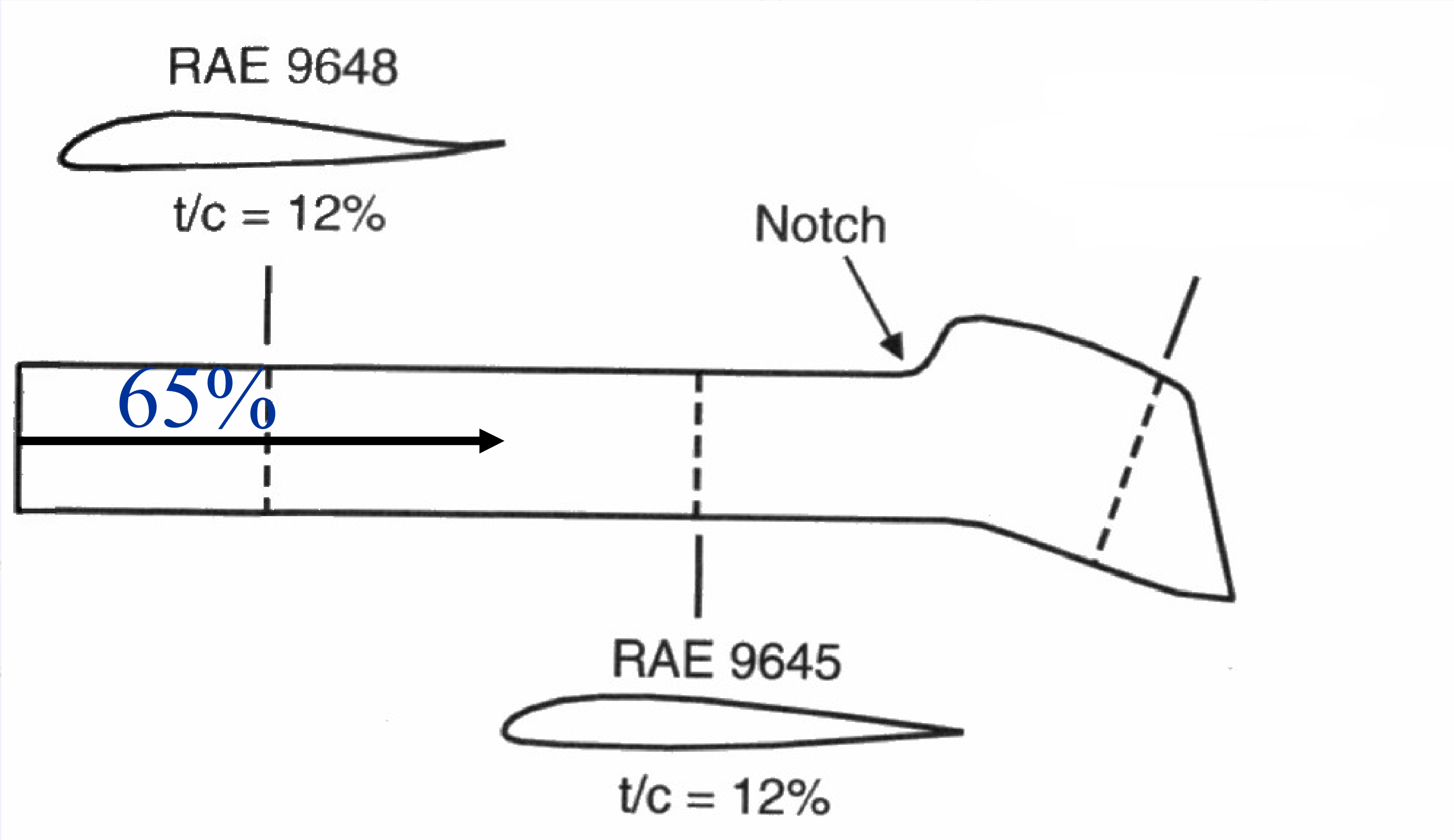
- British Experimental Rotor Program (BERP) rotor, was design to meet the conflicting requirements of the advancing and retreating blade conditions.
- The research paid off in 1986 when a Westland Lynx helicopter attained the absolute speed record for a conventional helicopter with a speed of 400.87 km/h

BERP rotor



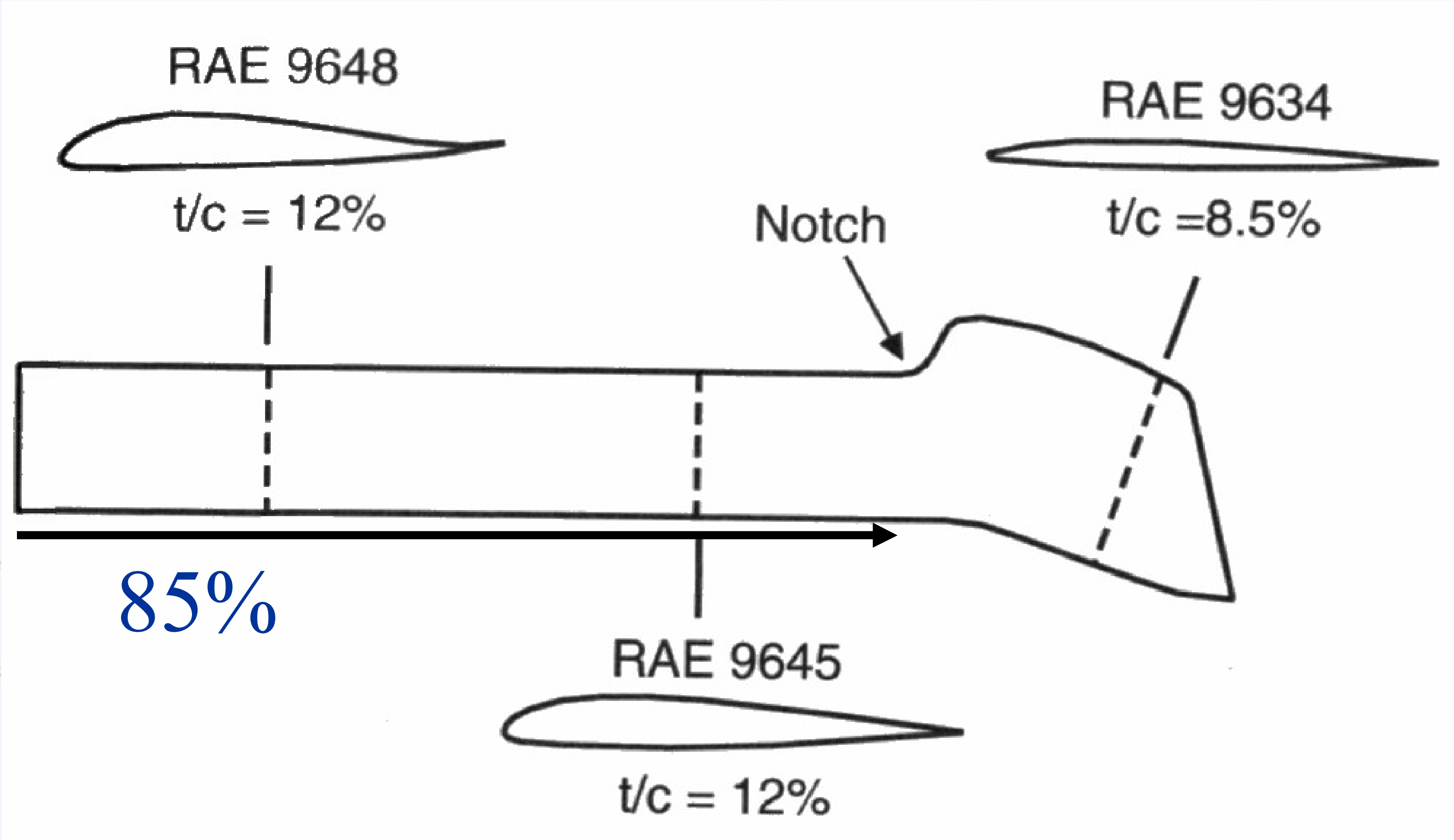
- High $C_{l_{max}}$ (1.55).
- High pitching moments

BERP rotor



- Lower $C_{l_{max}}$.
- Offset of the high pitching moments (RAE 9645)

BERP rotor

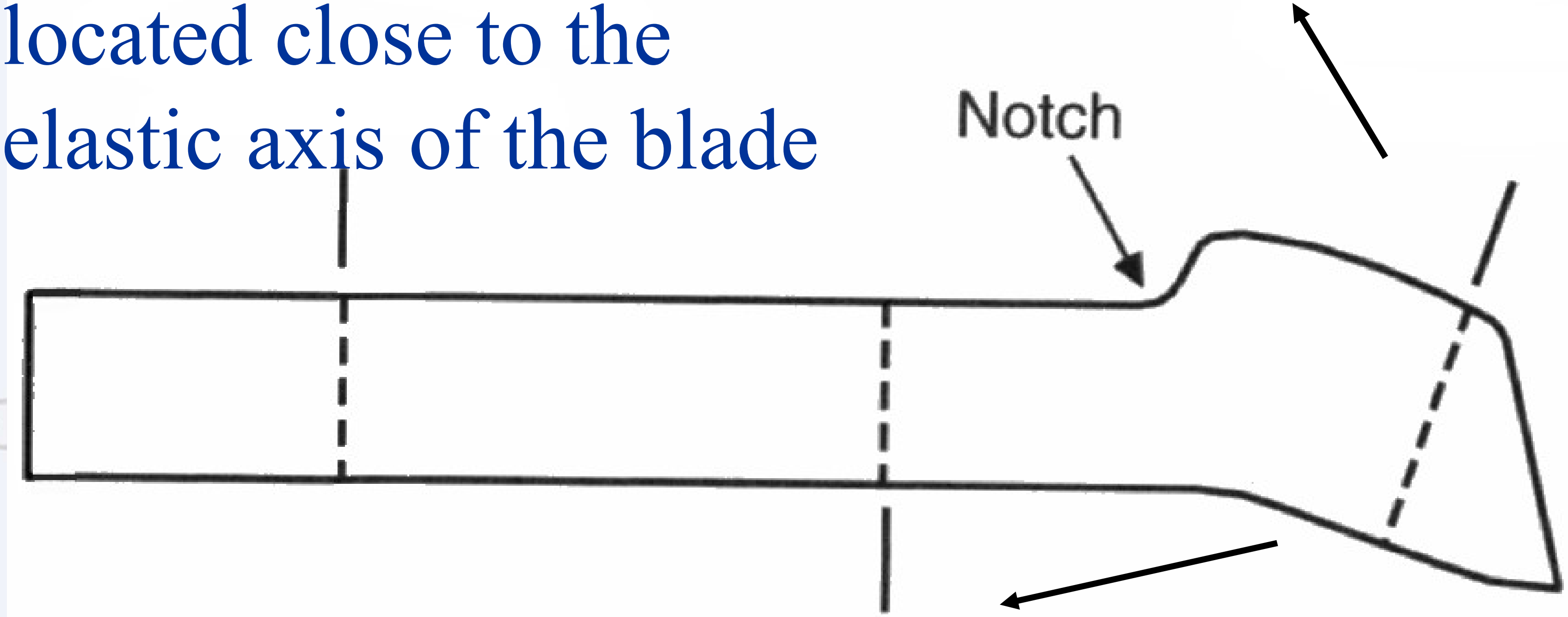


- Low *t/c* ratio (high M_{dd})
- Cambered to give weak shock wave and low pitching moments

BERP rotor

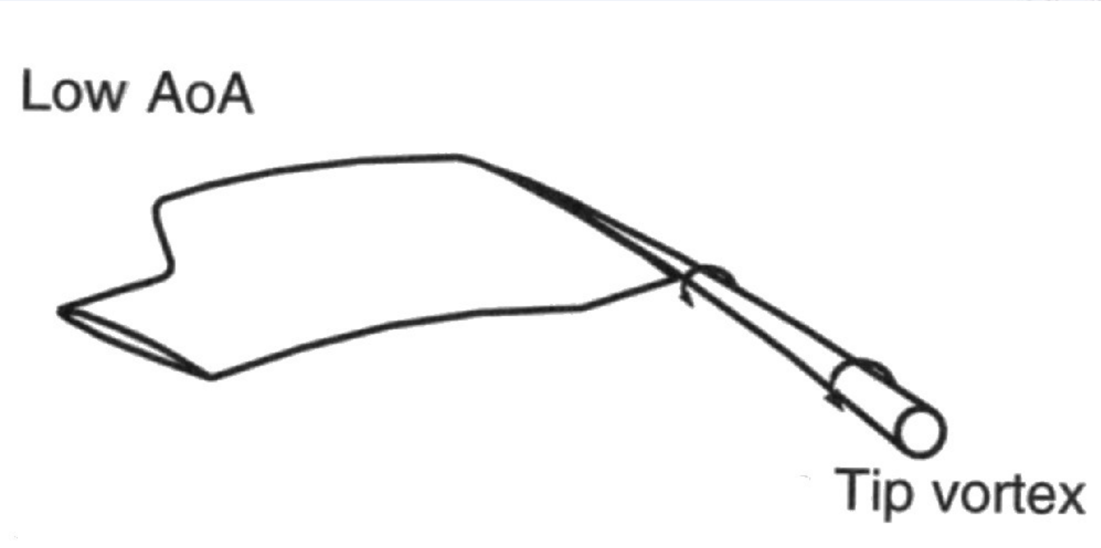
Mean centre of pressure located close to the elastic axis of the blade

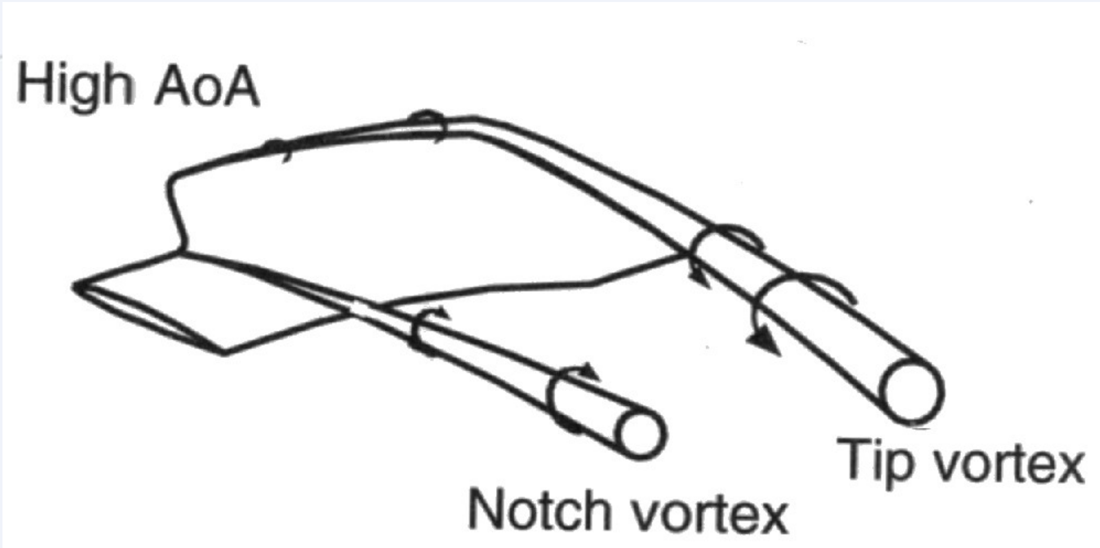
¼ chord offset forward



Sweep maintaining the M_n approximately constant

BERP rotor





Momentum Theory

- Let's simplify our first approach and develop a simple method capable of predicting the rotor thrust and power

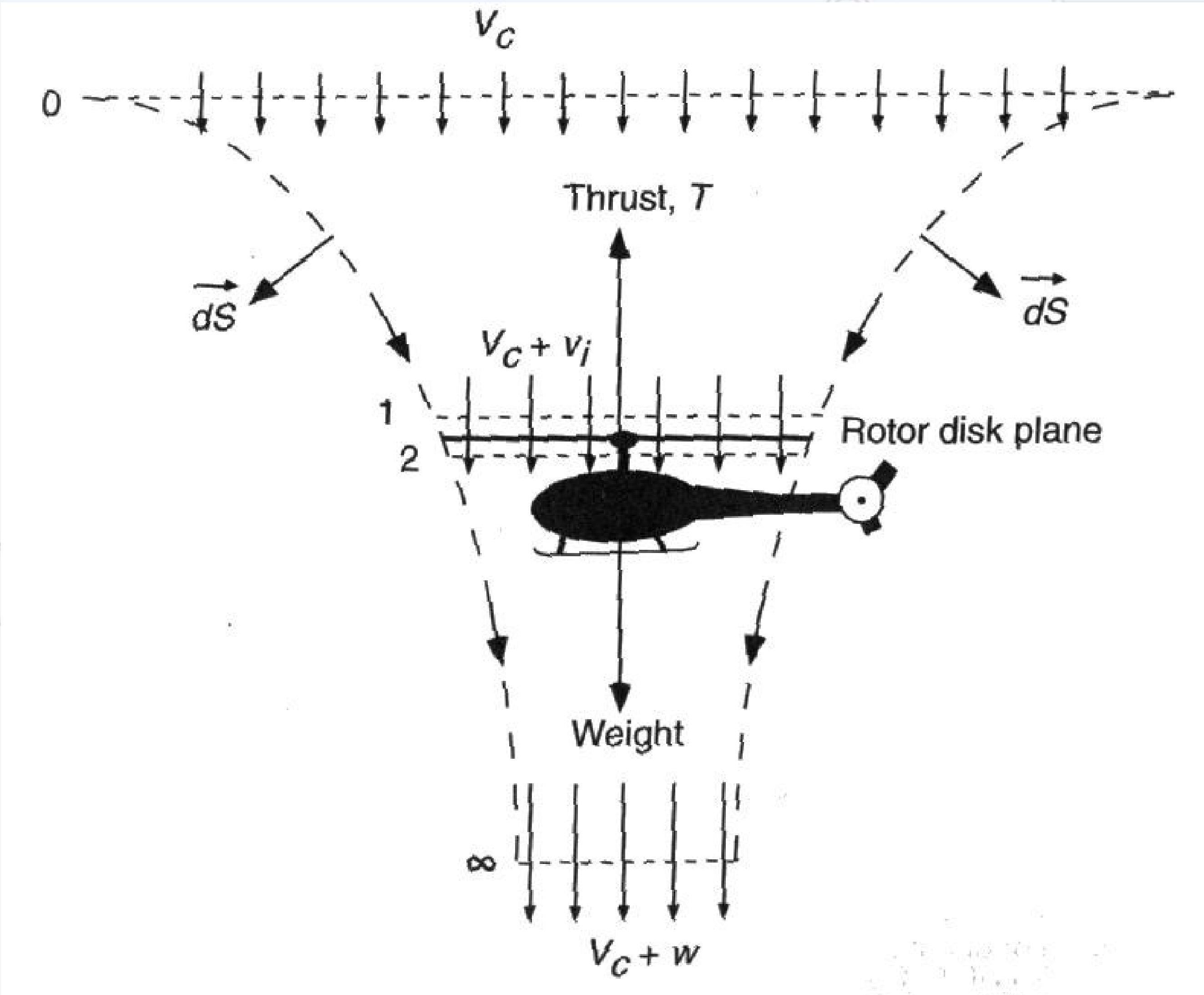
Momentum Theory

- First developed by Rankine (1895) for marine propellers and developed further and generalized by several other authors

Assumptions

- Conditions in hover:
  - No forward speed
  - No vertical speed
  - The flow field is axisymetrical
  - There is a wake boundary with the flow outside this boundary being quiescent
  - The flow velocities inside this boundary can be quite high

Representation and notation



Ideal Power Hover

- Power consumed=Energy rate flow out-Energy rate flow in and the following expression can be obtained

$$P = T\sqrt{\frac{T}{2\,A\,\rho}}$$

Or in terms of the induced velocity:

$$P = Tv_h = \left(2\rho A v_h^{\,2}\right)v_h = 2\rho A v_h^{\,3}$$

Non Ideal effects Hover

- The actual rotor power can then be expressed as:

$$C_{P_a} = C_{P_i} + C_{P_0} = \kappa \frac{C_T^{3/2}}{\sqrt{2}} + \frac{1}{8}\sigma C_{d0}$$

- Where to profile power coefficient is obtained :

$$P_0 = \frac{1}{8}\rho N_b \Omega^3 c C_{d0} R^4$$

$$C_{P_0} = \frac{1}{8}\left(\frac{N_b c R}{A}\right) C_{d0} = \frac{1}{8}\sigma C_{d0}$$

Power required for axial flight

- We can write the power ratio as:

$$\frac{P}{P_h} = \frac{V_c + v_i}{v_h} = \frac{V_c}{v_h} + \frac{v_i}{v_h}$$

- Using the previous equations obtained for climbing

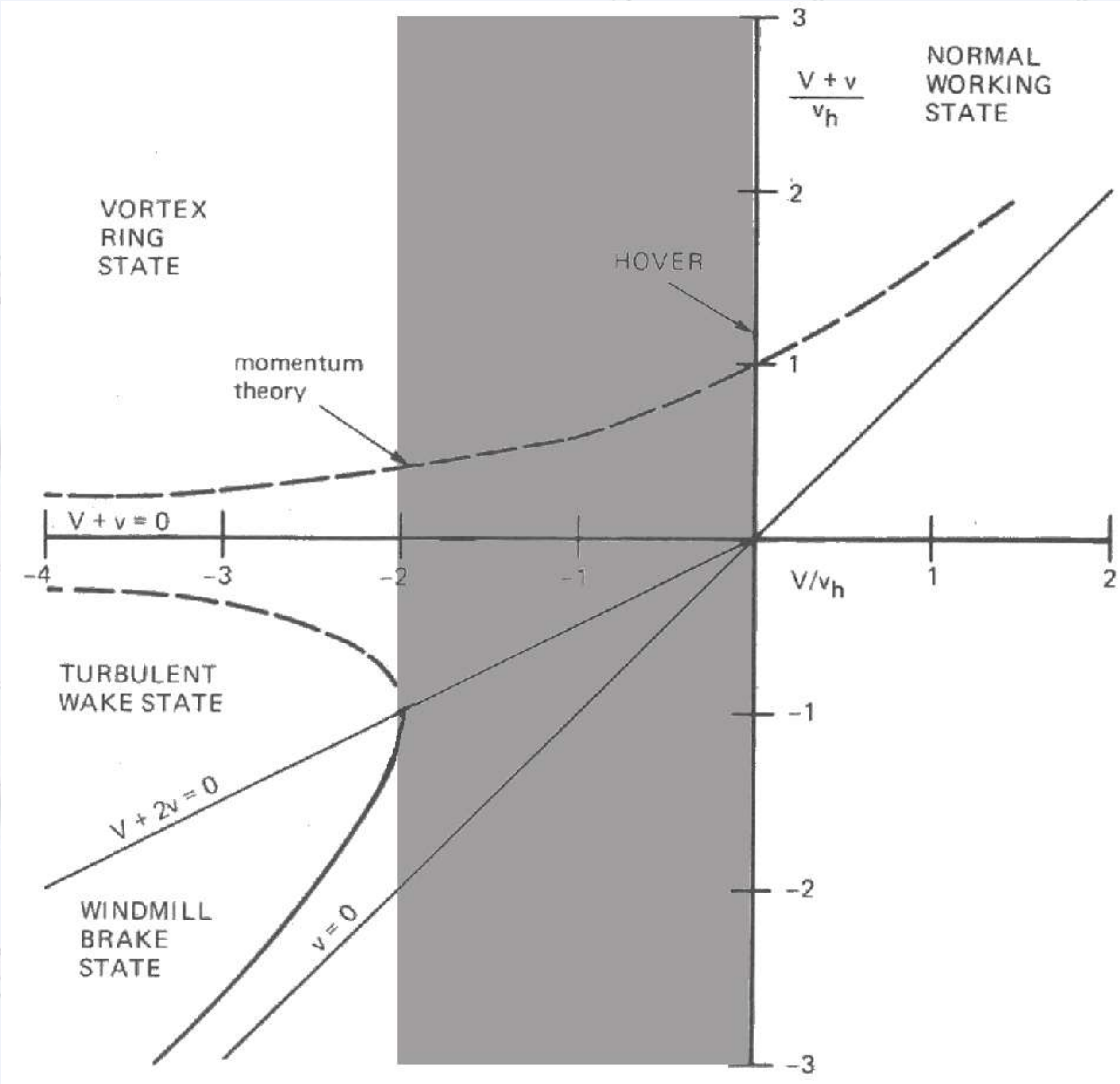
$$\frac{P}{P_h} = \frac{V_c}{v_h} - \left(\frac{V_c}{2v_h}\right) + \sqrt{\left(\frac{V_c}{2v_h}\right)^2 + 1} = \left(\frac{V_c}{2v_h}\right) + \sqrt{\left(\frac{V_c}{2v_h}\right)^2 + 1}$$

Power required for axial flight

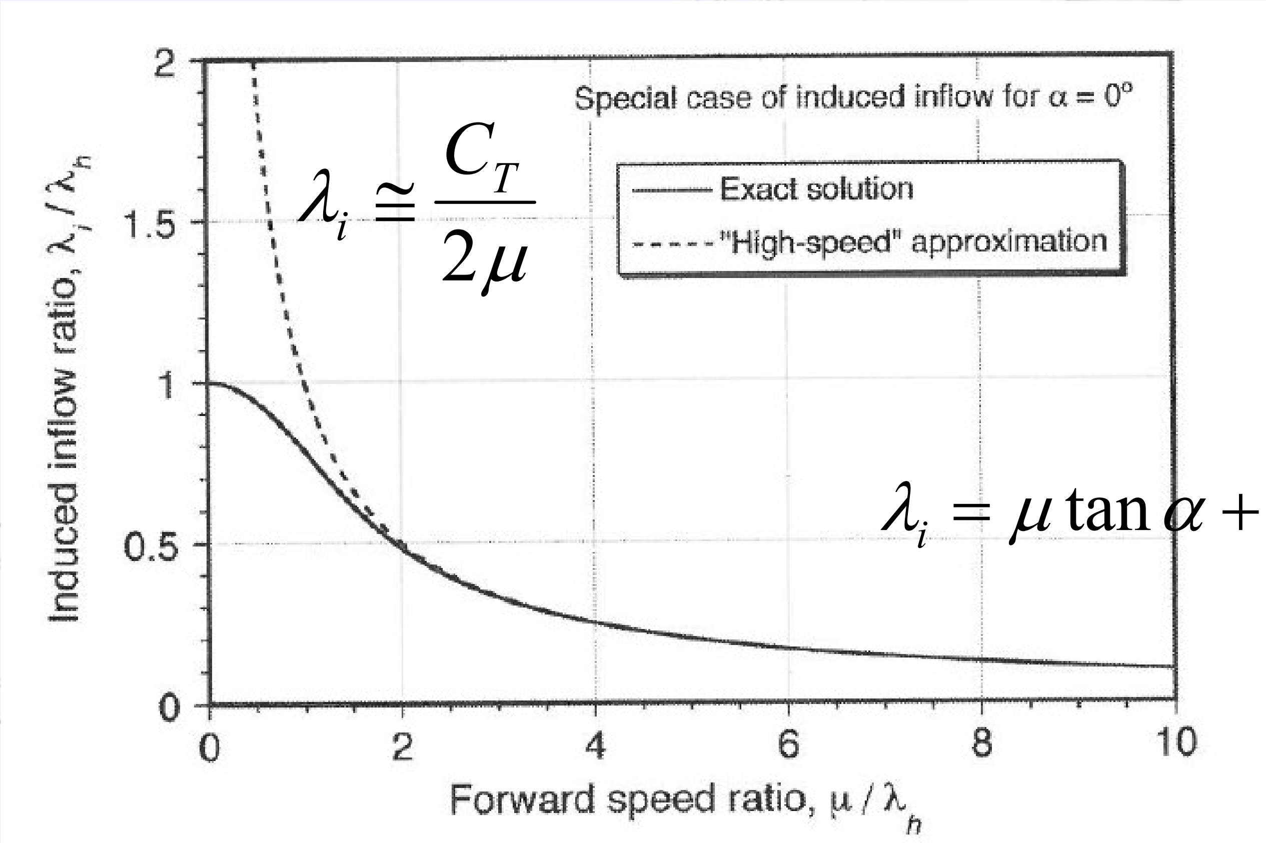
- Using the previous equations obtained for descent

$$\frac{P}{P_h} = \frac{V_c}{2v_h} - \sqrt{\left(\frac{V_c}{2v_h}\right)^2 - 1}$$

Power required for axial flight



Variation of Non-Dimensional Inflow with Advance Ratio



$$\lambda_i \cong \frac{C_T}{2\mu}$$

$$\lambda_i = \mu \tan\alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda_i^{\,2}}}$$

- Notice that inflow velocity rapidly decreases with advance ratio

Power Coefficient

$$C_{P_{\mathrm{Uncorrected}}} = C_T\lambda_i + \frac{1}{2}\frac{f}{A}\mu^3 + \frac{\sigma C_{d0}}{8}\left[1+3\mu^2\right]$$

Induced power

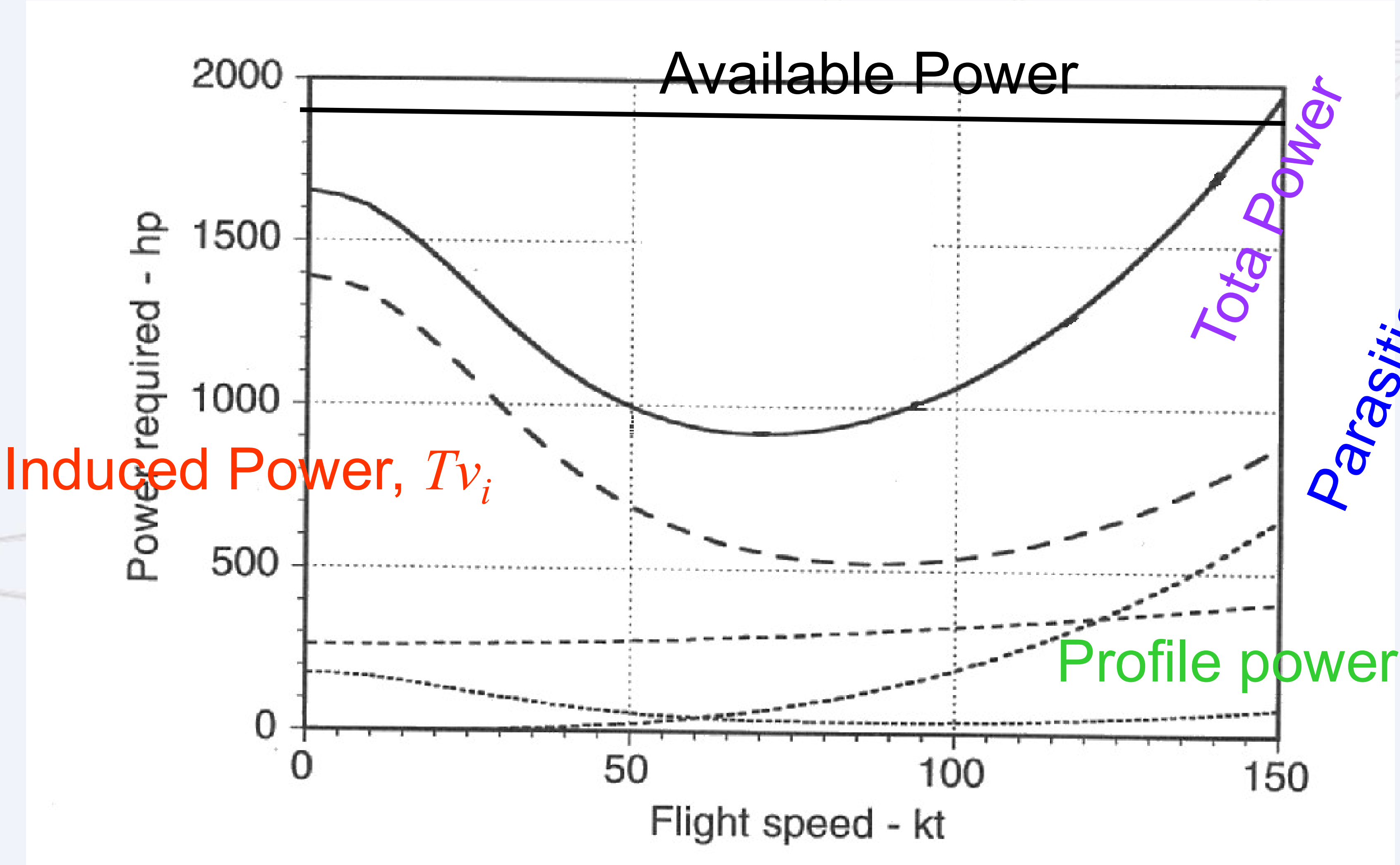
Parasite Power

Profile Power

$$C_{P_{\mathrm{Corrected}}} = \underset{1.15}{\kappa} C_T\lambda_i + \frac{1}{2}\frac{f}{A}\mu^3 + \frac{\sigma C_{d0}}{8}\left[1+4.6\mu^2\right]$$

C_D is the vehicle parasite drag coefficient and S the reference area. Because there is no agreement on a common reference area it is customary to supply the product $C_D S = f$ equivalent flat plate area

Power in Forward Flight



Steady aerodynamics

- What is the induced power in hover for a rotary wing aircraft with 2200kg, 11m diameter at sea level, using momentum theory?
- And with a forward speed of 15m/s?

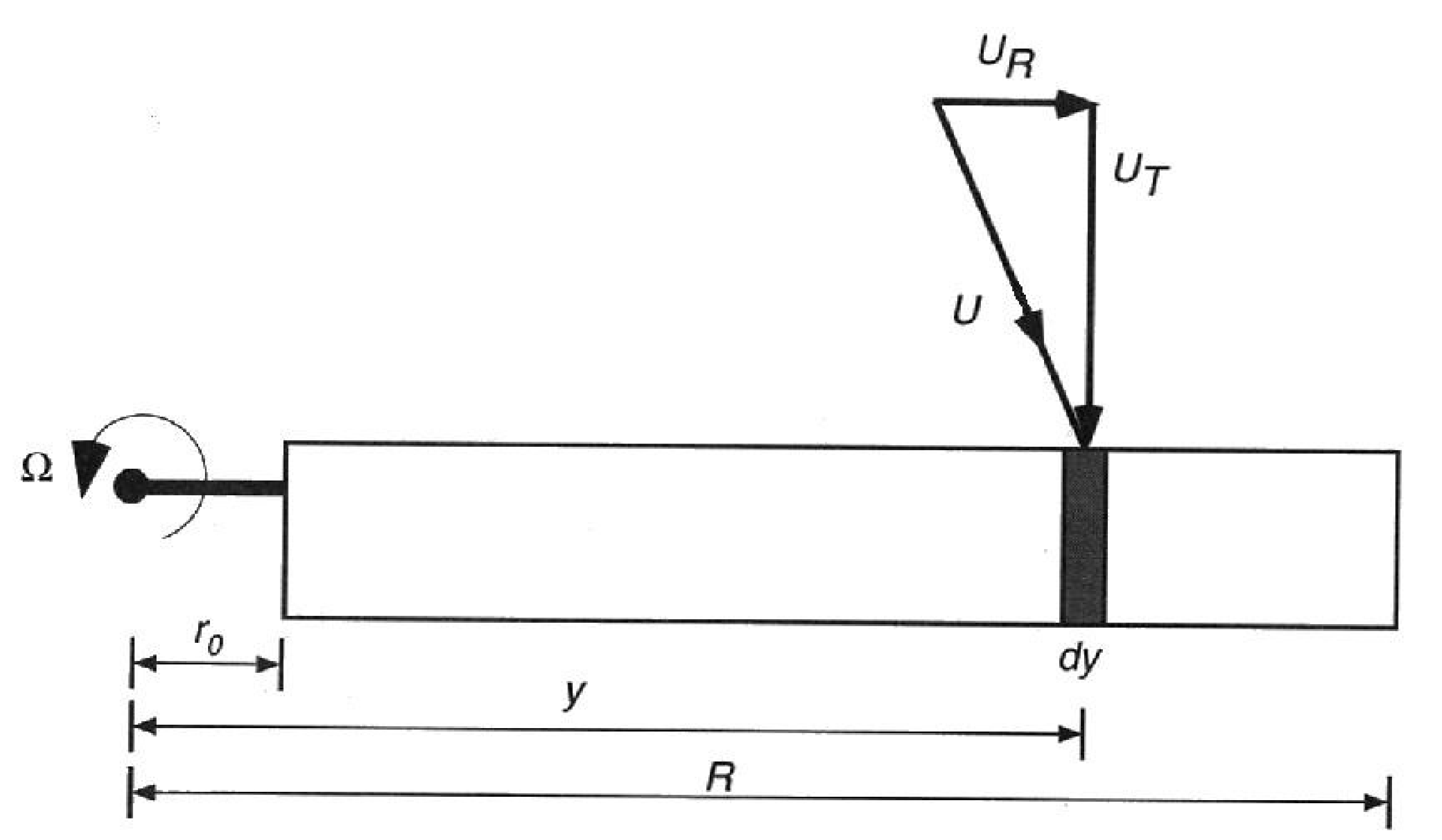
Blade Element Theory

- The momentum theory does not take into account
  - Number of blades
  - Airfoil characteristics (lift, drag, angle of zero lift)
  - Blade planform (taper, sweep, root cut-out)
  - Blade twist distribution
  - Compressibility effects

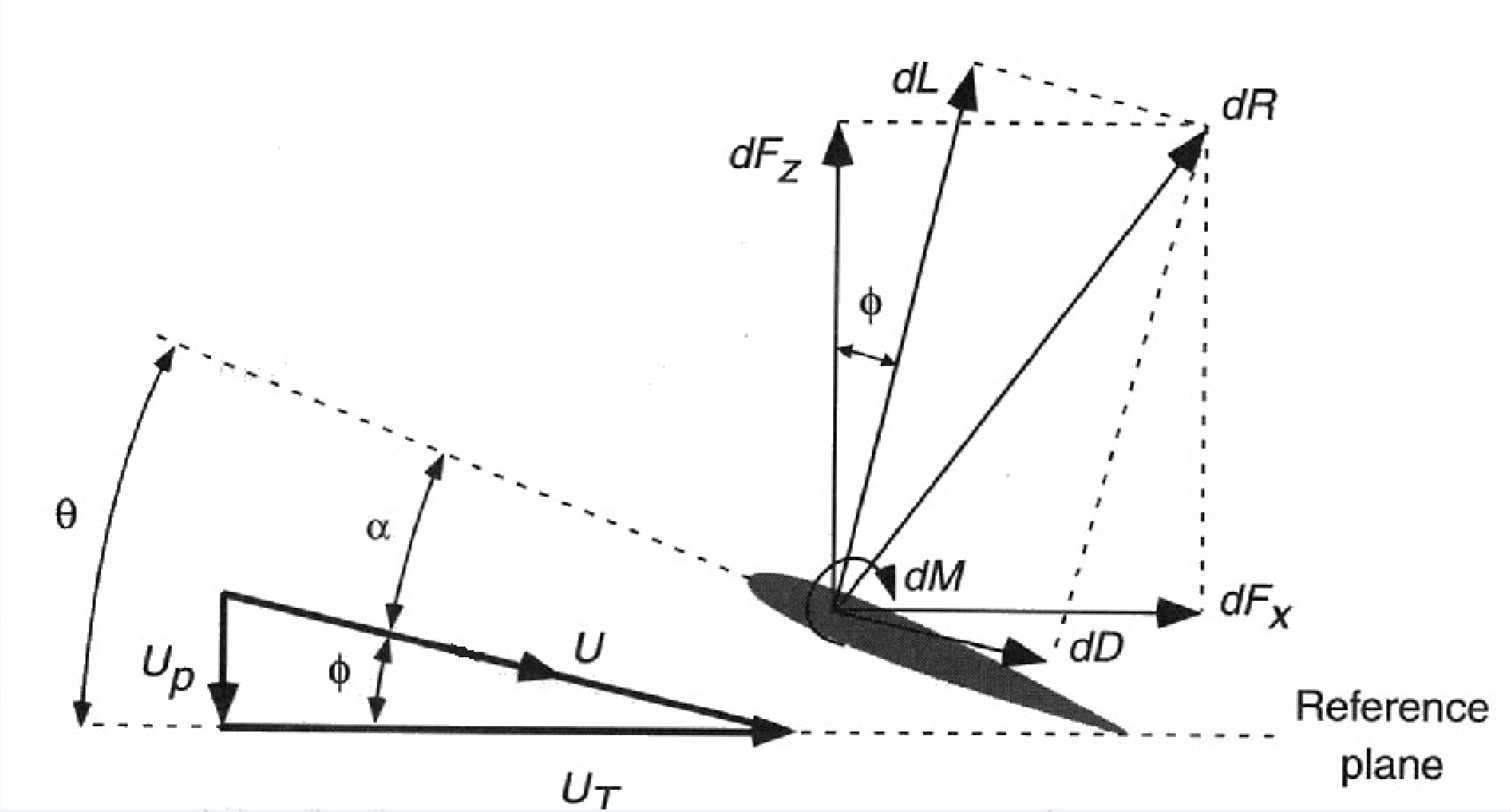
Blade Element Theory

- BET assumes that each blade section acts as a two-dimensional airfoil to produce aerodynamic forces
- The blade is then divided in non-interacting sections where all the computations are performed using 2-D aerodynamics
- An integration over the blade length gives the total thrust and total power

BET Model



BET Model



- The in plane Velocity $U_T = \Omega y$
- The out of plane Velocity $U_P = V_C + v_i$
- Therefore the total velocity is $U = \sqrt{U_T^2 + U_P^2}$

BET model

- The incremental lift per unit span:

$$dL = \tfrac{1}{2}\rho U^2 c C_l dy$$

- The incremental drag per unit span:

$$dD = \tfrac{1}{2}\rho U^2 c C_d dy$$

- Or in quantities parallel and perpendicular to the rotor disk plane:

$$\begin{cases} dF_z = dL\cos\phi - dD\sin\phi \\ dF_x = dL\sin\phi + dD\cos\phi \end{cases}$$

BET model

- We can then calculate the Thrust:

$$dT = N_b dF_z$$

- The Torque

$$dQ = N_b dF_x y$$

- The Power

$$dP = N_b dF_x \Omega y$$

- Remember N_b is the number of blades

BET model

- And we can relate all three with C_l and C_d

$$
\begin{cases}
dT = N_b\left(dL\cos\phi - dD\sin\phi\right) \\
dQ = N_b\left(dL\sin\phi + dD\cos\phi\right)y \\
dP = N_b\left(dL\sin\phi + dD\cos\phi\right)\Omega y
\end{cases}
$$

BET model assumptions

- The following assumptions are valid within the helicopter aerodynamics

$$U_T >> U_P \Rightarrow U = \sqrt{U_P^2 + U_T^2} \approx U_T$$

$$\phi \approx 0 \Rightarrow \begin{cases} \phi = \tan^{-1}\left(U_P/U_T\right) \approx U_P/U_T \\ \sin\phi = \phi \\ \cos\phi = 1 \end{cases}$$

$$dD << dL \Rightarrow dD\sin\phi \approx dD\phi \approx 0$$

Basic Equations

- The expression for Thrust, Torque and Power are:

$$\begin{cases} dT = N_b\left(dL\cos\phi - dD\sin\phi\right) = N_b\left(dL\right) \\ dQ = N_b\left(dL\sin\phi + dD\cos\phi\right)y = N_b\left(dL\phi + dD\right)y \\ dP = N_b\left(dL\sin\phi + dD\cos\phi\right)\Omega y = N_b\left(dL\phi + dD\right)\Omega y \end{cases}$$

- Let's now nondimensionalize using for length R and for speed $V_{tip} = \Omega R$

Nondimensional form

- $r=y/R$
- $U_T/\Omega R= \Omega y/ \Omega R=y/R=r$
- And the thrust, torque and power coefficients already defined:

$$dC_T=\frac{dT}{\rho A(\Omega R)^2},dC_Q=\frac{dQ}{\rho A(\Omega R)^2 R},dC_P=\frac{dP}{\rho A(\Omega R)^3}$$

- Now the inflow ratio is

$$\lambda=\frac{V_c+v_i}{\Omega R}=\frac{V_c+v_i}{\Omega y}\left(\frac{\Omega y}{\Omega R}\right)=\frac{U_P}{U_T}\left(\frac{y}{R}\right)=\phi r$$

Thrust coefficient (incremental)

- Substituting the previous equations in the Thrust coefficient equation:

$$dC_T = \frac{N_b dL}{\rho A(\Omega R)^2} = \frac{N_b\left(\frac{1}{2}\rho U_T{}^2 c C_l dy\right)}{\rho A(\Omega R)^2}$$

$$= \frac{1}{2}\left(\frac{N_b c}{\pi R}\right) C_l \left(\frac{y}{R}\right)^2 d\left(\frac{y}{R}\right) = \frac{1}{2}\sigma C_l r^2 dr$$

Power coefficient (incremental)

- Using the same analysis for the Power coefficient

$$dC_P = dC_Q = \frac{dQ}{\rho A(\Omega R)^2 R} = \frac{N_b(\phi dL + dD)y}{\rho A(\Omega R)^2 R}$$

$$= \frac{1}{2}\sigma(\phi C_l + C_D)\left(\frac{y}{R}\right)^3 d\left(\frac{y}{R}\right)$$

$$= \frac{1}{2}\sigma(\phi C_l + C_D)r^3 dr$$

Total Thrust and Power

- To find the total blade contribution for Thrust and power we have on integrate between the root and tip of the blade

$$C_T = \tfrac{1}{2}\int_0^1 \sigma C_l r^2 dr = \tfrac{1}{2}\sigma\int_0^1 C_l r^2 dr$$

- If the blade is rectangular *c=const*
- For the torque and power coefficient

$$C_Q = C_P = \tfrac{1}{2}\sigma\int_0^1 (\phi C_l + C_d) r^3 dr = \tfrac{1}{2}\sigma\int_0^1 \left(\lambda C_l r^2 + C_d r^3\right) dr$$

Total Thrust and Power

- To evaluate the previous expressions, we need:
- Inflow ratio $\lambda = \lambda(r)$
- Lift coefficient $C_l = C_l(\alpha, Re, M)$
- Drag coefficient $C_d = C_d(\alpha, Re, M)$
- AOA $\alpha = \alpha(V_C, \theta, v_i)$
- Induced Velocity $v_i = v_i(r)$

Numerical Solution needed

Approximations

- With certain assumptions and approximations it is possible to find closed form analytical solutions.
- The solutions are important because they serve to illustrate the fundamental form of the results in term of operational and geometric parameters of the rotor
- Let's the assume a rectangular blade $c=const.$ From the definition $\sigma=const.$ too.

Thrust approximation

- From the Steady linearized aerodynamics:

$$C_l = C_{l_\alpha}\left(\alpha-\alpha_0\right) = C_{l_\alpha}\left(\theta-\phi-\alpha_0\right)$$

- We can consider C_{l_α} constant without serious loss of accuracy
- Let's also assume symmetric airfoils $\alpha_0=0$
- We can then write:

$$C_T = \frac{1}{2}\int_0^1 \sigma C_l r^2 dr = \frac{1}{2}\sigma C_{l\alpha}\int_0^1 (\theta-\phi) r^2 dr$$

$$C_T = \frac{1}{2}\sigma C_{l\alpha}\int_0^1 \left(\theta r^2 - \lambda r\right) dr$$

Untwisted Blades

- For a blade with zero twist $\theta = const. = \theta_0$.
- Let's also assume uniform inflow velocity, as assumed in the momentum theory $\lambda = const.$
- The Thrust coefficient can be written as:

$$C_T = \tfrac{1}{2}\sigma C_{l\alpha}\int_0^1 \left(\theta_0 r^2 - \lambda r\right)dr = \tfrac{1}{2}\sigma C_{l\alpha}\left[\theta_0 \tfrac{r^3}{3} - \lambda \tfrac{r^2}{2}\right]_0^1$$

$$C_T = \tfrac{1}{2}\sigma C_{l\alpha}\left[\frac{\theta_0}{3} - \frac{\lambda}{2}\right]$$

Power approximations

- We have seen that the incremental power coefficient (that is equal to the torque coefficient):

$$dC_P = \tfrac{1}{2}\sigma\left(\phi C_l + C_d\right)r^3 dr = \tfrac{1}{2}\sigma\left(\lambda C_l r^2 + C_d r^3\right)dr =$$

$$= \tfrac{1}{2}\sigma\lambda C_l r^2 dr + \tfrac{1}{2}\sigma C_d r^3 dr =$$

$$= dC_{P_i} + dC_{P_0}$$

- Remembering that

$$dC_{P_i} = \lambda dC_T \Rightarrow \qquad dC_P = \lambda dC_T + dC_{P_0}$$

Power approximations

- Therefore the total power:

$$C_P = \int_{r=0}^{r=1} \lambda dC_T + \int_0^1 \tfrac{1}{2}\sigma C_d r^3 dr = \lambda C_T + \tfrac{1}{8}\sigma C_{d_0}$$

- Assuming uniform inflow and $C_d = C_{d0} = const.$
- Using once more the inflow expression obtained in hover:

$$C_P = \frac{C_T^{3/2}}{\sqrt{2}} + \tfrac{1}{8}\sigma C_{d_0}$$

- Expression already obtained in the momentum theory

Example

- Given the inflow distribution over a rotor with rectangular untwisted blades is approximately triangular, and assuming no tip-loss
  - Compute the variation with radius of inflow angle and section angle of attack for such rotor.
  - Derive the relation between the blade element lift coefficient and C_T/σ
  - Derive an expression for the hovering power of such a rotor in terms of C_T/σ and C_{d0}