Circuit Theory and Electronics Fundamentals

MEAer (Integrated Master In Aerospace Engineering), Técnico, University of Lisbon

Laboratory 2: RC Circuit analysis

Group 3

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Contents

1 Introduction

The objective of this laboratory assignment is to study the behaviour of an RC circuit, as seen in Figure 1. In order to analyse it, we will make use of different techniques such as Thevenin and Phasor analysis in order to establish the natural, forced and total solutions of the circuit. A frequency response analysis then follows, where we look at the magnitude and phase as a function of frequency. Then, we compare the theoretical and simulation data.

In Section 2, we go through the various steps that are needed to obtain the total solution of the circuit (natural plus forced responses) and present the theoretical frequency response analysis. In Section 3, we present the circuit response as simulated in NGSpice. The results in operating point and transient analysis are compared to the theoretical results obtained in Section 2. The conclusions of this study are laid out in Section 4.

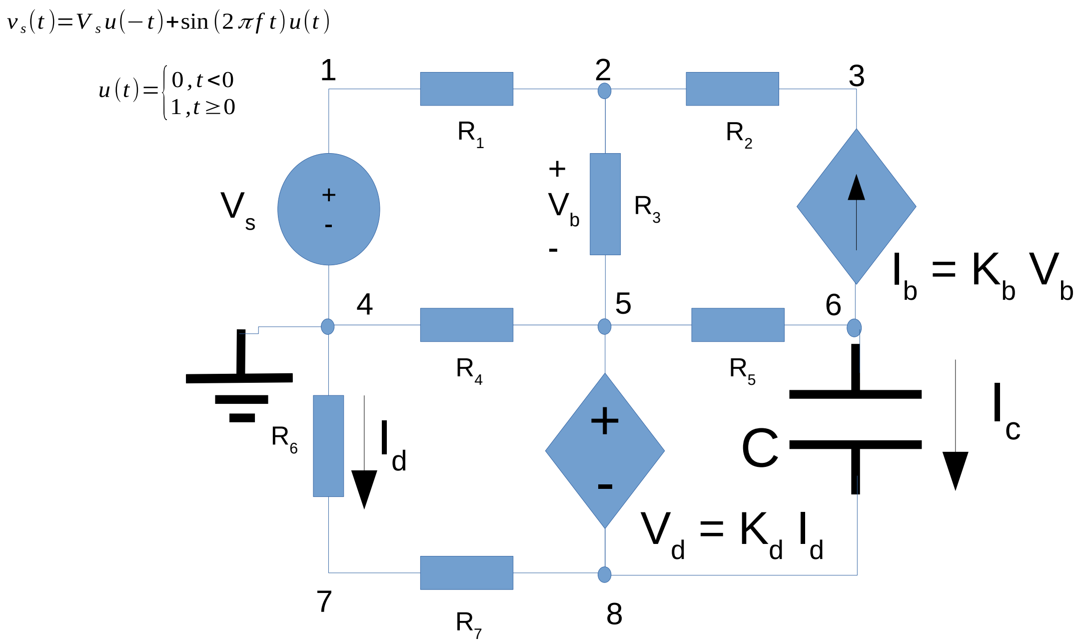

Figure 1: RC circuit.

2 Theoretical Analysis

We analyse the circuit shown in Figure 1 for t¡0 using the nodal method. The nodes are numbered according to what's shown in the picture. In this first instance we are working in a steady state, where no current is flowing through the capacitor: we can replace it with an open circuit. After doing this, it is clear that all the components we are working with are linear and so we will need to solve a system of linear equations to determine the initial values for the subsequent analysis. This way, we've ran the nodal method and solved the linear system on GNU Octave.

2.1 Node Analysis for t<0

Knowing that V4=0 since it is connected to the ground:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & -G3 & 0 & 0 & 0 \\ 0 & Kb+G2 & -G2 & -Kb & 0 & 0 & 0 \\ -G1 & G1 & 0 & G4 & 0 & G6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G6-G7 & G7 \\ 0 & 0 & 0 & 1 & 0 & G6*Kd & -1 \\ 0 & -G3 & 0 & G3+G4+G5 & -G5 & G6 & 0 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} Vs \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

We can also easily obtain the current values in the various branches through Ohm's law. This yields the following results:

Name	Value [mA and V]
@I_{R1}	0.219167
@I_{R2}	0.229771
@I_{R3}	0.010604
@I_{R4}	1.185502
@I_{R5}	0.229771
@I_{R6}	0.966335
@I_{R7}	0.966335
@I_b	-0.229771
@I_C	0.000000
@I_{Vs}	0.219167
@I_{Kd}	0.966335
V_1	5.113399
V_2	4.889447
V_3	4.428712
V_4	0.000000
V_5	4.921444
V_6	5.620907
V_7	-1.953900
V_8	-2.927814

Table 1: Theoretical analysis results for time, t, inferior to 0 . (A variable preceded by @ is of type *current*)

2.2 Equivalent Resistor

In order to compute the equivalent resistor we ran a nodal analysis, making Vs=0 and replacing the capacitor with a voltage source Vx=V6-V8 as calculated in the previous step. This is made to ensure that the voltage in the capacitor is continuous since it does so in reality: this is a capacitor discharging through a resistance, any discontinuity in voltage would require an infinite amount of current.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & -G3 & 0 & 0 & 0 \\ 0 & Kb+G2 & -G2 & -Kb & 0 & 0 & 0 \\ -G1 & G1 & 0 & G4 & 0 & G6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G6-G7 & G7 \\ 0 & 0 & 0 & 1 & 0 & G6*Kd & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Vx \end{pmatrix} \tag{2}$$

With the following definitions:

$$V_x = V_6 - V_8; \tag{3}$$

$$I_x = \frac{V_6 - V_5}{R_5} + \frac{V_3 - V_2}{R_2}; \tag{4}$$

$$Req = \frac{V_x}{I_x} \tag{5}$$

We were able to obtain the following values, laying the foundation for the natural solution:

Name	Value
V_x	8.548721 V
V_6	8.548721 V
V_8	0.000000 V
@I_x	2.808224 mA
R_{eq}	3.044174 kOhm
tau	3.059522 ms

Table 2: Theoretical analysis calculations for R_{eq} and tau.

2.3 Natural solution

The general solution for RC circuits, making use of the computations in the previous section, yields:

$$V_{6n}(t) = V_x exp(-\frac{t}{RC}); \tag{6}$$

From this equation, we obtained the following plot, which depicts the first twenty milisseconds of the circuit's natural response.

Name	Value [V]
V_1	0.000000
V_2	0.000000
V_3	-0.000000
V_4	0.000000
V_5	0.000000
V_6	-0.000000
V_7	-0.000000
V_8	0.000000

Table 3: Node voltages for t=∞.

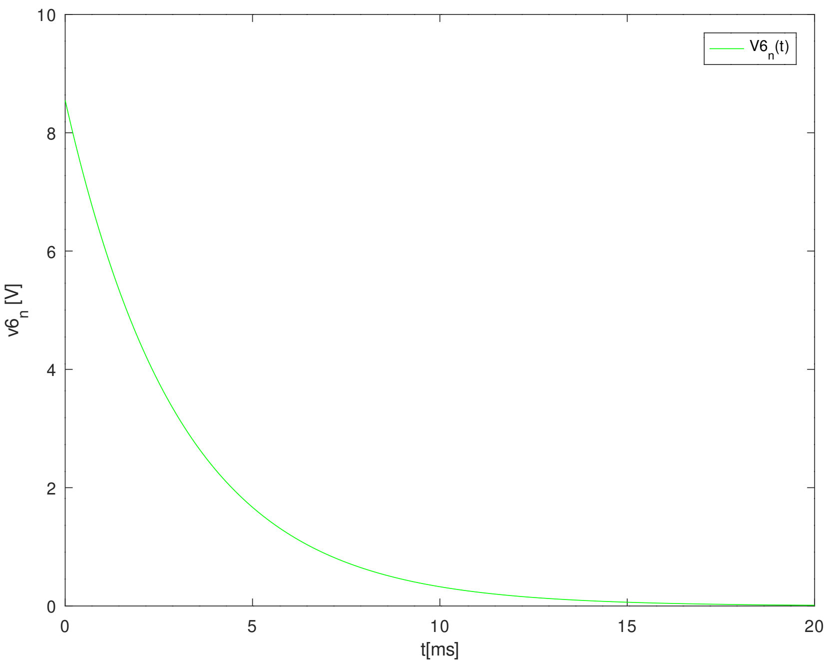

Figure 2: Natural solution to V_6 node voltage.

2.4 Forced and final total solution

In order to study the forced response of the system, we performed a nodal analysis to obtain the phasor voltage in every node by solving the following system:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & -G3 & 0 & 0 & 0 \\ 0 & Kb+G2 & -G2 & -Kb & 0 & 0 & 0 \\ -G1 & G1 & 0 & G4 & 0 & G6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G6-G7 & G7 \\ 0 & 0 & 0 & 1 & 0 & G6*Kd & -1 \\ 0 & -G3 & 0 & G3+G4+G5 & -G5-jwC & G6 & jwC \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} -j \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

We then get these values:

Name	Amplitude [V]	Phase [Degrees]
V_1	1.000000	-90.000000
V_2	0.956203	-90.000000
V_3	0.866099	-90.000000
V_4	0.000000	0.000000
V_5	0.962460	-90.000000
V_6	0.574648	98.680964
V_7	0.382114	90.000000
V_8	0.572577	90.000000

Table 4: Phasor voltage for forced regime in every node.

We're now able to make sense of the equation that describes the forced solution to the voltage at node 6.

$$V_{6f}(t) = V_{6r}cos(wt + V_{6\phi}) = -0.086733cos(2000\pi * t + 1.7223); \tag{8}$$

2.5 Final total solution

Now we convert the phasors to real time functions and consider an angular frequency of 2000π. By superimposing both natural and forced responses we get the total solution:

$$V_6(t) = V_{6f}(t) + V_{6n}(t); \tag{9}$$

By plotting both $V_s(t)$ and $V_6(t)$ from -5ms to 20ms we can see that both plots are constant before t=0. The evolution of V6 is as expected, as we can clearly see the negative exponential behaviour as well as the induced frequency.

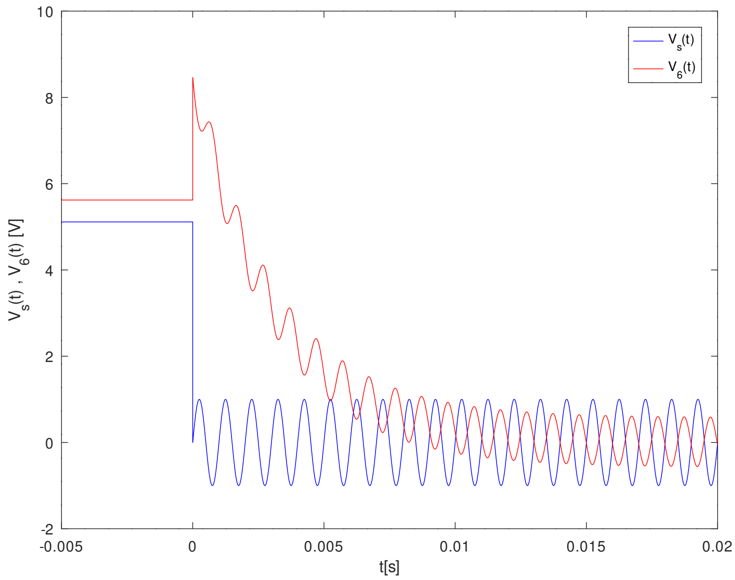

Figure 3: Total solution to V_6 node voltage, compared to V_s voltage source.

2.6 Frequency response

As we can see in the following plots, for low frequencies (<1Hz) every voltage is in phase. The capacitor is given enough time to charge up (asymptotically) to the same voltage as the voltage source, whose phase and magnitude remain constant throughout the analysed period. This means that at 0.1 Hz (period of 10 seconds), the capacitor charges and discharges in a fraction of a second and behaves like an open circuit for the rest of the cycle. We start seeing divergences in the graphs around the "cut-off frequency", defined as $\frac{1}{2\pi\tau}$, which in our case is around 52Hz, explaining the changes happening between 10 and 10²Hz. When we consider a time interval in the same order of magnitude of τ, the time it takes for a capacitor to deplete 36.8% of its charge through a resistance, we are no longer giving the capacitor enough time to charge or discharge (almost) all the way. For much higher frequencies than the cut off, the capacitor starts behaving like a short circuit since it can no longer oppose the change of current (its reactance drops). The apparent phase discontinuity is only due to the use of the arctan function, whose output is bounded by [-180,180] until it circles back around to the top. So the phase is in fact continuous. The following equations help describe the behaviour shown in the plots. They come from reducing the circuit to a sinusoidal voltage source, equivalent resistance and a capacitor.

$$V_c = \frac{V_s}{\sqrt{1 + ((R_{eq}C)2\pi * f)^2}} \tag{10}$$

$$\phi V_c = -\frac{\pi}{2} + arctan(R_eqC2\pi * f) \tag{11}$$

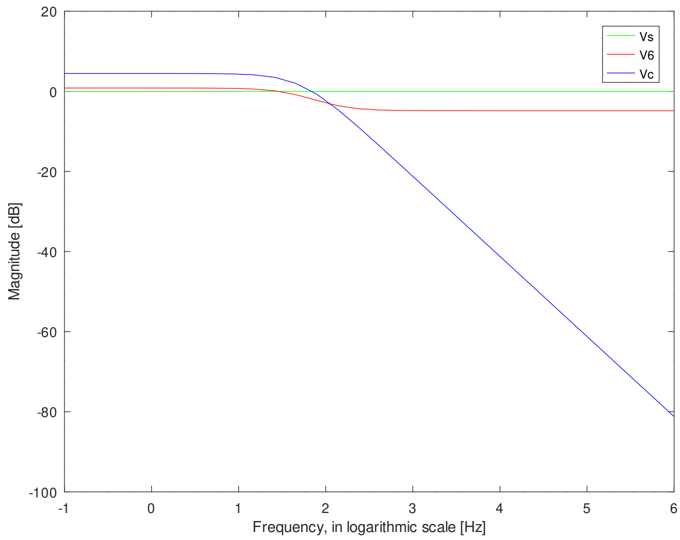

Figure 4: Variation with frequency of V_s, V_6 and V_c voltage magnitudes (in dB).

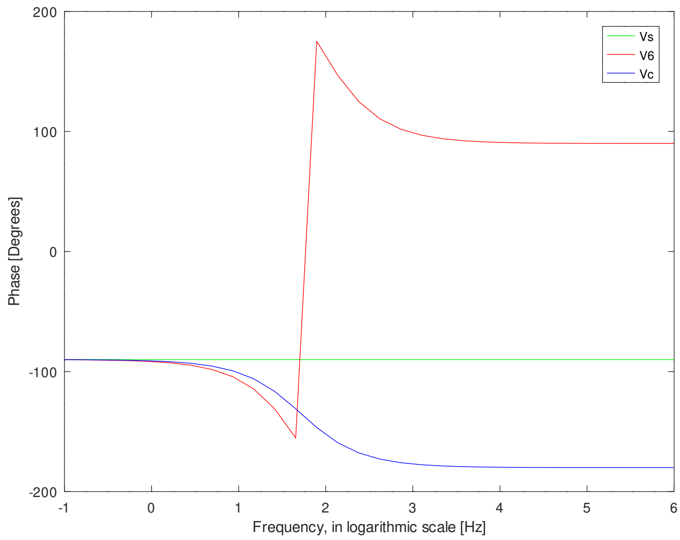

Figure 5: Variation with frequency of V_s, V_6 and V_c voltage phases.

3 Simulation Analysis

Since this circuit has a sinusoidal voltage source, the voltage and current values of the various components vary in time. Therefore, we must perform a transient analysis to simulate the circuit's total response. We also ran operating point analysis for both t¡0 and t=0 to determine what the initial conditions were and establish the boundary conditions.

3.1 Operating Point Analysis

The tables below show the simulated operating point results for both $t < 0$ (where we assume no current is flowing through the capacitor) and t=0 where we replace Vs for a short-circuit.

Name	Value [A or V]
@c[i]	0.000000e+00
@g[i]	-2.29771e-04
@r1[i]	2.191669e-04
@r2[i]	2.297712e-04
@r3[i]	1.060424e-05
@r4[i]	1.185502e-03
@r5[i]	2.297712e-04
@r6[i]	9.663348e-04
@r7[i]	9.663348e-04
v(1)	5.113399e+00
v(2)	4.889448e+00
v(3)	4.428712e+00
v(4)	-1.95390e+00
v(5)	4.921444e+00
v(6)	5.620908e+00
v(7)	-1.95390e+00
v(8)	-2.92781e+00

Table 5: Operating point analysis for $t < 0$. A variable preceded by @ is of type *current* other variables are of type *voltage* and expressed in Volt.

Name	Value [A or V]
@g[i]	-4.25263e-18
@r1[i]	4.056361e-18
@r2[i]	4.252625e-18
@r3[i]	1.962643e-19
@r4[i]	-8.55795e-19
@r5[i]	2.808224e-03
@r6[i]	-8.67362e-19
@r7[i]	-1.78493e-18
v(1)	0.000000e+00
v(2)	-4.14491e-15
v(3)	-1.26722e-14
v(4)	1.753779e-15
v(5)	-3.55271e-15
v(6)	8.548722e+00
v(7)	1.753779e-15
v(8)	3.552714e-15

Table 6: Operating point analysis for $t = 0$. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

4 Conclusion

In this laboratory assignment the objective of studying the presented circuit has been achieved. All the calculations for the Mesh and Node methods have been carried out using the Octave Maths tool and the circuit simulation has been done using the Ngspice tool. In a real, presential lab class, all sorts of experimental errors would be present: the resistance of the wiring, internal resistance of the sources, the temperature, external noise, etc. Besides that, reading and assembly errors would be expected. In this case, the simulation results matched the theoretical results: the reason for this match is the fact that this is a straightforward circuit containing only linear components, so if done correctly, the theoretical and simulation models cannot have noticeable differences. For more complex components, the theoretical and simulation models could differ greatly but this is not the case in this work. Despite the proximity of the values, there is still some difference between them (around the 6th decimal place). This can be considered negligible for the experiment in case, which doesn't require a higher degree of accuracy since the whole purpose is just to study a simple circuit (it could be a problem if this belonged to a bigger functional structure where every small discrepancy between the expected and real data mattered). Nevertheless, these differences may have been caused by the employment of different numerical methods for solving the linear system and for different rounding criteria employed by the two different tools.

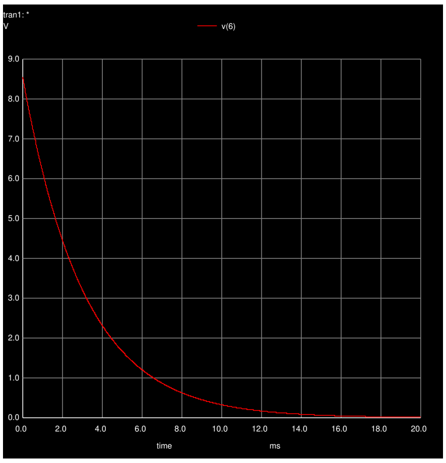

Figure 6: Simulated natural response for V_6 node voltage.

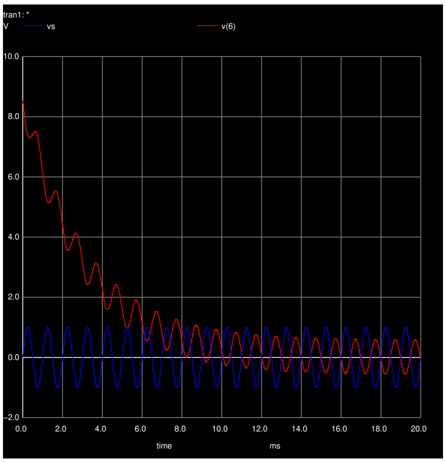

Figure 7: Simulated response for V_6 node voltage and stimulated voltage V_S.

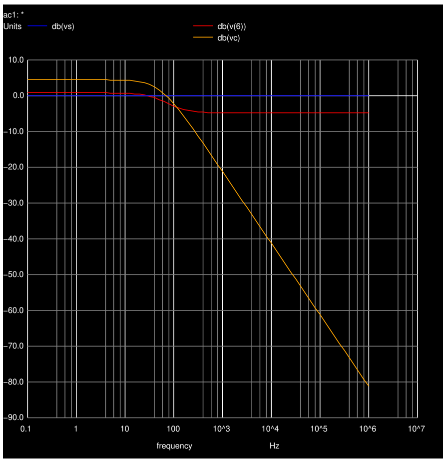

Figure 8: ...

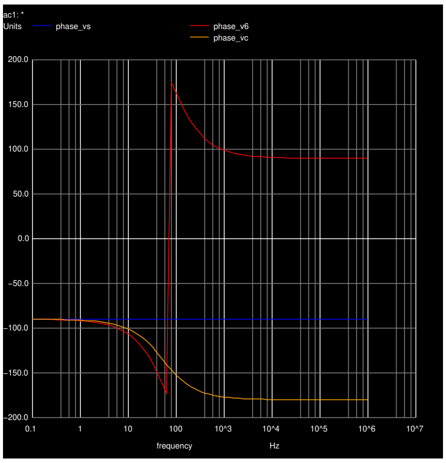

Figure 9: ...