45. Clebsch-Gordan Coefficients, Spherical Harmonics, and d Functions

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2	Coefficients		
m_1	m_2			
.	.			
.	.			

$1/2\times1/2$

m_1	m_2	$J=1$, $M=+1$
+1/2	+1/2	1

m_1	m_2	1, 0	0, 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	1, -1
−1/2	−1/2	1

$1\times1/2$

m_1	m_2	3/2, +3/2
+1	+1/2	1

m_1	m_2	3/2, +1/2	1/2, +1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	3/2, −1/2	1/2, −1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	3/2, −3/2
−1	−1/2	1

2×1

m_1	m_2	3, +3
+2	+1	1

m_1	m_2	3, +2	2, +2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	3, +1	2, +1	1, +1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	3, 0	2, 0	1, 0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	3, −1	2, −1	1, −1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	3, −2	2, −2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	3, −3
−2	−1	1

1×1

m_1	m_2	2, +2
+1	+1	1

m_1	m_2	2, +1	1, +1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	2, 0	1, 0	0, 0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	2, −1	1, −1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	2, −2
−1	−1	1

$$Y_1^0=\sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1=-\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0=\sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta-\frac{1}{2}\Big)$$

$$Y_2^1=-\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2=\frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$2\times1/2$

m_1	m_2	5/2, +5/2
+2	+1/2	1

m_1	m_2	5/2, +3/2	3/2, +3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	5/2, +1/2	3/2, +1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	5/2, −1/2	3/2, −1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	5/2, −3/2	3/2, −3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	5/2, −5/2
−2	−1/2	1

$3/2\times1/2$

m_1	m_2	2, +2
+3/2	+1/2	1

m_1	m_2	2, +1	1, +1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	2, 0	1, 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	2, −1	1, −1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	2, −2
−3/2	−1/2	1

$3/2\times1$

m_1	m_2	5/2, +5/2
+3/2	+1	1

m_1	m_2	5/2, +3/2	3/2, +3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	5/2, −3/2	3/2, −3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	5/2, −5/2
−3/2	−1	1

$$Y_\ell^{-m}=(-1)^m Y_\ell^{m*}\qquad d^{\,\ell}_{m,0}=\sqrt{\frac{4\pi}{2\ell+1}}\,Y_\ell^m\,e^{-im\phi}$$

$$\langle j_1j_2m_1m_2|j_1j_2JM\rangle = (-1)^{J-j_1-j_2}\langle j_2j_1m_2m_1|j_2j_1JM\rangle$$

$$d^{\,j}_{m',m}=(-1)^{m-m'}d^{\,j}_{m,m'}=d^{\,j}_{-m,-m'}$$

$$d^{\,1}_{0,0}=\cos\theta\qquad d^{\,1/2}_{1/2,1/2}=\cos\frac{\theta}{2}\qquad d^{\,1/2}_{1/2,-1/2}=-\sin\frac{\theta}{2}$$

$$d^{\,1}_{1,1}=\frac{1+\cos\theta}{2}\qquad d^{\,1}_{1,0}=-\frac{\sin\theta}{\sqrt{2}}\qquad d^{\,1}_{1,-1}=\frac{1-\cos\theta}{2}$$

$2\times3/2$

m_1	m_2	7/2, +7/2
+2	+3/2	1

m_1	m_2	7/2, +5/2	5/2, +5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	7/2, +3/2	5/2, +3/2	3/2, +3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	7/2, +1/2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	7/2, −1/2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	7/2, −3/2	5/2, −3/2	3/2, −3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	7/2, −5/2	5/2, −5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	7/2, −7/2
−2	−3/2	1

$3/2\times3/2$

m_1	m_2	3, +3
+3/2	+3/2	1

m_1	m_2	3, +2	2, +2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	3, +1	2, +1	1, +1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	3, 0	2, 0	1, 0	0, 0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	3, −1	2, −1	1, −1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	3, −2	2, −2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	3, −3
−3/2	−3/2	1

2×2

m_1	m_2	4, +4
+2	+2	1

m_1	m_2	4, +3	3, +3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	4, +2	3, +2	2, +2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	4, +1	3, +1	2, +1	1, +1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	4, 0	3, 0	2, 0	1, 0	0, 0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	4, −1	3, −1	2, −1	1, −1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	4, −2	3, −2	2, −2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	4, −3	3, −3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	4, −4
−2	−2	1

$$d^{\,3/2}_{3/2,3/2}=\frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,1/2}=-\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-1/2}=\sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-3/2}=-\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,1/2}=\frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,-1/2}=-\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^{\,2}_{2,2}=\Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{\,2}_{2,1}=-\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,0}=\frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^{\,2}_{2,-1}=-\frac{1-\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,-2}=\Big(\frac{1-\cos\theta}{2}\Big)^2$$

$$d^{\,2}_{1,1}=\frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{\,2}_{1,0}=-\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{\,2}_{1,-1}=\frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^{\,2}_{0,0}=\Big(\frac{3}{2}\cos^2\theta-\frac{1}{2}\Big)$$

Figure 45.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).

1. Physical Constants (a major revision)

Table 1.1. Revised 2019 by C.G. Wohl (LBNL). Reviewed by P.J. Mohr and D.B. Newell (NIST). Mainly from "CODATA Recommended Values of the Fundamental Physical Constants: 2018," E. Tiesinga, D.B. Newell, P.J. Mohr, and B.N. Taylor, NIST SP961 (May 2019). The electron charge magnitude e, and the Planck, Boltzmann, and Avogadro constants h, k, and N_A, now join c as having defined values; the free-space permittivity and permeability constants ϵ_0 and μ_0 are no longer exact. These changes affect practically everything else in the Table. Figures in parentheses after the values are the 1-standard-deviation uncertainties in the last digits; the fractional uncertainties in parts per 10^9 (ppb) are in the last column. The full 2018 CODATA Committee on Data for Science and Technology set of constants are found at `https://physics.nist.gov/constants`. The last set of constants (beginning with the Fermi coupling constant) comes from the Particle Data Group. See also "The International System of Units (SI)," 9th ed. (2019) of the International Bureau of Weights and Measures (BIPM), `https://www.bipm.org/utils/common/pdf/si-brochure/SI-Brochure-9-EN.pdf`.

Quantity	Symbol, equation	Value	Uncertainty (ppb)
speed of light in vacuum	c	299 792 458 m s^{-1}	exact
Planck constant	h	6.626 070 15$\times 10^{-34}$ J s (or J/Hz)$^\sharp$	exact
Planck constant, reduced	$\hbar \equiv h/2\pi$	1.054 571 817... $\times 10^{-34}$ J s	exact*
		= 6.582 119 569... $\times 10^{-22}$ MeV s	exact*
electron charge magnitude	e	1.602 176 634$\times 10^{-19}$ C	exact
conversion constant	$\hbar c$	197.326 980 4... MeV fm	exact*
conversion constant	$(\hbar c)^2$	0.389 379 372 1... GeV^2 mbarn	exact*
electron mass	m_e	0.510 998 950 00(15) MeV/c^2 = 9.109 383 7015(28)$\times 10^{-31}$ kg	0.30
proton mass	m_p	938.272 088 16(29) MeV/c^2 = 1.672 621 923 69(51)$\times 10^{-27}$ kg	0.31
		= 1.007 276 466 621(53) u = 1836.152 673 43(11) m_e	0.053, 0.060
neutron mass	m_n	939.565 420 52(54) MeV/c^2 = 1.008 664 915 95(49) u	0.57, 0.48
deuteron mass	m_d	1875.612 942 57(57) MeV/c^2	0.30
unified atomic mass unit**	u = (mass ^{12}C atom)/12	931.494 102 42(28) MeV/c^2 = 1.660 539 066 60(50)$\times 10^{-27}$ kg	0.30
permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	8.854 187 8128(13) $\times 10^{-12}$ F m^{-1}	0.15
permeability of free space	$\mu_0/(4\pi \times 10^{-7})$	1.000 000 000 55(15) N A^{-2}	0.15
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297 352 5693(11)$\times 10^{-3}$ = 1/137.035 999 084(21)†	0.15
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 3262(13)$\times 10^{-15}$ m	0.45
(e^- Compton wavelength)/2π	$\lambda\!\!\!^{-}_e = \hbar/m_e c = r_e \alpha^{-1}$	3.861 592 6796(12)$\times 10^{-13}$ m	0.30
Bohr radius ($m_{\rm nucleus} = \infty$)	$a_\infty = 4\pi\epsilon_0\hbar^2/m_e e^2 = r_e\alpha^{-2}$	0.529 177 210 903(80)$\times 10^{-10}$ m	0.15
wavelength of 1 eV/c particle	$hc/(1\ \rm eV)$	1.239 841 984... $\times 10^{-6}$ m	exact*
Rydberg energy	$hcR_\infty = m_e e^4/2(4\pi\epsilon_0)^2\hbar^2 = m_e c^2\alpha^2/2$	13.605 693 122 994(26) eV	1.9$\times 10^{-3}$
Thomson cross section	$\sigma_T = 8\pi r_e^2/3$	0.665 245 873 21(60) barn	0.91
Bohr magneton	$\mu_B = e\hbar/2m_e$	5.788 381 8060(17)$\times 10^{-11}$ MeV T^{-1}	0.3
nuclear magneton	$\mu_N = e\hbar/2m_p$	3.152 451 258 44(96)$\times 10^{-14}$ MeV T^{-1}	0.31
electron cyclotron freq./field	$\omega^e_{\rm cycl}/B = e/m_e$	1.758 820 010 76(53)$\times 10^{11}$ rad s^{-1} T^{-1}	0.30
proton cyclotron freq./field	$\omega^p_{\rm cycl}/B = e/m_p$	9.578 833 1560(29)$\times 10^{7}$ rad s^{-1} T^{-1}	0.31
gravitational constant‡	G_N	6.674 30(15)$\times 10^{-11}$ m^3 kg^{-1} s^{-2}	2.2×10^4
		= 6.708 83(15)$\times 10^{-39}$ $\hbar c$ (GeV/c^2)$^{-2}$	2.2×10^4
standard gravitational accel.	g_N	9.806 65 m s^{-2}	exact
Avogadro constant	N_A	6.022 140 76$\times 10^{23}$ mol^{-1}	exact
Boltzmann constant	k	1.380 649$\times 10^{-23}$ J K^{-1}	exact
		= 8.617 333 262... $\times 10^{-5}$ eV K^{-1}	exact*
molar volume, ideal gas at STP	$N_A k$ (273.15 K)/(101 325 Pa)	22.413 969 54... $\times 10^{-3}$ m^3 mol^{-1}	exact*
Wien displacement law constant	$b = \lambda_{\rm max} T$	2.897 771 955... $\times 10^{-3}$ m K	exact*
Stefan-Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	5.670 374 419... $\times 10^{-8}$ W m^{-2} K^{-4}	exact*
Fermi coupling constant‡‡	$G_F/(\hbar c)^3$	1.166 378 7(6)$\times 10^{-5}$ GeV^{-2}	510
weak-mixing angle	$\sin^2\widehat{\theta}(M_Z)$ ($\overline{\rm MS}$)	0.231 21(4)††	1.7×10^5
$W^\pm$ boson mass	m_W	80.379(12) GeV/c^2	1.5×10^5
Z^0 boson mass	m_Z	91.1876(21) GeV/c^2	2.3×10^4
strong coupling constant	$\alpha_s(m_Z)$	0.1179(10)	8.5×10^6

$\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ldots$	$e = 2.718\ 281\ 828\ 459\ 045\ 235\ldots$	$\gamma = 0.577\ 215\ 664\ 901\ 532\ 860\ldots$

1 in $\equiv$ 0.0254 m	1 G $\equiv 10^{-4}$ T	1 eV = 1.602 176 634 $\times 10^{-19}$ J (exact)	kT at 300 K = [38.681 740(22)]$^{-1}$ eV
1 Å $\equiv$ 0.1 nm	1 dyne $\equiv 10^{-5}$ N	(1 kg)c^2 = 5.609 588 603 ... $\times 10^{35}$ eV (exact*)	0 °C $\equiv$ 273.15 K
1 barn $\equiv 10^{-28}$ m^2	1 erg $\equiv 10^{-7}$ J	1 C = 2.997 924 58 $\times 10^9$ esu	1 atmosphere $\equiv$ 760 Torr $\equiv$ 101 325 Pa

$^\sharp$ CODATA recommends that the unit be J/Hz to stress that in $h = E/\nu$ the frequency ν is in cycles/sec (Hz), not radians/sec.
* These are calculated from exact values and are exact to the number of places given (*i.e.* no rounding).
** The molar mass of ^{12}C is 11.999 999 9958(36) g.
† At $Q^2 = 0$. At $Q^2 \approx m_W^2$ the value is $\sim 1/128$.
‡ Absolute laboratory measurements of G_N have been made only on scales of about 1 cm to 1 m.
‡‡ See the discussion in Sec. 10, "Electroweak model and constraints on new physics."
†† The corresponding $\sin^2\theta$ for the effective angle is 0.23153(4).

2. Astrophysical Constants and Parameters

Table 2.1: Revised August 2019 by D.E. Groom (LBNL) and D. Scott (U. of British Columbia). The figures in parentheses after some values give the 1-σ uncertainties in the last digit(s). Physical constants are from Ref. [1]. While every effort has been made to obtain the most accurate current values of the listed quantities, the table does not represent a critical review or adjustment of the constants, and is not intended as a primary reference. The values and uncertainties for the cosmological parameters depend on the exact data sets, priors, and basis parameters used in the fit. Many of the derived parameters reported in this table have non-Gaussian likelihoods. Parameters may be highly correlated, so care must be taken in propagating errors. Unless otherwise specified, cosmological parameters are derived from a 6-parameter ΛCDM cosmology fit to *Planck* cosmic microwave background 2018 temperature (TT) + polarization (TE,EE+lowE) + lensing data [2]. For more information see Ref. [3] and the original papers.

Quantity	Symbol, equation.	Value	Reference,footnote
Newtonian constant of gravitation	G_N	$6.674\,30(15) \times 10^{-11} \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$	[1]
Planck mass	$M_P = \sqrt{\hbar c/G_N}$	$1.220\,890(14) \times 10^{19}$ GeV$/c^2 = 2.176\,434(24) \times 10^{-8}$ kg	[1]
Planck length	$l_P = \sqrt{\hbar G_N/c^3}$	$1.616\,255(18) \times 10^{-35}$ m	[1]
tropical year (equinox to equinox, 2020)	yr	$31\,556\,925.1$ s $= 365.242\,189$ days	[4]
sidereal year (period of Earth around Sun relative to stars)		$31\,558\,149.8$ s $\approx \pi \times 10^7$ s	[4]
mean sidereal day (Earth rotation period relative to stars)		$23^{\mathrm{h}}\,56^{\mathrm{m}}\,04\overset{\mathrm{s}}{.}090\,53$	[4]
astronomical unit	au	$149\,597\,870\,700$ m	exact [5]
parsec (1 au/1 arc sec)	pc	$3.085\,677\,581\,49 \times 10^{16}$ m $= 3.261\,56\ldots$ ly	exact [6]
light year (deprecated unit)	ly	$0.306\,601\ldots$ pc $= 0.946\,073\ldots \times 10^{16}$ m	[7]
solid angle	deg^2	$(\pi/180)^2$ sr $= 3.046\,17\ldots \times 10^{-4}$ sr	[8]
Schwarzschild radius of the Sun	$2G_N M_\odot/c^2$	$2.953\,250\,076\,100\,25$ km	[9]
Solar mass	$M_\odot$	$1.988\,41(4) \times 10^{30}$ kg	[10]
nominal Solar equatorial radius	$\mathcal{R}_\odot$	6.957×10^8 m	exact [11]
nominal Solar constant	$\mathcal{S}_\odot$	1361 W m^{-2}	exact [11,12]
nominal Solar photosphere temperature	$\mathcal{T}_\odot$	5772 K	exact [11]
nominal Solar luminosity	$\mathcal{L}_\odot$	3.828×10^{26} W	exact [11,13]
Schwarzschild radius of the Earth	$2G_N M_\oplus/c^2$	$8.870\,055\,940$ mm	[9]
Earth mass	$M_\oplus$	$5.972\,17(13) \times 10^{24}$ kg	[10]
nominal Earth equatorial radius	$\mathcal{R}_\oplus$	6.3781×10^6 m	exact [11]
Chandrasekhar mass	M_{Ch}	$3.097\,972\,\mu^{-2}\,M_P^3/m_H^2 = 1.433\,77(6)\,(\mu/2)^{-2}\,M_\odot$	[14,15]
Eddington luminosity	L_{Ed}	$1.257\,065\,179\,8(12) \times 10^{31}\,(M/M_\odot)$ W	[16,17]
		$= 3.283\,869\,330\,8(31) \times 10^4\,(M/M_\odot)\,\mathcal{L}_\odot$	
jansky (flux density)	Jy	10^{-26} W m^{-2} Hz^{-1}	definition
luminosity conversion	f_0	$3.0128 \times 10^{28} \times 10^{-0.4\,M_{\mathrm{Bol}}}$ W	exact [18]
		(M_{Bol} = absolute bolometric magnitude = bolometric magnitude at 10 pc)	
flux conversion	$\mathcal{F}$	$2.518\,021\,002 \times 10^{-8} \times 10^{-0.4\,m_{\mathrm{Bol}}}$ W m^{-2}	exact [18]
		(m_{Bol} = apparent bolometric magnitude)	
ABsolute monochromatic magnitude	AB	$-2.5\,\log_{10} f_\nu - 56.10$ (for f_ν in W m^{-2} Hz^{-1})	[19]
		$= -2.5\,\log_{10} f_\nu + 8.90$ (for f_ν in Jy)	
Solar angular velocity around Galactic center	Θ_0/R_0	27.1(5) km s^{-1} kpc^{-1}	[20]
Solar distance from Galactic center	R_0	8.178 ± 0.013(stat.) $\pm\,0.022$(sys.) kpc	[21,22]
circular velocity at R_0	v_0 or Θ_0	240(8) km s^{-1}	[22,23]
escape velocity from the Galaxy	v_{esc}	492 km s^{-1} $< v_{\mathrm{esc}} <$ 587 km s^{-1} (90%)	[24]
local disk density	ρ_{disk}	$6.6(9) \times 10^{-24}$ g cm^{-3} = 3.7(5) GeV$/c^2$ cm^{-3}	[25]
local dark matter density	ρ_χ	canonical value 0.3 GeV$/c^2$ cm^{-3} within factor 2–3	[26]
present-day CMB temperature	T_0	2.7255(6) K	[27,28]
present-day CMB dipole amplitude	d	3.3621(10) mK	[27,29]
Solar velocity with respect to CMB	$v_\odot$	369.82(11) km s^{-1} towards $(l, b) = (264.021(11)^\circ, 48.253(5)^\circ)$	[29]
Local Group velocity with respect to CMB	v_{LG}	620(15) km s^{-1} towards $(l, b) = (271.9(20)^\circ, 29.6(14)^\circ)$	[29]
number density of CMB photons	n_γ	$410.7(3)\,(T/2.7255)^3$ cm^{-3}	[30]
density of CMB photons	ρ_γ	$4.645(4)\,(T/2.7255)^4 \times 10^{-34}$ g cm^{-3} ≈ 0.260 eV cm^{-3}	[30]
entropy density/Boltzmann constant	s/k	$2\,891.2\,(T/2.7255)^3$ cm^{-3}	[30]
present-day Hubble expansion rate	H_0	$100\,h$ km s^{-1} Mpc^{-1} $= h \times (9.777\,752$ Gyr$)^{-1}$	[31]
scaling factor for Hubble expansion rate	h	0.674(5)	[2,32]
Hubble length	c/H_0	$0.925\,0629 \times 10^{26}\,h^{-1}$ m $= 1.372(10) \times 10^{26}$ m	
scaling for cosmological constant	$c^2/3H_0^2$	$2.85247 \times 10^{51}\,h^{-2}$ m^2 $= 6.21(9) \times 10^{51}$ m^2	

Quantity	Symbol, equation.	Value	Reference,footnote
critical density of the Universe	$\rho_{\rm crit} = 3H_0^2/8\pi G_N$	$1.878\,34(4) \times 10^{-29}\ h^2$ g cm^{-3}	
		$= 1.053\,672(24) \times 10^{-5}\ h^2$ (GeV/c^2) cm^{-3}	
		$= 2.77536627 \times 10^{11}\ h^2\ M_\odot$Mpc^{-3}	
baryon-to-photon ratio (from BBN)	$\eta = n_{\rm b}/n_\gamma$	$5.8 \times 10^{-10} \le \eta \le 6.5 \times 10^{-10}$ (95% CL)	[33]
number density of baryons	$n_{\rm b}$	$2.515(17) \times 10^{-7}$ cm^{-3}	[2,3,34,35]
		$(2.4 \times 10^{-7} < n_{\rm b} < 2.7 \times 10^{-7})$ cm^{-3} (95% CL, $\eta \times n_\gamma$)	
CMB radiation density of the Universe	$\Omega_\gamma = \rho_\gamma/\rho_{\rm crit}$	$2.473 \times 10^{-5}(T/2.7255)^4\ h^{-2} = 5.38(15) \times 10^{-5}$	[30]
- - - *Planck* 2018 6-parameter fit to flat ΛCDM cosmology - - -			
baryon density of the Universe	$\Omega_{\rm b} = \rho_{\rm b}/\rho_{\rm crit}$	$^\ddagger\, 0.02237(15)\, h^{-2} = {}^\dagger\, 0.0493(6)$	[2,3,27]
cold dark matter density of the Universe	$\Omega_{\rm c} = \rho_c/\rho_{\rm crit}$	$^\ddagger\, 0.1200(12)\, h^{-2} = {}^\dagger\, 0.265(7)$	[2,3,27]
100 × approx to $r_*/D_{\rm A}$	$100 \times \theta_{\rm MC}$	$^\ddagger\, 1.04092(31)$	[2,3,27]
reionization optical depth	τ	$^\ddagger\, 0.054(7)$	[2,3,27]
ln(power prim. curv. pert.) ($k_0 = 0.05$ Mpc^{-1})	$\ln(10^{10}\Delta^2_{\mathcal{R}})$	$^\ddagger\, 3.044(14)$	[2,3,27]
scalar spectral index	$n_{\rm s}$	$^\ddagger\, 0.965(4)$	[2,3,27]
pressureless matter parameter	$\Omega_{\rm m} = \Omega_{\rm c} + \Omega_{\rm b}$	$^\dagger\, 0.315(7)$	[2,3]
dark energy density parameter	Ω_Λ	$^\dagger\, 0.685(7)$	[2,3]
energy density of dark energy	ρ_Λ	$^\dagger\, 5.83(16) \times 10^{-30}$ g cm^{-3}	[2]
cosmological constant	Λ	$^\dagger\, 1.088(30) \times 10^{-56}$ cm^{-2}	[2]
fluctuation amplitude at 8 h^{-1} Mpc scale	σ_8	$^\dagger\, 0.811(6)$	[2,3]
redshift of matter-radiation equality	$z_{\rm eq}$	$^\dagger\, 3402(26)$	[2,36]
age at matter-radiation equality	$t_{\rm eq}$	$^\dagger\, 51.1(8)$ kyr	[2,37]
redshift at which optical depth equals unity	z_*	$^\dagger\, 1089.92(25)$	[2]
comoving size of sound horizon at z_*	r_*	$^\dagger\, 144.43(26)$ Mpc	[2,38]
age when optical depth equals unity	t_*	$^\dagger\, 372.9(10)$ kyr	[2,37]
redshift at half reionization	$z_{\rm i}$	$^\dagger\, 7.7(7)$	[2,39]
age at half reionization	$t_{\rm i}$	$^\dagger\, 690(90)$ Myr	[2]
redshift when acceleration was zero	z_q	$^\dagger\, 0.636(18)$	[2,37]
age when acceleration was zero	t_q	$^\dagger\, 7.70(10)$ Gyr	[2]
age of the Universe today	t_0	$^\dagger\, 13.797(23)$ Gyr	[2]
effective number of neutrinos	$N_{\rm eff}$	$^\sharp\, 2.99(17)$	[2,40,41]
sum of neutrino masses	Σm_ν	$^\sharp < 0.12$ eV (95%, CMB + BAO); ≥ 0.06 eV (mixing)	[2,41–43]
neutrino density of the Universe	$\Omega_\nu = h^{-2}\Sigma m_{\nu_j}/93.14$ eV	$^\sharp < 0.003$ (95%, CMB + BAO); ≥ 0.0012 (mixing)	[2,42,43]
curvature	Ω_K	$^\sharp\, 0.0007(19)$	[2]
running spectral index, $k_0 = 0.05$ Mpc^{-1}	$dn_{\rm s}/d\ln k$	$^\sharp\, -0.004(7)$	[2]
tensor-to-scalar field perturbations ratio,	$r_{0.002} = T/S$	$^\sharp < 0.058$ (95% CL, $k_0 = 0.002$ Mpc^{-1}, no running)	[2,44,45]
dark energy equation of state parameter	w	$-1.028(31)$	[2,46]
primordial helium fraction	$Y_{\rm p}$	$0.245(4)$	[47]

‡ Parameter in 6-parameter ΛCDM fit; † Derived parameter in 6-parameter ΛCDM fit; $^\sharp$ Extended model parameter, *Planck* + BAO data [2].

References

[1] CODATA recommended 2018 values of the fundamental physical constants: `https://physics.nist.gov/cuu/Constants/index.html`.

[2] Planck Collab. 2018 Results VI (2018), [arXiv:1807.06209].

[3] O. Lahav & A.R. Liddle, "The Cosmological Parameters," Sec. 25.1 in this *Review*.

[4] *The Astronomical Almanac for the year 2020*.

[5] The astronomical unit of length (au) in meters is re-defined (IAU XXVIII General Assembly 2012, Resolution B2) to be a conventional unit of length in agreement with the value adopted in IAU XXVII 2009 Resolution B2. It is to be used with all time scales.

[6] The distance at which 1 au subtends 1 arc sec: 1 au divided by $\pi/648\,000$.

[7] IAU XVI GA 1976, Recommendations.

[8] The number of square degrees on a sphere is $360^2/\pi = 41\,259.9\ldots$.

[9] Observationally determined mass parameter $G_N M \times 2/c^2$ [1] for either the Sun or the Earth, where $\mathcal{GM}_\odot = 1.327\,124\,4 \times 10^{20}$ m^3 s^{-2} and $\mathcal{GM}_\oplus = 3.986\,004 \times 10^{14}$ m^3 s^{-2} [48].

[10] $G_N M \div G_N$ [1].

[11] IAU XXIX GA, 2015, Resolution B3, "on recommended nominal conversion constants . . . " Calligraphic symbol indicates recommended nominal value.

[12] See also G. Kopp & J.L. Lean, Geophys. Res. Lett. **38**, L01706 (2011), who give (1360.8 ± 0.6) W m^{-2}; see paper for caveats and other measurements.

[13] $4\pi\,(1\ \text{au})^2 \times \mathcal{S}_\odot$, assuming isotropic irradiance.

[14] S. Chandrasekhar, Astrophys. J. **74**, 81 (1931).