Circuit Theory and Electronics Fundamentals

MEAer (Integrated Master In Aerospace Engineering), Técnico, University of Lisbon

Laboratory 2: RC Circuit analysis

Group 3

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1 Introduction

The objective of this laboratory assignment is to study the behaviour of an RC circuit, as seen in Figure 1. In order to analyse it, we will make use of different techniques such as Thevenin and Phasor analysis in order to establish the natural, forced and total solutions of the circuit. A frequency response analysis then follows, where we look at the magnitude and phase as a function of frequency. Then, we compare the theoretical and simulation data.

In Section 2, we go through the various steps that are needed to obtain the total solution of the circuit (natural plus forced responses) and present the theoretical frequency response analysis. In Section 3, we present the circuit response as simulated in NGSpice. The results in operating point and transient analysis are compared to the theoretical results obtained in Section 2. The conclusions of this study are laid out in Section 4.

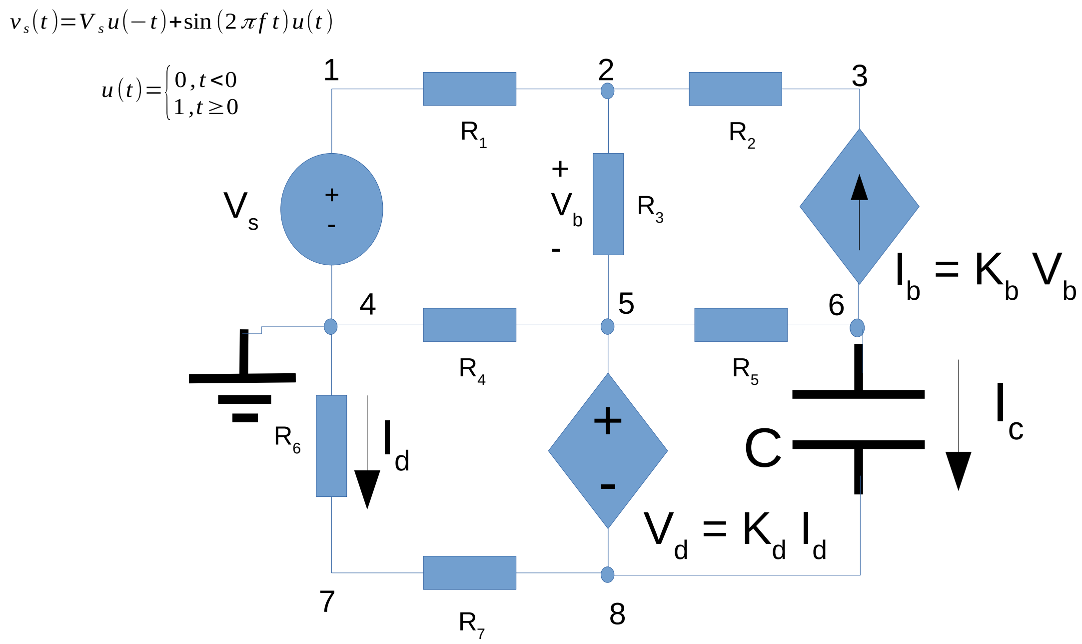

Figure 1: RC circuit.

2 Theoretical Analysis

We analyse the circuit shown in Figure 1 for t¡0 using the nodal method. The nodes are numbered according to what's shown in the picture. In this first instance we are working in a steady state, where no current is flowing through the capacitor: we can replace it with an open circuit. After doing this, it is clear that all the components we are working with are linear and so we will need to solve a system of linear equations to determine the initial values for the subsequent analysis. This way, we've ran the nodal method and solved the linear system on GNU Octave.

2.1 Node Analysis for t<0

Knowing that V4=0 since it is connected to the ground:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & -G3 & 0 & 0 & 0 \\ 0 & Kb+G2 & -G2 & -Kb & 0 & 0 & 0 \\ -G1 & G1 & 0 & G4 & 0 & G6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G6-G7 & G7 \\ 0 & 0 & 0 & 1 & 0 & G6*Kd & -1 \\ 0 & -G3 & 0 & G3+G4+G5 & -G5 & G6 & 0 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} Vs \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

We can also easily obtain the current values in the various branches through Ohm's law. This yields the following results:

Name	Value [mA and V]
@I_{R1}	0.260667
@I_{R2}	0.273260
@I_{R3}	0.012594
@I_{R4}	1.205334
@I_{R5}	0.273260
@I_{R6}	0.944667
@I_{R7}	0.944667
@I_{b}	-0.273260
@I_{C}	0.000000
@I_{Vs}	0.260667
@I_{Kd}	0.944667
V_1	5.077034
V_2	4.803907
V_3	4.231486
V_4	0.000000
V_5	4.841795
V_6	5.695822
V_7	-1.958859
V_8	-2.935159

Table 1: Theoretical analysis results for time, t, inferior to 0 . (A variable preceded by @ is of type *current*)

2.2 Equivalent Resistor

In order to compute the equivalent resistor we ran a nodal analysis, making Vs=0 and replacing the capacitor with a voltage source Vx=V6-V8 as calculated in the previous step. This is made to ensure that the voltage in the capacitor is continuous since it does so in reality: this is a capacitor discharging through a resistance, any discontinuity in voltage would require an infinite amount of current.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & -G3 & 0 & 0 & 0 \\ 0 & Kb+G2 & -G2 & -Kb & 0 & 0 & 0 \\ -G1 & G1 & 0 & G4 & 0 & G6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G6-G7 & G7 \\ 0 & 0 & 0 & 1 & 0 & G6*Kd & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Vx \end{pmatrix} \tag{2}$$

With the following definitions:

$$V_x = V_6 - V_8; \tag{3}$$

$$I_x = \frac{V_6 - V_5}{R_5} + \frac{V_3 - V_2}{R_2}; \tag{4}$$

$$Req = \frac{V_x}{I_x} \tag{5}$$

We were able to obtain the following values, laying the foundation for the natural solution:

Name	Value
V_x	8.630981 V
V_6	8.630981 V
V_8	0.000000 V
$@I_x$	2.761627 mA
R_{eq}	3.125325 kOhm
tau	3.220013 ms

Table 2: Theoretical analysis calculations for R_{eq} and tau.

2.3 Natural solution

The general solution for RC circuits, making use of the computations in the previous section, yields:

$$V_{6n}(t) = V_x exp(-\frac{t}{RC}); \tag{6}$$

From this equation, we obtained the following plot, which depicts the first twenty milisseconds of the circuit's natural response.

Name	Value [V]
V_1	0.000000
V_2	0.000000
V_3	-0.000000
V_4	0.000000
V_5	0.000000
V_6	-0.000000
V_7	-0.000000
V_8	0.000000

Table 3: Node voltages for t=∞.

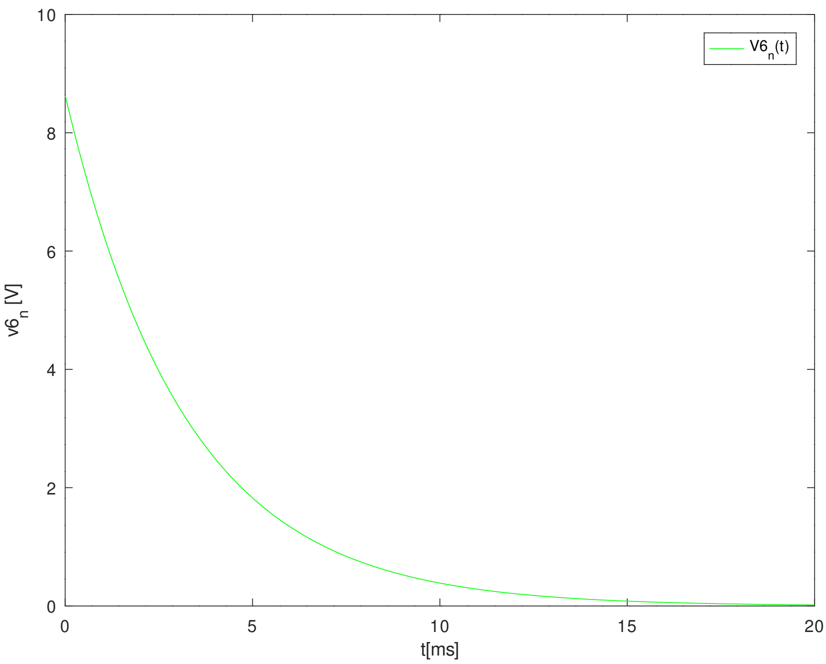

Figure 2: Natural solution to V_6 node voltage.

2.4 Forced and final total solution

In order to study the forced response of the system, we performed a nodal analysis to obtain the phasor voltage in every node by solving the following system:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & -G3 & 0 & 0 & 0 \\ 0 & Kb+G2 & -G2 & -Kb & 0 & 0 & 0 \\ -G1 & G1 & 0 & G4 & 0 & G6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G6-G7 & G7 \\ 0 & 0 & 0 & 1 & 0 & G6*Kd & -1 \\ 0 & -G3 & 0 & G3+G4+G5 & -G5-jwC & G6 & jwC \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V5 \\ V6 \\ V7 \\ V8 \end{pmatrix} = \begin{pmatrix} -j \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

We then get these values:

Name	Amplitude [V]	Phase [Degrees]
V_1	1.000000	-90.000000
V_2	0.946203	-90.000000
V_3	0.833456	-90.000000
V_4	0.000000	0.000000
V_5	0.953666	-90.000000
V_6	0.580070	98.308416
V_7	0.385827	90.000000
V_8	0.578125	90.000000

Table 4: Phasor voltage for forced regime in every node.

We're now able to make sense of the equation that describes the forced solution to the voltage at node 6.

$$V_{6f}(t) = V_{6r}cos(wt + V_{6\phi}) = -0.086733cos(2000\pi * t + 1.7223); \tag{8}$$

2.5 Final total solution

Now we convert the phasors to real time functions and consider an angular frequency of 2000π. By superimposing both natural and forced responses we get the total solution:

$$V_6(t) = V_{6f}(t) + V_{6n}(t); \tag{9}$$

By plotting both $V_s(t)$ and $V_6(t)$ from -5ms to 20ms we can see that both plots are constant before t=0. The evolution of V6 is as expected, as we can clearly see the negative exponential behaviour as well as the induced frequency.

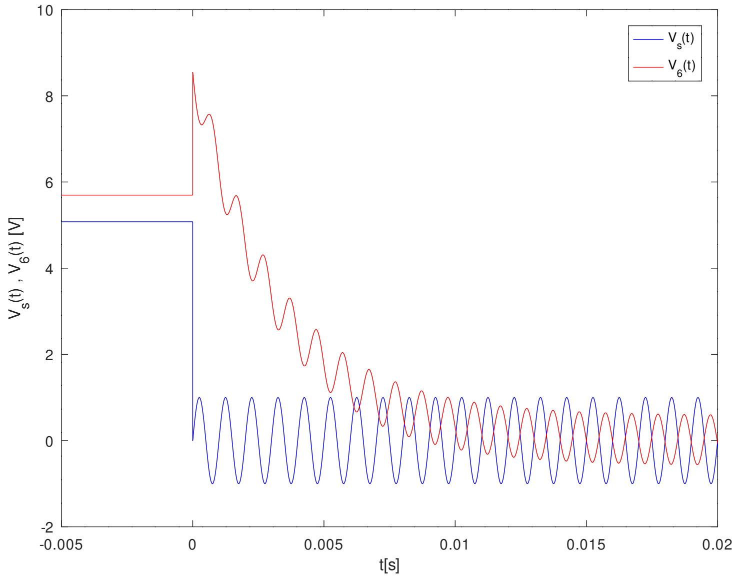

Figure 3: Total solution to V_6 node voltage, compared to V_s voltage source.

2.6 Frequency response

As we can see in the following plots, for low frequencies (<1Hz) every voltage is in phase. The capacitor is given enough time to charge up (asymptotically) to the same voltage as the voltage source, whose phase and magnitude remain constant throughout the analysed period. This means that at 0.1 Hz (period of 10 seconds), the capacitor charges and discharges in a fraction of a second and behaves like an open circuit for the rest of the cycle. We start seeing divergences in the graphs around the "cut-off frequency", defined as $\frac{1}{2\pi\tau}$, which in our case is around 52Hz, explaining the changes happening between 10 and 10^2Hz. When we consider a time interval in the same order of magnitude of τ, the time it takes for a capacitor to deplete 36.8% of its charge through a resistance, we are no longer giving the capacitor enough time to charge or discharge (almost) all the way. For much higher frequencies than the cut off, the capacitor starts behaving like a short circuit since it can no longer oppose the change of current (its reactance drops). The apparent phase discontinuity is only due to the use of the arctan function, whose output is bounded by [-180,180] until it circles back around to the top. So the phase is in fact continuous. The following equations help describe the behaviour shown in the plots. They come from reducing the circuit to a sinusoidal voltage source, equivalent resistance and a capacitor.

$$V_c = \frac{V_s}{\sqrt{1 + ((R_{eq}C)2\pi * f)^2}} \tag{10}$$

$$\phi V_c = -\frac{\pi}{2} + arctan(R_eqC2\pi * f) \tag{11}$$

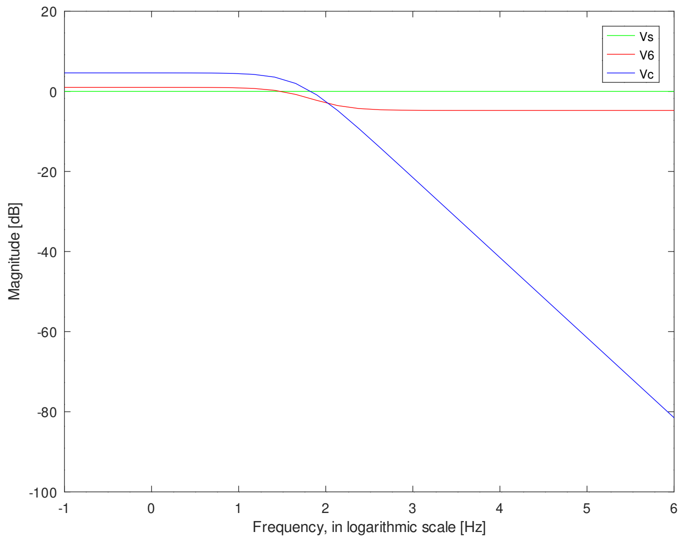

Figure 4: Variation with frequency of V_s, V_6 and V_c voltage magnitudes (in dB).

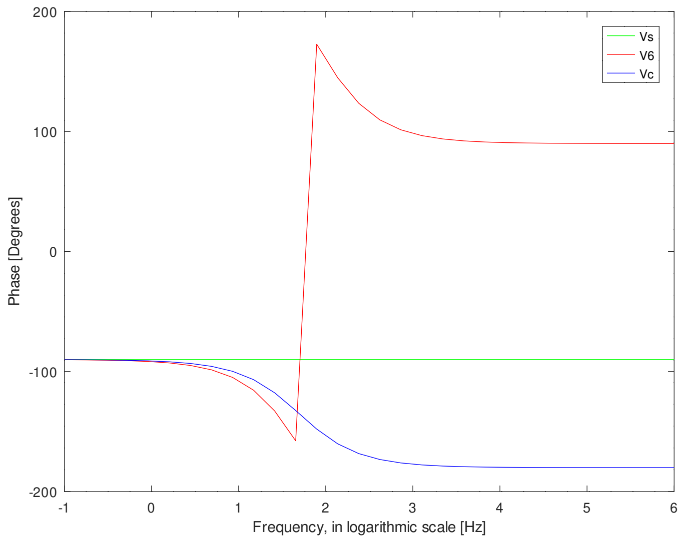

Figure 5: Variation with frequency of V_s, V_6 and V_c voltage phases.

3 Simulation Analysis

Since this circuit has a sinusoidal voltage source, the voltage and current values of the various components vary in time. Therefore, we must perform a transient analysis to simulate the circuit's total response. We also ran operating point analysis for both $t < 0$ and $t = 0$ to determine what the initial conditions were and establish the boundary conditions.

3.1 Operating Point Analysis

The tables below show the simulated operating point results for both $t < 0$ (where we assume no current is flowing through the capacitor) and t=0 where we replace Vs for a short-circuit.

Name	Value [A or V]
@c[i]	0.000000e+00
@g[i]	-2.73260e-04
@r1[i]	2.606667e-04
@r2[i]	2.732603e-04
@r3[i]	1.259361e-05
@r4[i]	1.205334e-03
@r5[i]	2.732603e-04
@r6[i]	9.446670e-04
@r7[i]	9.446670e-04
v(1)	5.077034e+00
v(2)	4.803907e+00
v(3)	4.231486e+00
v(4)	-1.95886e+00
v(5)	4.841795e+00
v(6)	5.695822e+00
v(7)	-1.95886e+00
v(8)	-2.93516e+00

Table 5: Operating point analysis for $t < 0$. A variable preceded by @ is of type *current* other variables are of type *voltage* and expressed in Volt.

Name	Value [A or V]
@g[i]	-2.06346e-18
@r1[i]	1.968366e-18
@r2[i]	2.063464e-18
@r3[i]	9.509784e-20
@r4[i]	-4.42213e-19
@r5[i]	2.761627e-03
@r6[i]	-4.33681e-19
@r7[i]	-8.48659e-19
v(1)	0.000000e+00
v(2)	-2.06246e-15
v(3)	-6.38497e-15
v(4)	8.992793e-16
v(5)	-1.77636e-15
v(6)	8.630981e+00
v(7)	8.992793e-16
v(8)	1.776357e-15

Table 6: Operating point analysis for $t = 0$. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.2 Natural response

In this section we simulate the natural response of the circuit using the computations from the previous section.

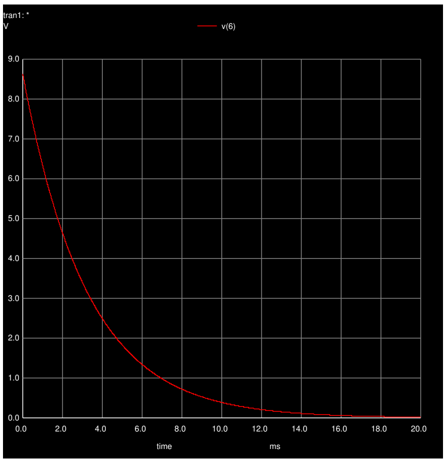

Figure 6: Simulated natural response for V_6 node voltage.

3.3 Natural and forced response

We repeat the previous step with vs(t) considering a frequency of 1kHz. Below is the plot for both the stimulus and the response.

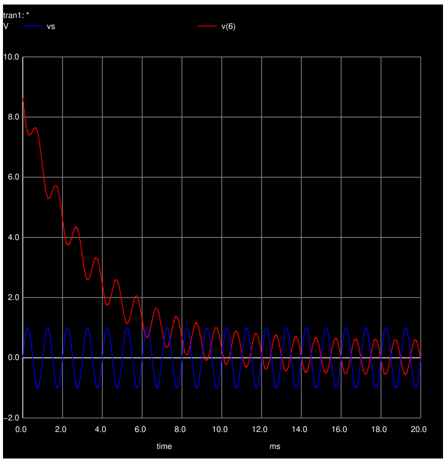

Figure 7: Simulated response for V_6 node voltage and stimulated voltage V_S.

3.4 Frequency response

After simulating the frequency response in node 6 we plotted the magnitude and phase of V6, Vc and Vs side by side.

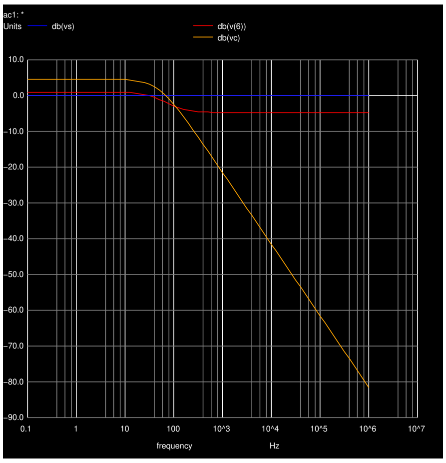

Figure 8: Magnitude of Vs, Vc and V6 as a function of frequency

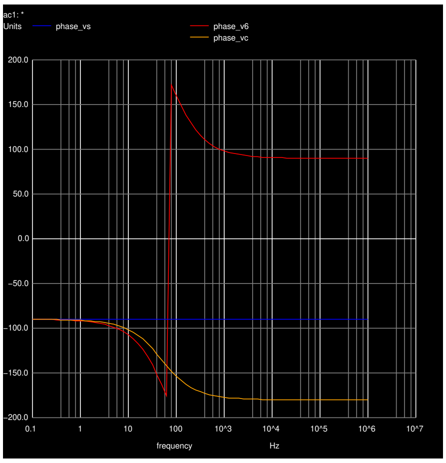

Figure 9: Phase of Vs, Vc and V6 as a function of frequency

4 Conclusion

In this laboratory assignment we were successful in producing coherent calculations using the Octave Maths tool and the circuit simulation, done using the Ngspice tool. Static and transient analysis were performed both theoretically and using a circuit simulation. Some results in the simulation that were predicted to yield zero current or voltage were actually very slightly off - in the magnitude of 1e-15. This could be do to the writing and reading of the different text files truncating the numbers at specific decimal places. The whole report has been automatised and we're confident that it would yield consistent results with a new set of data.