Circuit Theory and Electronics Fundamentals

MEAer (Integrated Master In Aerospace Engineering), Técnico, University of Lisbon

Laboratory 5: OP-AMP bandpass filter

Group 3

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1 Introduction

The objective of this laboratory assignment is to build a BandPass Filter (BPF) circuit whose specifications are: a central frequency at 1000Hz and a gain at central frequency of 40dB. In order to achieve this, we used a selected number of components whose properties were predefined. Our circuit is comprised of a μA741 OPAMP connected to two resistors, creating a non-inverting amplifier, and a combination of resistors and capacitors to take care of the filtering. These components are arranged according to figure 1.

The signal goes through a first stage where we have a capacitor whose function is to block unwanted DC current and to filter out lower frequencies, hence this stage being a high pass filter, since the rest of the circuit is connected to the terminals of the resistor. Now the signal goes through a combination of resistors and a μ741 OPAMP. This arrangement creates a non-inverting amplifier. In this configuration, the output signal is "in-phase" with the input signal. Feedback control of the non-inverting OPAMP is achieved by applying a small part of the output voltage back to the inverting (-) terminal via R3-R4 voltage divider network. And finally, the signal is subjected to a voltage divider network comprised of resistor R2 and capacitor C2. The desired voltage corresponds to the voltage drop at the terminals of the capacitor which, as a result, is subjected to a low pass filtering. For this to run, we need two supply DC voltage sources overlapped with our main circuit in order to power the transistors inside the OPAMP.

For this laboratory assignment there is a figure of merit, depending on the results obtained in the NGSpice simulation. This figure takes into account the cost of the components used, and the results they provide - a desirably low gain deviation and low central frequency deviation. The figure is calculated using the formula given by equation 1. The cost englobes the cost of transistors (0.1 units per transistor), diodes (0.1 units per diode), resistors (1 unit per KOhm) and capacitors (1 unit per uF). The objective is to achieve the highest merit, so we tried different configurations of components data until we achieved our greatest figure of merit.

$$M = \frac{1}{Cost * (GainDeviation + CentralFrequencyDeviation + 10^{-6})} \tag{1}$$

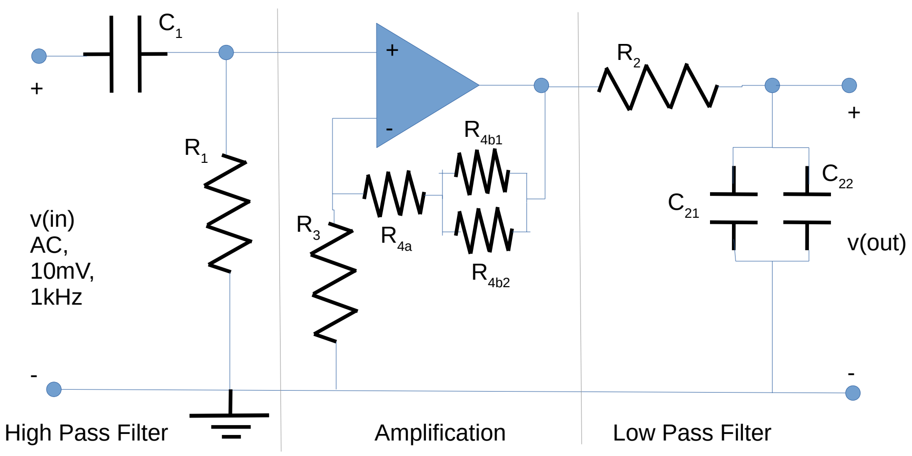

Figure 1: Geometry of our Band-Pass filter circuit.

To analyse this circuit theoretically we computed the transfer function for each stage in order to obtain the overall transfer function. Using the incremental model, we determined the input and output impedances of the overall circuit. That being said, in Section 2 we present the theoretical models and calculations used to determine the transfer function and, therefore, the frequency response of the circuit, in Section 3 we introduce the results obtained in the simulation. Finally, in Section 4 we compare the two sets of results, looking for possible discrepancies and we lay out our conclusions. In Table 1, we list the numeric values of the components used:

Name	**Values**
R_1	1.000000 kOhm
C_1	0.220000 uFarad
R_2	1.000000 kOhm
C_{21}	0.220000 uFarad
C_{22}	0.220000 uFarad
R_3	1.000000 kOhm
R_{4a}	100.000000 kOhm
R_{4b1}	100.000000 kOhm
R_{4b2}	100.000000 kOhm

Table 1: Values of components used in our analysis and simulation.

2 Theoretical Analysis

2.1 Circuit frequency response

With the following equations we determined both cutoff frequencies for the band pass circuit (the lower cut-off frequency appears from the high-pass stage, and the upper cut-off frequency appears from the low-pass stage):

$$w_L = \frac{1}{R_1 C_1} \tag{2}$$

$$w_H = \frac{1}{R_2 C_2} \tag{3}$$

This was the definition used to determine the central frequency, which is meant to be 1 kHz.

$$w_O = \sqrt{w_H w_L} \tag{4}$$

Name	**Values**
Lower Cut-Off Frequency	723.431560 Hz
Upper Cut-Off Frequency	1446.863119 Hz
Central Frequency	1023.086723 Hz

Table 2: Cut off frequencies and central frequency.

This central frequency result represents a 2.309% relative error, which is a relatively good result (given the components we could choose from), that we hope to replicate in the simulation.

As wanted, the plot has the trait of a narrower band-pass filter, with its top gain arriving in the 1 kHz neighbourhood, and the gain itself is on the 40 dB as pretended.

The phase drops from 90 degrees to -90 degrees, which is cause of the 2 poles of the transfer function (which we will present in the next subsection), one pole introduced by the high-pass and the other pole introduced by the low pass. Each pole causes a 90 degree drop, 45º the decade before and 45º the decade after. At the central frequency, the phase is zero, which means the output voltage is in phase with the input voltage.

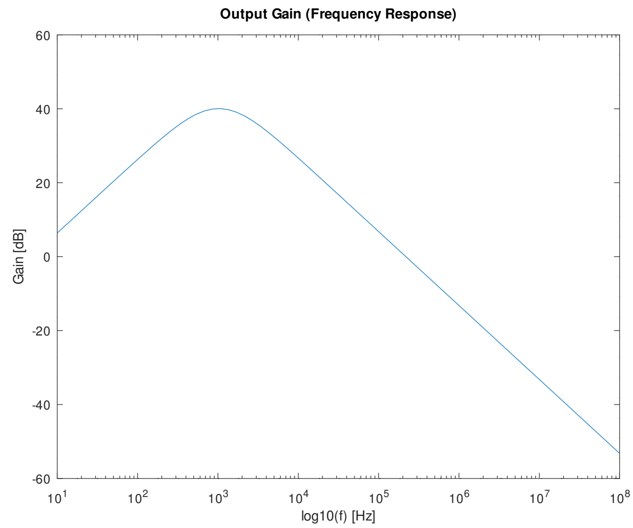

Figure 2: Voltage gain frequency response.

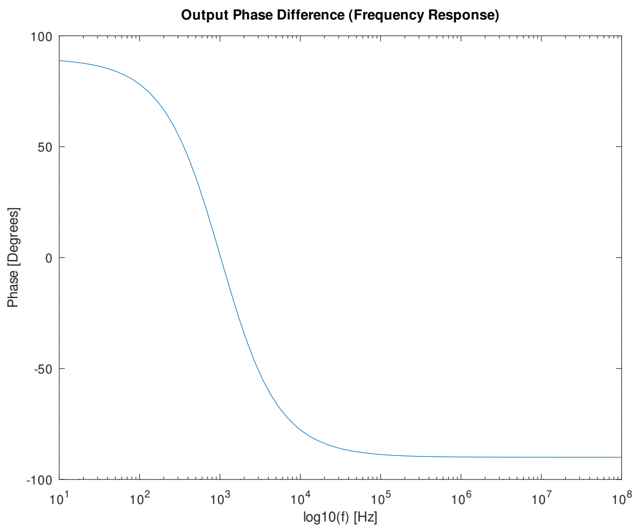

Figure 3: Voltage phase frequency response.

2.2 Central frequency results

For a frequency of 1 KHz we computed the circuit gain and input and output impedances for the amplifier. Also, we determined a theoretical figure of merit based on the predicted results with the set of values we chose.

The gain value for the entire circuit is obtained by multiplying the individual gains of each of the three stages of the circuit. This is an ideal situation, where we don't take into account the loss of signal between stages, or charge effects between the same stages. These are the equations for the gain values:

$$HighPassGain = |\frac{R_1 C_1 j w_O}{1 + R_1 C_1 j w_O}| \tag{5}$$

$$AmplifierGain = 1 + \frac{R_4}{R_3} \tag{6}$$

$$LowPassGain = |\frac{1}{1 + R_2 C_2 j w_O}| \tag{7}$$

Using the incremental model for the circuit (the input impedance of the OP-AMP is infinite and the output impedance of the OP-AMP is null, according to the model studied in class), we determined the following equations for the input and output impedances of the circuit:

$$Z_i = R_1 + \frac{1}{jw_O C_1} \tag{8}$$

$$Z_o = \frac{R_2}{1 + jw_O R_2 C_2} \tag{9}$$

Name	Values
Gain (1 KHz)	100.643363
Gain (1 KHz)(dB)	40.055703 dB
Gain (calculated central frequency) (dB)	40.057714 dB
Z_{in}	1000.000000 -723.431560j Ohm
Z_{in} modulus	1234.241962 Ohm
Z_{in} phase	-35.883164 Degrees
Z_{out}	676.732451 -467.723894j Ohm
Z_{out} modulus	822.637497 Ohm
Z_{out} phase	-34.650304 Degrees
Cost	13626.952040MU
Merit	3.092445$*10^{-6}$

Table 3: Gain, input and output impedances at the central frequency.

This way we can see that the gain for a frequency of 1KHz is almost the same as the gain for the calculated central frequency, meaning the 1KHz frequency is still very much within the band pass region. We obtained then a relative error of 0.643% for the gain at central frequency, which is meant to be 100, so this a fantastic theoretical result. The input impedance value is quite high, which is good, so, depending on the resistance of the input, most of the input voltage will flow through ahead to the OP-AMP as pretended. The output impedance value though is quite large, so this circuit will not be suited to loads with very low resistance values, but given the components we had available there wasn't much room for improvement. Also, we don't know the load that would be linked to this circuit, so we can't fully know how good or bad this value is, we just know it is clearly not the most desirable.

3 Simulation Analysis

Since the input voltage source in this circuit is sinusoidal, the voltage and current values of the various components vary in time, and we are interested therefore in analysing how they evolve in time and obviously we want to picture the transformation of our AC input voltage source from the input to the output of the circuit. We will run a transient analysis which will help us measuring the input and output impedances. We will also run a frequency response analysis in order to determine the gain and central frequency of our output amplified sinusoidal signal.

3.1 Frequency response and impedances

We measured the input impedance of the circuit, seen through the perspective of the source, and the output impedance, seen through the perspective of our output (using a dummy test source). With the frequency response, we measured the voltage gain in our output and the lower and upper cut-off frequencies. Using the same equation from the theoretical analysis we determined the central frequency, then extracting the output gain for such frequency. In table 4, we present the results of our calculations. In figures 4and 5 we can see the frequency response for our output's gain and phase, respectively. With this figures we can notice a slightly narrow band-pass filter, as expected and wanted. Regarding the phase plot though, we notice a full circle phase drop until it reaches the 90 degrees back again, whereas in the theoretical analysis the phase drops from 90 degrees to -90 degrees, stabilizing there. This difference is cause of the aproximation used to study the OP-AMP behaviour in the theoretical section. For study purposes, we considered the OP-AMP did not introduce any phase difference in its output compared to its input, which is not true in reality. Due to the two transistors used in the uA741 OP-AMP, the transfer function in this sector actually presents 2 poles, which causes a double phase drop of 90 degrees (45 degrees the decade before and 45 degrees the decade after) each, which then means the phase actually drops 180 degrees due to the OP-AMP, hence stabilizing not in -90 degrees, but in 90 (-270) degrees.

Name	**Values**
Gain	99.7361
Gain(dB)	39.977
Lower cut-off frequency	403.611 Hz
Upper cut-off frequency	2386.17 Hz
Bandwidth	1982.56 Hz
Central frequency	981.37 Hz
Input impedance	0.999979 + j*-0.723583 kOhm
Input impedance modulus	1.23431 kOhm
Input impedance phase	-35.8894 degrees
Output impedance	0.68172 + j*-0.46675 kOhm
Output impedance modulus	0.826194 kOhm
Output impedance phase	-34.3981 degrees

Table 4: Output gain, center frequency, and input and output impedances of the OP-AMP band-pass filter.

Our input impedance value is reasonably high, so, depending on the input signal's inner resistance, it does allow the great majority of the input voltage to flow through ahead to the OP-AMP. That being said, the output impedance value is also considerably high, which in this case is not desirable. This means that for loads with low resistance values, most of the voltage will be consumed by the circuit's output impedance itself, undoing its very purpose. This band-pass filter is then better suited for loads with greater resistance values. Given the components we had at our disposal, it was very difficult to accomplish a lower value for the output impedance, specially without comprimising the most important goal of this laboratory assignment. Looking now to the central frequency obtained, and the gain for such frequency, we had very slight deviations from the targeted values - 1 kHz and 40 dB. That being said, we obtained a 1.863% relative error for the central frequency value, and a 0.264% error for the output voltage gain, which we consider to be fantastic results, when we take into account the limited components available, which made optimization more difficult than usual.

3.2 Final product and merit

In figure 6, we can compare the output signal to the input. As normal, the output voltage does not possess a DC component. Since the wanted gain was 40 dB, which is the same as 100$\times$ the initial amplitude, we can see the clear evolution from the starting 10mV to the final 1V of amplitude (more or less). For the first few milisseconds there is some small variation in the output sinusoidal wave due to a small transient regime. In table 5, we present the cost and

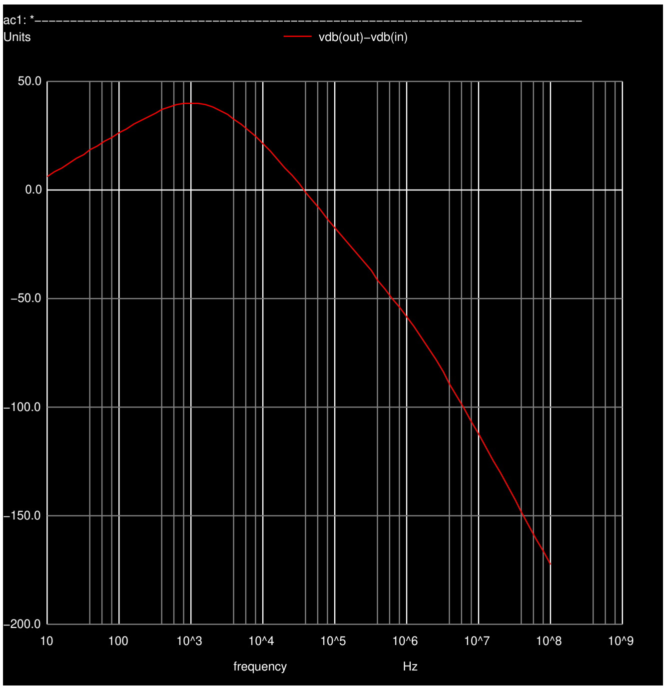

Figure 4: Output voltage gain (frequency response).

merit of our circuit. As we've already discusseed, we obtained a central frequency and gain very close to the targeted ones, which is expressed by very low relative errors, and therefore very small deviations. The overall cost of the circuit is considerably high, but that is mainly due to the high cost of the OP-AMP subcircuit itself. Because of this, using higher cost components in the rest of the circuit became more tolerable, since it wouldn't create as much of an effect in the overall cost value. Because of this we used the three 100 kOhm resistors available in the amplification stage, which allowed us to get an almost perfect gain for the central frequency, without harming the merit figure. The merit figure itself is clearly a very low number, but we believe that is only because of the restraints of the merit formula itself.

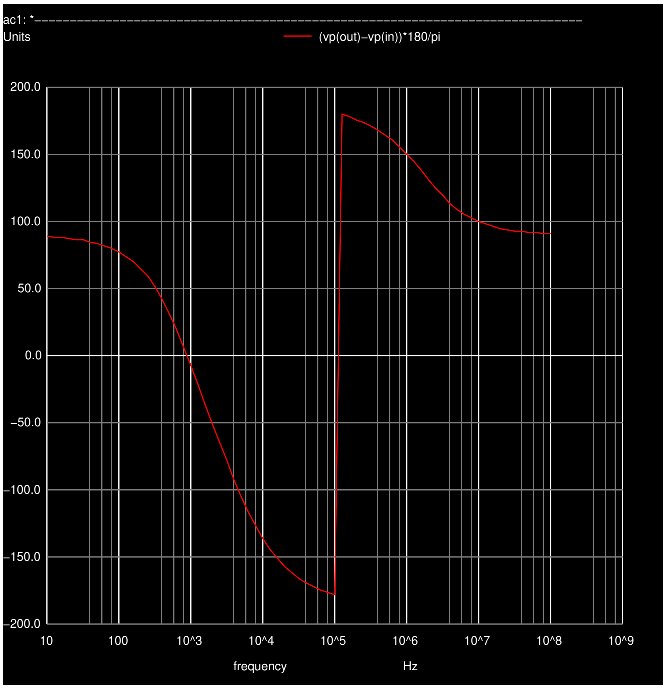

Figure 5: Output voltage phase difference (frequency response).

Name	Values
cost	1.362695e+04
merit	3.883972e-06

Table 5: Cost and merit of the OP-AMP band-pass filter.

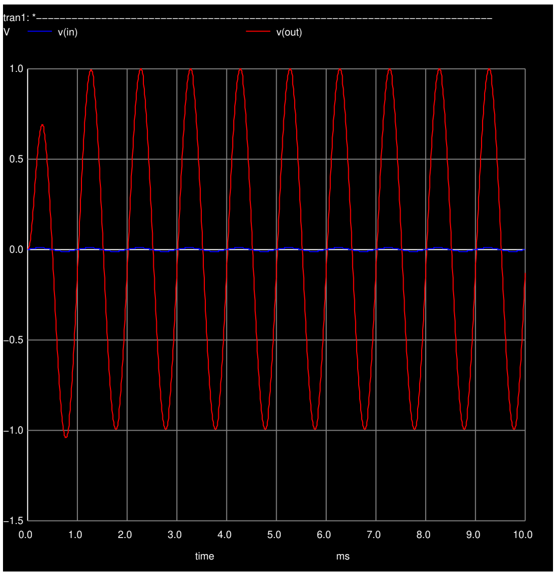

Figure 6: Comparison between the input and the output sinusoidal signals.

4 Conclusion

Theoretical	**Value**	**Simulation**	**Value**
Frequency response and impedances			
$Gain$	100.643363	Gain	99.7361
$Gain(dB)$	40.055703 dB	Gain(dB)	39.977 dB
$LowerCut-offFreq$	723.431560 Hz	Lower cut-off freq	403.611 Hz
$UpperCut-offFreq$	1446.863119 Hz	Upper cut-off freq	2386.17 Hz
$CentralFreq$	1023.086723 Hz	Central freq	981.37 Hz
$Z_{in}Modulus$	1234.241962 Ohm	Zin modulus	1.23431 kOhm
$Z_{in}Phase$	-35.883164 Degrees	Zin phase	-35.8894 Degrees
$Z_{out}Modulus$	822.637497 Ohm	Zout modulus	0.826194 kOhm
$Z_{out}Phase$	-34.650304 Degrees	Zout phase	-34.3981 Degrees
$Cost$	13626.952040	Cost	1.362695e+04
$Merit$	3.092445*10^{-6}	Merit	3.883972e-06

Table 6: Comparison of the theoretical and simulated data results, regarding the frequency response and impedances.

In this laboratory assignment, we managed to build a BandPass Filter (BPF) circuit which is represented in Figure 1. The first step of our analysis was to determine the frequency response by computing the transfer function of the whole circuit, followed by the determination of the input and output impedances.

In the last report we explained how the theoretical model of the transistor lacked the complexity needed to yield closer results to the simulation. In this assignment we dealt again with 2 transistors inside the OP-AMP. This alone means that the theoretical model used to analyse the circuit can differ significantly since it isn't expected to take into account the non-linearity of transistors. Adding to that is the complexity of the OP-AMP model used in Ngspice, especially the use of various capacitors and diodes which weren't taken into account in the octave analysis. Furthermore, another thing that was not taken into account was the parasitic capacitance of the 2 ransistors themselves, inside de OP-AMP. All these factors combined can cause discrepancies between the theoretical and simulation analysis.

That being said, almost every set of data value predicted in the theoretical section is matched in the simulation, with the only exception being the lower and upper cut-off frequency values, that caused our central frequency value to deviate a little. The bandwidth, despite still being approximately centered around the 1 KHz frequency, was much bigger (double the width). We've seen in the previous report that this might have been a source of error because we were not able to get a value for the upper cut off frequency. We believe that the issue with the bandwith this time around might have had to do with the parasitic capacitance of the transistors as a source. Another factor, which caused differences in the phase frequency response plots, is the two poles introduced by the OP-AMP itself that weren't considered in the simplified theoretical model. For theoretical purpose, the transfer function for the circuit only has two poles (one by the high pass, other by the low pass), and each one of the poles symbolizes (is directly correlated) with one of the cut-off frequencies. If in reality we have 4, not 2, poles, this can not be the case, so obviously the cut-off frequencies will be different, even if the central frequency mantains itself around the 1 kHz zone, which is the most important thing since it is the set of data being evaluated here, not the bandwidth.

Despite all of this, the merit figures are somewhat similar, especially if we consider the significant differences from previous assignments. Finally, we were this way able to design a OP-AMP band-pass filter which operates for a central frequency in the neighbourhood of the desired 1 kHz, with a gain of 40 dB as pretended. Despite the higher cost of the components used (aside from the OP-AMP sub-circuit itself), it allowed us to achieve better and more precise results, and even improving the figure of merit, so we do consider this laboratory assignment to be a success.