RAU Independence Cup 2019

RUSSIAN-ARMENIAN
UNIVERSITY

Problems

Time limit for all problems: 2 seconds
Memory limit for all problems: 256MB

Problems are not ordered by difficulty.
Do not open before the contest has started.

Problem A
Bitonic Skyline

The city has n buildings, which are arranged along the coast on one line and numbered from 1 to n. The King wants to improve the view for the visitors from the sea, by making the city's skyline *bitonic*. The skyline is called bitonic if, when viewing from the sea from left to right, the heights of buildings strictly increases until the tallest building, and then strictly decreases. The King does not want to remove floors from the buildings, but he can add some floors. Of course, adding a new floor to any building is expensive. The King has asked you to calculate the smallest total number of floors that can be added to the buildings so that the skyline becomes bitonic. We assume that all the floors in all buildings have the same height.

Input

The first line of the input contains the integer n ($1 \le n \le 10^5$), the number of the buildings in the city. The next line contains n space-separated integers. The ith number corresponds to the number of floors in the ith building when viewed from the sea. No building has more than 10^9 floors.

Output

The output contains a single integer, the smallest number of floors that the King needs to add to make the skyline bitonic.

Sample Input 1	Sample Output 1
5 2 1 5 3 3	3

Sample Input 2	Sample Output 2
6 1 2 3 4 2 1	0

Problem B
Watering Cacti

Armen has n cacti, arranged in one line from left to right, one metre apart from one another. After many years' of experience he has determined the optimal method of watering his cacti. First, he waters the leftmost cactus, then he waters the rightmost cactus, then he waters the leftmost unwatered cactus, then the rightmost unwatered cactus, and so on. Even if the method is best for cacti, Armen feels that he gets tired. He asks you to find out the distance that he travels every time he needs to water all his cacti.

Input

The first and only line of the input contains one integer n ($1 \leq n \leq 10^4$), the number of Armen's cacti.

Output

The output contains the distance, in meters, that Armen travels in order to water all n cacti.

Sample Input 1	Sample Output 1
3	3

Problem C
Scavenger Hunt

Your friend Armen is playing a game of scavenger hunt. The game takes place on the horizontal x axis, and initially Armen is at coordinate 0. In one second he can move one coordinate to the left or to the right. The organisers of the game place n treasures, at integer coordinates $x_1, \ldots, x_n$, respectively. These coordinates are unknown to you and Armen. Initially, Armen is given the description of treasure 1, and when he reaches x_1, he will recognise it. At that point he will be given the description of treasure 2, but, once again, not its coordinate. Then his goal would be to find treasure 2. When he reaches treasure 2, he is given the description of treasure 3, and so on, until he reaches all the treasures.

The judges assess his performance based on how long it took him to find the items, compared to the minimum possible time, $T_{min} = \sum_{i=0}^{n-1} |x_{i+1} - x_i|$, where $x_0 = 0$. He receives a prize if he collects all the treasures in at most $8 \cdot T_{min}$ seconds. Write a program that will give instructions to Armen so that he wins the prize.

The interaction protocol between you and Armen

This is an interactive problem. First, your program must read one line containing an integer, the number of treasures n ($100 \leq n \leq 1000$).

Then it needs to print instructions to reach treasure 1. Each instruction must be one integer h, printed on a separate line. It means that you are instructing Armen to move from his current position x to $x + h$. After each instruction, your program must read the feedback from Armen in one line. The line will be either "`Yes` x" or "`No` x". This will indicate whether Armen found the treasure, and also give you his current coordinate x. Once you receive a "`Yes` x" from Armen, you need to give him instructions to find the second treasure, and so on until the last treasure. Note that when Armen finds the treasure during the instruction, he stops at the treasure.

Don't forget to flush the output after each request! Also your program is required to complete its execution after receiving the last `Yes` output, meaning Armen has found all the treasures.

It is guaranteed that no treasure is more than 10^9 seconds away from the origin, thus, you are not allowed to give Armen instructions that will make him go farther. Below is an example interaction, for $n = 2$ to save space. In actual tests $100 \leq n \leq 1000$.

Armen’s feedback 1	Your instructions 1
2	
	4
No 4	
	-10
No -6	
	100
Yes 10	
	-10
No 0	
	5
No 5	
	-10
Yes -5	

Note that, Armen does not need to finish the third instruction because he finds the first treasure at location 10 before finishing it.

Problem D
Quiz

Armen is taking a math quiz, which tests his knowledge of divisions. The quiz contains a string of digits 0 to 9 and symbols ‘?’. The ‘?’ represents unknown digits in a number. Armen’s goal is to replace the ‘?’ symbols with digits 0 to 9 such that the resulting string is the smallest possible integer divisible by 8. The resulting integer should not have leading 0’s. Help Armen solve the quiz.

Input

The first and only line of the input contains a non-empty string of up to 1000 characters. Each character is a digit from 0 to 9 or ‘?’. The string will not start with a 0.

Output

The only line in the output contains the answer to the quiz given in the input, or -1 if it is not possible to find an answer.

Sample Input 1	Sample Output 1
`1?23`	`-1`

Sample Input 2	Sample Output 2
`3?2?`	`3024`

Problem E
Feed the Cactus

Armen is worried that his cactus does not get all the nutrients it needs, so he decides to go to different towns to find the necessary items. He lives in a country with n towns, which are connected by bidirectional roads. The road networks is such that there is exactly one route that goes between any two towns without visiting the same town twice.

Each town supplies a subset of all possible k items that Armen may need. When he visits a town, he buys all the available items that he does not yet have. Armen wants to make one trip between two towns and make sure that he has at least l out of the possible k items. Help Armen find the number of different routes he can take to find at least l items. Two routes are different, if there is a town that he visits in one but not in the other.

Input

The first line of the input contains three integers n ($1 \leq n \leq 10^5$), k and l ($1 \leq l \leq k \leq 8$), the number of towns in the country, the number of possible items, and the number of items Armen wants to get, respectively.

The following n lines contains binary strings of length k. The ith string describes the items available in town i: If there is a 1 at position j of the string, then the jth item is available at town i, otherwise it is not.

Each of the following $n - 1$ lines contain two integers u and v, indicating that there is a road between towns u and v. It is guaranteed that the road network satisfies the requirements in the statement.

Output

The output contains the number of different routes that Armen can take.

Sample Input 1

```
6 3 2
100
001
110
010
000
010
1 2
3 2
3 4
4 5
4 6
```

Sample Output 1

```
13
```

Problem F
Homework

In the math homework you are given a sequence of n positive integers. You take the sum of every consecutive pair of numbers, to create a new sequence of $n - 1$ positive integers. You repeat the process until you have one number left, which is the final answer to the homework. For example, if you start with the sequence $(2, 1, 4, 5)$, you get sequences $(3, 5, 9)$ and $(8, 14)$, and finally 22, the final answer to the homework.

You have lost all of your math homework, except the final answer! But the teacher wants to see your full answer that includes the steps of your calculations. Write a program that will recover the full answer.

Input

The only line of the input contains two integers, n $(1 \leq n \leq 20)$ and a $(1 \leq a \leq 10^9)$, the number of elements in the original sequence and the final answer, respectively.

Output

If you made a mistake and it is not possible to find an integer sequence of length n with an answer a print "-1" in the only line of the output. Otherwise, print n lines, where the ith line contains the ith sequence of your full answer. Each sequence should be printed as space-separated integers.

Sample Input 1

```
4 22
```

Sample Output 1

```
1 1 1 15
2 2 16
4 18
22
```

Sample Input 2

```
2 1
```

Sample Output 2

```
-1
```

Problem G
Decoration

Armen wants to start a business of decorating houses with plants. Each house he decorates must have exactly p plants of *unique* types. He wants to know the largest number of houses he can decorate using his n plants.

Input

The first line of the input contains two integers, n and p ($1 \leq n, p \leq 10^6$), the total number of plants, and the number of plants in every decorated house. The next line of the input contains n space-separated integers between 1 and 10^6. The ith integer represents the type of the ith plant.

Output

The only line of the output contains the largest number of houses that Armen can decorate.

Sample Input 1	Sample Output 1
6 3 1 2 1 2 3 4	2

Sample Input 2	Sample Output 2
14 3 3 4 1 1 1 2 3 1 2 1 1 5 6 7	4

Sample Input 3	Sample Output 3
8 5 1 1 2 2 3 3 4 4	0

Problem H
Renovation

To renovate his home, Armen has bought n tools which he will put in his garage. The garage is long and narrow, with the entrance at one end. We will represent the garage as the x axis, with the door being at coordinate 0. He can place the tools at positive integer coordinates, only one tool per location. While Armen is working, he always has exactly one tool with him, and the remaining $n - 1$ tools are in the garage. When he comes to the garage to get a new tool, he enters at coordinate 0, walks directly to where his required tool is, swaps it with the old tool, and walks back to the entrance. This means that the tools move in the garage throughout the renovation.

Armen knows exactly in what order he will use the tools (some tools may get used more than once). He wants to know what is the minimal possible distance that he needs to walk in the garage to finish he renovation, if he arranges his tools optimally at the start.

Input

The first line of the input contains two integers, n and m ($1 \le n, m \le 10^5$), the total number of tools, and the length of his renovation. The next line of the input contains m space-separated integers between 1 and n. Armen uses the tools in the order given by this sequence. At the start of the work, Armen already has the first tool of the sequence in his hand. It is guaranteed that the sequence does not have the same tool listed consecutively.

Output

The only line of the output contains the least possible distance that Armen needs to walk in his garage to finish the renovation.

Sample Input 1	Sample Output 1

`3 5`
`1 3 2 3 2`
	`10`

Sample Input 2	Sample Output 2

`5 9`
`1 4 2 3 5 2 5 2 5`
	`28`

Problem I
Interpreter

After mastering mathematics and gardening, Armen wants to learn programming. He wants to start from the basics and learn Basic. He does not find it easy, and has asked you to write an interpreter for a simple version of Basic prgramming language.

Below is the formal description of a language:

```
var := 'a' .. 'z'
int := '0' .. '65535'
arg := <var> | <int>
op := '+' | '-' | '*' | '/'
comp := '<' | '=' | '>'
test := <arg> <comp> <arg>
expr := <arg> | <arg> <op> <arg>
command := 'PRINT' <arg> |
           'GOTO' <int> |
           'END' |
           'IF' <test> 'THEN' <command> |
           <var> = <expr>
```

The program consists of $n \geq 1$ lines, numbered from 1 to n. The string on each line is an instance of a `command`, defined formally above. The last command is always `END`. Commands are executed sequentially starting from first command unless a `GOTO` command is encountered.

Below is the description of the commands:

`PRINT <arg>` – prints the value of the constant or variable, followed by a line break. It is guaranteed that the value of the variable is defined at the point when it is being used.

`GOTO <int>` – continue the execution to the command starting from the given line number. It is guaranteed that the argument of a command is natural number between 1 and n.

`IF <test> THEN <command>` – if after substituting the values of the variables in `test`, the statement becomes true, then the given command is executed. Otherwise, the next command is executed. It is guaranteed that the values of variables presented in the statement `test` are defined.

`END` – the command that finishes the execution of the program. After this command the interpreter stops.

`<var> = <expr>` – value of the expression `expr` is calculated and assigned to the variable `var`. It is guaranteed that the values of the variables in `expr` are defined.

Input

The first line of the input contains an integer n, ($1 \leq n \leq 1000$), the number of commands. The next n lines contain the source code of the program starting from the first command.

It is guaranteed that

- all elements of commands are separated by a space;
- all division operations are integer divisions, and the remainder is discarded;
- after any arithmetic calculations the value of variables will be between 0 and 65535, and there will be no division by 0;
- the program is syntactically correct;
- that all commands contain no more than 60 symbols;
- the program will finish the execution at a command `END` after no more than 10000 commands.

Output

The output should contain the result of the execution of the program given in the input, i.e. what the program will print to the output.

Sample Input 1

```
12
p = 2
i = 2
IF i = p THEN GOTO 9
j = p / i
j = j * i
IF j = p THEN GOTO 10
i = i + 1
GOTO 3
PRINT p
p = p + 1
IF p < 100 THEN GOTO 2
END
```

Sample Output 1

```
2
3
5
7
11
13
17
19
23
29
31
37
41
43
47
53
59
61
67
71
73
79
83
89
97
```

Problem J
Quality of the Alphabet

Little Armen and Vartan are studying strings and their properties. Once they create a string S, they write it on a long piece of paper from left to right and cut it in two parts. Armen takes the left part, Vartan takes the right part. The parts may contain all the string or none of it.

Armen and Vartan are afraid of string X_A and X_V, respectively. In particular, Armen does not want to get a piece of string, in which X_A appears more than k_A times as a contiguous substring. Otherwise he is afraid of his piece. Similarly, Vartan does not want to get a piece of string, where X_V appears more than k_V times as a contiguous substring.

For a string S, Armen and Vartan define its quality as the number of ways they can split S into two pieces such that none of them is afraid from their piece. Then, they quality of the English alphabet is the sum of the qualities of all strings of length n that are composed of lowercase English letters. Help them find the quality of the English alphabet.

Input

The first line of the input contains a single integer n ($1 \leq n \leq 10^9$), the length of the strings. The second line contains a string and an integer, X_A and k_A, respectively, that are space-separated. The third line contains X_V and k_V, in the same format. It is guaranteed that for $P \in \{A, V\}$, the string X_P is composed of lowercase English letters, and that $1 \leq k_P \cdot len(X_P) \leq 40$.

Output

The only line of the output contains the quality of the English alphabet. Since this number may be very large, you are required to output the answer modulo $10^9 + 7$.

Sample Input 1	Sample Output 1
3 aa 1 aa 1	70302

Sample Input 2	Sample Output 2
10 aaa 5 bbb 5	41309314

Problem K
Equality

The villages of your country are located on a straight road. Each village has an integer wealth, which is negative if the village is in debt. The make the country look better financially, the King has decided to merge some villages into larger ones. Since there is only one road in the country, he can only merge two villages i and j if all the villages between i and j are also merged with them. The wealth of a merged village is the sum of the original wealths of the villages it consists of.

The *equality-index* of two villages is the product of their wealths. The total equality-index of the country is the sum of the equality-indices of all unordered pairs of villages. The King has asked you to calculate the largest possible equality-index that his country can have, if he merges the villages wisely.

Input

The first line of the input contains one integer, n ($1 \leq n \leq 10^5$), the number of villages. The next line contains n space-separated integers between -1000 and 1000. The ith integer is the wealth of the ith village on the road, when counting from one of its ends.

Output

The only line of the output contains the largest possible equality-index of the country.

Sample Input 1	Sample Output 1
6 3 -2 1 -4 5 2	9

Sample Input 2	Sample Output 2
5 1 2 3 2 1	31

Problem L
Starry Sky

It was dark, one of the beautiful nights of Yerevan. The weather was wonderful and Armen decided to sleep outside. He lied down to sleep, but the view of the sky was so interesting that he started to research the starry sky. He found Ursa Major, Ursa Minor and several other constellations. Then he realized that several stars form a rectangle. He was amazed at that, and decided to find some other 4 stars that form a square. After searching for several minutes he found a square as well. He then decided to take any 4 stars in the sky and quickly determine type of a quadrangle that they form. It is often not easy to determine the type of the quadrange quickly, so he is asking you to help him with this interesting task.

Input

The input contains 4 lines, each describing coordinates of a star in the sky with 2 integers x and y ($-1000 \leq x, y \leq 1000$). Armen is able to quickly detect if three points are on a single line, thus, in his examples no 3 points lie on a same line.

Output

The output should contain one of the following words:

```
Square
Rectangle
Rhomb
Trapezium
Parallelogram
Quadrangle
```

If the given quadrangle is of several types at the same time, you should consider the priority given above. For example, if the given quadrangle is a square then you should print “Square,” even though it is also a rectangle, a rhomb and a parallelogram.

Sample Input 1

```
2 0
2 4
0 2
4 2
```

Sample Output 1

```
Square
```

Sample Input 2

```
5 2
6 4
6 0
7 2
```

Sample Output 2

```
Rhomb
```

Problem M
Messages

While playing in the garden, Armen finds two ancient rocks which contain inscriptions. He discovers that the first rock contains n secret binary code words, each written on a separate line on the rock. The second rock contains m messages, each one is written on a single line on of the rock, using exactly one code word. Unfortunately, the rocks are broken and Armen can see only the prefixes of the original lines of the rocks. Thus, he may not be able to uniquely identify which code word matches each message. Nevertheless, he wants to know, for each message, how many code words can match it.

Input

The first line of the input contains two integers n and m ($1 \leq n, m \leq 50000$), the number of code words and the number of messages, respectively. The next n lines of the input contain binary strings, one per line, that represent the code words. After that, the next m lines contain the binary strings that represent the messages. Each line on both of the rocks contains at least one character, and at most 10000 characters. The total number of characters on the two rocks does not exceed 500000.

Output

The output contains m lines. The ith line contains one integer, the number of code words that the ith message could match.

Sample Input 1

```
4 5
010
1
100
110
0
1
01
01001
11
```

Sample Output 1

```
1
3
1
1
2
```