

Circuit Theory and Electronics Fundamentals

MEAer (Integrated Master In Aerospace Engineering), Técnico, University of Lisbon

Laboratory 5: OP-AMP bandpass filter

Group 3

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Contents

| | | |
|----------|---|----------|
| 1 | Introduction | 2 |
| 2 | Theoretical Analysis | 3 |
| 2.1 | Circuit frequency response | 3 |
| 2.2 | Central frequency results | 4 |
| 3 | Simulation Analysis | 5 |
| 3.1 | Frequency response and impedances | 5 |
| 3.2 | Final product and merit | 6 |
| 4 | Conclusion | 9 |

1 Introduction

The objective of this laboratory assignment is to build a BandPass Filter (BPF) circuit whose specifications are: a central frequency at 1000Hz and a gain at central frequency of 40dB. In order to achieve this, we used a selected number of components whose properties were predefined. Our circuit is comprised of a $\mu A741$ OPAMP connected to two resistors, creating a non-inverting amplifier, and a combination of resistors and capacitors to take care of the filtering. These components are arranged according to figure 1.

The signal goes through a first stage where we have a capacitor whose function is to block unwanted DC current and to filter out lower frequencies, hence this stage being a high pass filter, since the rest of the circuit is connected to the terminals of the resistor. Now the signal goes through a combination of resistors and a $\mu 741$ OPAMP. This arrangement creates a non-inverting amplifier. In this configuration, the output signal is "in-phase" with the input signal. Feedback control of the non-inverting OPAMP is achieved by applying a small part of the output voltage back to the inverting (-) terminal via R3-R4 voltage divider network. And finally, the signal is subjected to a voltage divider network comprised of resistor R2 and capacitor C2. The desired voltage corresponds to the voltage drop at the terminals of the capacitor which, as a result, is subjected to a low pass filtering. For this to run, we need two supply DC voltage sources overlapped with our main circuit in order to power the transistors inside the OPAMP.

For this laboratory assignment there is a figure of merit, depending on the results obtained in the NGSpice simulation. This figure takes into account the cost of the components used, and the results they provide - a desirably low gain deviation and low central frequency deviation. The figure is calculated using the formula given by equation 1. The cost englobes the cost of transistors (0.1 units per transistor), diodes (0.1 units per diode), resistors (1 unit per KOhm) and capacitors (1 unit per uF). The objective is to achieve the highest merit, so we tried different configurations of components data until we achieved our greatest figure of merit.

$$M = \frac{1}{Cost * (GainDeviation + CentralFrequencyDeviation + 10^{-6})} \quad (1)$$

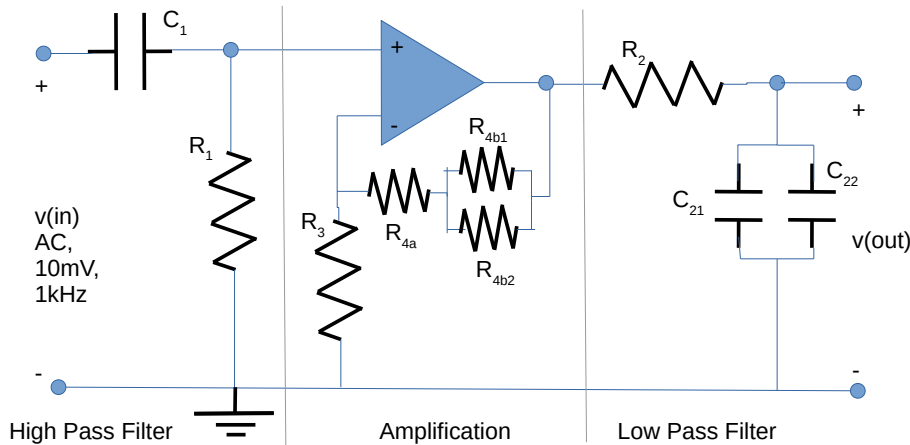


Figure 1: Geometry of our Band-Pass filter circuit.

To analyse this circuit theoretically we computed the transfer function for each stage in order to obtain the overall transfer function. Using the incremental model, we determined the input and output impedances of the overall circuit. That being said, in Section 2 we present the theoretical models and calculations used to determine the transfer function and, therefore, the

frequency response of the circuit, in Section 3 we introduce the results obtained in the simulation. Finally, in Section 4 we compare the two sets of results, looking for possible discrepancies and we lay out our conclusions. In Table 1, we list the numeric values of the components used:

| Name | Values |
|-----------|-----------------|
| R_1 | 1.000000 kOhm |
| C_1 | 0.220000 uFarad |
| R_2 | 1.000000 kOhm |
| C_{21} | 0.220000 uFarad |
| C_{22} | 0.220000 uFarad |
| R_3 | 1.000000 kOhm |
| R_{4a} | 100.000000 kOhm |
| R_{4b1} | 100.000000 kOhm |
| R_{4b2} | 100.000000 kOhm |

Table 1: Values of components used in our analysis and simulation.

2 Theoretical Analysis

2.1 Circuit frequency response

With the following equations we determined both cutoff frequencies for the band pass circuit (the lower cut-off frequency appears from the high-pass stage, and the upper cut-off frequency appears from the low-pass stage):

$$w_L = \frac{1}{R_1 C_1} \quad (2)$$

$$w_H = \frac{1}{R_2 C_2} \quad (3)$$

This was the definition used to determine the central frequency, which is meant to be 1 kHz.

$$w_O = \sqrt{w_H w_L} \quad (4)$$

| Name | Values |
|-------------------------|----------------|
| Lower Cut-Off Frequency | 723.431560 Hz |
| Upper Cut-Off Frequency | 1446.863119 Hz |
| Central Frequency | 1023.086723 Hz |

Table 2: Cut off frequencies and central frequency.

This central frequency result represents a 2.309% relative error, which is a relatively good result (given the components we could choose from), that we hope to replicate in the simulation.

As wanted, the plot has the trait of a narrower band-pass filter, with its top gain arriving in the 1 kHz neighbourhood, and the gain itself is on the 40 dB as pretended.

The phase drops from 90 degrees to -90 degrees, which is cause of the 2 poles of the transfer function (which we will present in the next subsection), one pole introduced by the high-pass and the other pole introduced by the low pass. Each pole causes a 90 degree drop, 45° the decade before and 45° the decade after. At the central frequency, the phase is zero, which means the output voltage is in phase with the input voltage.

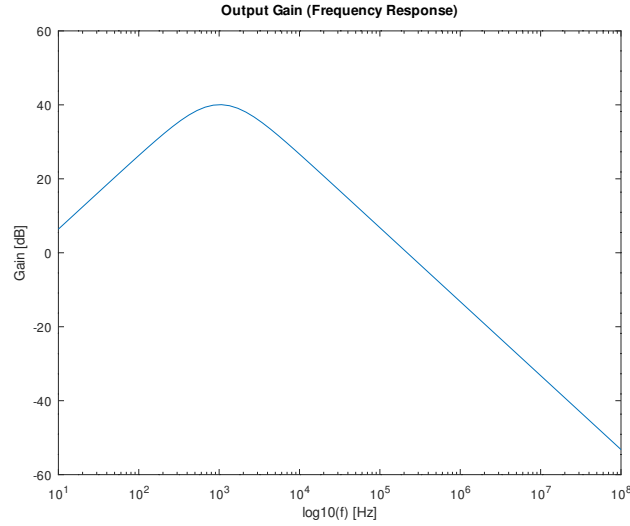


Figure 2: Voltage gain frequency response.

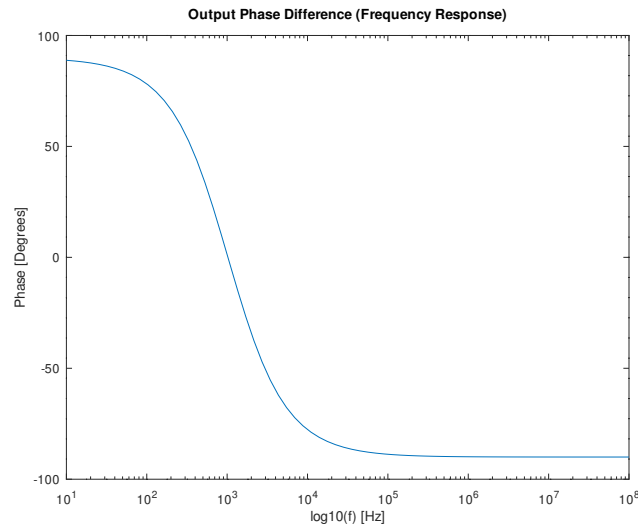


Figure 3: Voltage phase frequency response.

2.2 Central frequency results

For a frequency of 1 KHz we computed the circuit gain and input and output impedances for the amplifier. Also, we determined a theoretical figure of merit based on the predicted results with the set of values we chose.

The gain value for the entire circuit is obtained by multiplying the individual gains of each of the three stages of the circuit. This is an ideal situation, where we don't take into account the loss of signal between stages, or charge effects between the same stages. These are the equations for the gain values:

$$HighPassGain = \left| \frac{R_1 C_1 j\omega_O}{1 + R_1 C_1 j\omega_O} \right| \quad (5)$$

$$AmplifierGain = 1 + \frac{R_4}{R_3} \quad (6)$$

$$LowPassGain = \left| \frac{1}{1 + R_2 C_2 j\omega_O} \right| \quad (7)$$

Using the incremental model for the circuit (the input impedance of the OP-AMP is infinite and the output impedance of the OP-AMP is null, according to the model studied in class), we determined the following equations for the input and output impedances of the circuit:

$$Z_i = R_1 + \frac{1}{j\omega_O C_1} \quad (8)$$

$$Z_o = \frac{R_2}{1 + j\omega_O R_2 C_2} \quad (9)$$

| Name | Values |
|--|------------------------------|
| Gain (1 KHz) | 100.643363 |
| Gain (1 KHz)(dB) | 40.055703 dB |
| Gain (calculated central frequency) (dB) | 40.057714 dB |
| Z_{in} | 1000.000000 -723.431560j Ohm |
| Z_{in} modulus | 1234.241962 Ohm |
| Z_{in} phase | -35.883164 Degrees |
| Z_{out} | 676.732451 -467.723894j Ohm |
| Z_{out} modulus | 822.637497 Ohm |
| Z_{out} phase | -34.650304 Degrees |
| Cost | 13626.952040MU |
| Merit | 3.092445×10^{-6} |

Table 3: Gain, input and output impedances at the central frequency.

This way we can see that the gain for a frequency of 1KHz is almost the same as the gain for the calculated central frequency, meaning the 1KHz frequency is still very much within the band pass region. We obtained then a relative error of 0.643% for the gain at central frequency, which is meant to be 100, so this a fantastic theoretical result. The input impedance value is quite high, which is good, so, depending on the resistance of the input, most of the input voltage will flow through ahead to the OP-AMP as pretended. The output impedance value though is quite large, so this circuit will not be suited to loads with very low resistance values, but given the components we had available there wasn't much room for improvement. Also, we don't know the load that would be linked to this circuit, so we can't fully know how good or bad this value is, we just know it is clearly not the most desirable.

3 Simulation Analysis

Since the input voltage source in this circuit is sinusoidal, the voltage and current values of the various components vary in time, and we are interested therefore in analysing how they evolve in time and obviously we want to picture the transformation of our AC input voltage source from the input to the output of the circuit. We will run a transient analysis which will help us measuring the input and output impedances. We will also run a frequency response analysis in order to determine the gain and central frequency of our output amplified sinusoidal signal.

3.1 Frequency response and impedances

We measured the input impedance of the circuit, seen through the perspective of the source, and the output impedance, seen through the perspective of our output (using a dummy test source). With the frequency response, we measured the voltage gain in our output and the lower and upper cut-off frequencies. Using the same equation from the theoretical analysis

we determined the central frequency, then extracting the output gain for such frequency. In table 4, we present the results of our calculations. In figures 4 and 5 we can see the frequency response for our output's gain and phase, respectively. With these figures we can notice a slightly narrow band-pass filter, as expected and wanted. Regarding the phase plot though, we notice a full circle phase drop until it reaches the 90 degrees back again, whereas in the theoretical analysis the phase drops from 90 degrees to -90 degrees, stabilizing there. This difference is caused by the approximation used to study the OP-AMP behaviour in the theoretical section. For study purposes, we considered the OP-AMP did not introduce any phase difference in its output compared to its input, which is not true in reality. Due to the two transistors used in the uA741 OP-AMP, the transfer function in this sector actually presents 2 poles, which causes a double phase drop of 90 degrees (45 degrees the decade before and 45 degrees the decade after) each, which then means the phase actually drops 180 degrees due to the OP-AMP, hence stabilizing not in -90 degrees, but in 90 (-270) degrees.

| Name | Values |
|--------------------------|---|
| Gain | 99.7361 |
| Gain(dB) | 39.977 |
| Lower cut-off frequency | 403.611 Hz |
| Upper cut-off frequency | 2386.17 Hz |
| Bandwidth | 1982.56 Hz |
| Central frequency | 981.37 Hz |
| Input impedance | $0.999979 + j^*-0.723583 \text{ k}\Omega$ |
| Input impedance modulus | 1.23431 k Ω |
| Input impedance phase | -35.8894 degrees |
| Output impedance | $0.68172 + j^*-0.46675 \text{ k}\Omega$ |
| Output impedance modulus | 0.826194 k Ω |
| Output impedance phase | -34.3981 degrees |

Table 4: Output gain, center frequency, and input and output impedances of the OP-AMP band-pass filter.

Our input impedance value is reasonably high, so, depending on the input signal's inner resistance, it does allow the great majority of the input voltage to flow through ahead to the OP-AMP. That being said, the output impedance value is also considerably high, which in this case is not desirable. This means that for loads with low resistance values, most of the voltage will be consumed by the circuit's output impedance itself, undoing its very purpose. This band-pass filter is then better suited for loads with greater resistance values. Given the components we had at our disposal, it was very difficult to accomplish a lower value for the output impedance, specially without compromising the most important goal of this laboratory assignment. Looking now to the central frequency obtained, and the gain for such frequency, we had very slight deviations from the targeted values - 1 kHz and 40 dB. That being said, we obtained a 1.863% relative error for the central frequency value, and a 0.264% error for the output voltage gain, which we consider to be fantastic results, when we take into account the limited components available, which made optimization more difficult than usual.

3.2 Final product and merit

In figure 6, we can compare the output signal to the input. As normal, the output voltage does not possess a DC component. Since the wanted gain was 40 dB, which is the same as $100\times$ the initial amplitude, we can see the clear evolution from the starting 10mV to the final 1V of amplitude (more or less). For the first few milliseconds there is some small variation in the output sinusoidal wave due to a small transient regime. In table 5, we present the cost and

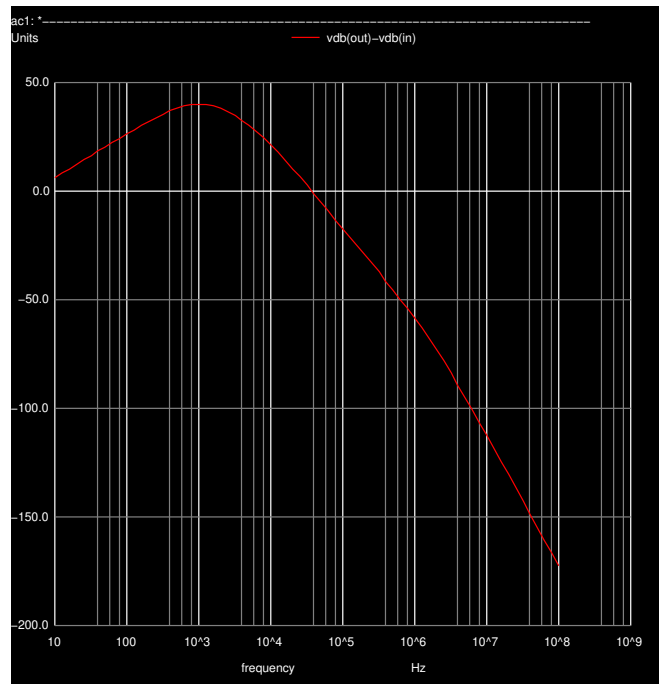


Figure 4: Output voltage gain (frequency response).

merit of our circuit. As we've already discussed, we obtained a central frequency and gain very close to the targeted ones, which is expressed by very low relative errors, and therefore very small deviations. The overall cost of the circuit is considerably high, but that is mainly due to the high cost of the OP-AMP subcircuit itself. Because of this, using higher cost components in the rest of the circuit became more tolerable, since it wouldn't create as much of an effect in the overall cost value. Because of this we used the three 100 kOhm resistors available in the amplification stage, which allowed us to get an almost perfect gain for the central frequency, without harming the merit figure. The merit figure itself is clearly a very low number, but we believe that is only because of the restraints of the merit formula itself.

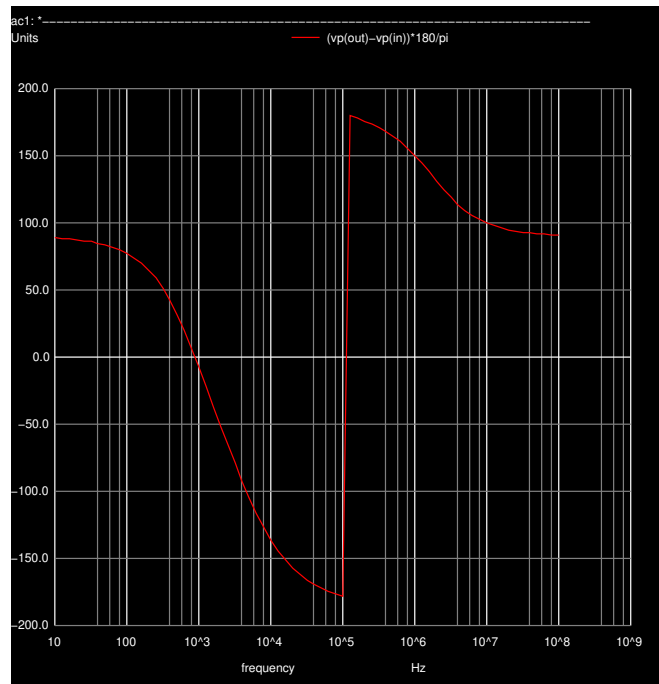


Figure 5: Output voltage phase difference (frequency response).

| Name | Values |
|-------|--------------|
| cost | 1.362695e+04 |
| merit | 3.883972e-06 |

Table 5: Cost and merit of the OP-AMP band-pass filter.

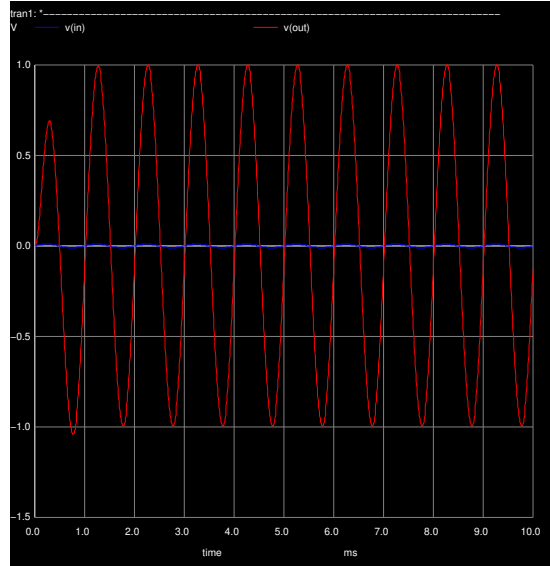


Figure 6: Comparison between the input and the output sinusoidal signals.

4 Conclusion

| Theoretical | Value | Simulation | Value |
|-----------------------------------|---------------------------|--------------------|------------------|
| Frequency response and impedances | | | |
| <i>Gain</i> | 100.643363 | Gain | 99.7361 |
| <i>Gain(dB)</i> | 40.055703 dB | Gain(dB) | 39.977 dB |
| <i>LowerCut – offFreq</i> | 723.431560 Hz | Lower cut-off freq | 403.611 Hz |
| <i>UpperCut – offFreq</i> | 1446.863119 Hz | Upper cut-off freq | 2386.17 Hz |
| <i>CentralFreq</i> | 1023.086723 Hz | Central freq | 981.37 Hz |
| <i>Z_{in}Modulus</i> | 1234.241962 Ohm | Zin modulus | 1.23431 kOhm |
| <i>Z_{in}Phase</i> | -35.883164 Degrees | Zin phase | -35.8894 Degrees |
| <i>Z_{out}Modulus</i> | 822.637497 Ohm | Zout modulus | 0.826194 kOhm |
| <i>Z_{out}Phase</i> | -34.650304 Degrees | Zout phase | -34.3981 Degrees |
| <i>Cost</i> | 13626.952040 | Cost | 1.362695e+04 |
| <i>Merit</i> | 3.092445*10 ⁻⁶ | Merit | 3.883972e-06 |

Table 6: Comparison of the theoretical and simulated data results, regarding the frequency response and impedances.

In this laboratory assignment, we managed to build a BandPass Filter (BPF) circuit which is represented in Figure 1. The first step of our analysis was to determine the frequency response by computing the transfer function of the whole circuit.

In the last report we explained how the theoretical model of the transistor lacked the complexity needed to yield closer results to the simulation. In this assignment we are dealing with

22 separate transistors inside the OP-AMP. This alone means that theoretical model used to analyse the circuit can differ significantly since it isn't expected to take into account the non-linearity of transistors. Adding to that is the complexity of the OP-AMP model used in Ngspice, especially the use of various capacitors and diodes which weren't taken into account in the octave analysis.

Due to this, the theoretical and simulation results differ significantly in some aspects. The bandwidth, despite still being approximately centered around the 1 KHz frequency, was much bigger (double the width).

Despite all of this, the merit figures are somewhat similar, especially if we consider the significant differences from previous assignments.