01 Sept 2020

Scalar tomography, single tomo axis: Derivation of $\frac{\partial \varepsilon}{\partial A}$ (1)

$\vec{r}$ - lab coord.

$\vec{r}_s$ - sample coord

Integer Model?

$\hat{R}(\vec{r}_s) = A(\vec{r}_s)$ - voxel int. model

$$\hat{I}_n(x,y) = \sum_{z}^{3D} A(\vec{r}_s) \quad \text{- forward model}$$

3D

↳ depends on orientation

$$\varepsilon = \sum_n \{\hat{I}_n(x,y) - I_n(x,y)\}^2 \quad \text{- cost function}$$

↳ Edit: has to sum to scalar

$$\vec{r}_s = \mathbb{R}_n^{exp} \vec{r}$$

$$r = (R_n^{exp})^{-1} \vec{r}_s$$

$$R_n^{exp} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

Rotation around y

(Note that this is different from the meeting with Manuel where rotation was around z!!

$$(R_n^{exp})^{-1} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

(2)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix}$$

$$= \begin{bmatrix} x_s\cos\alpha - z_s\sin\alpha \\ y_s \\ x_s\sin\alpha + z_s\cos\alpha \end{bmatrix}$$

$$\frac{\partial \varepsilon}{\partial A} = \sum_{n,x,y} \frac{\partial}{\partial A(\vec{r}_s')} \left\{ \hat{I}_n(x,y) - I_n(x,y) \right\}^2$$

Given pd
$r_s' = (x', y')$

$$= \sum_{n,x,y} 2 \left\{ \hat{I}_n(x,y) - I_n(x,y) \right\} \frac{\partial}{\partial A(\vec{r}_s')} \hat{I}_n(x,y)$$

$$= \sum_{n,x,y} 2 \left\{ \hat{I}_n(x,y) - I_n(x,y) \right\} \frac{\partial}{\partial A(\vec{r}_s')} \sum_{z} A(\vec{r}_s)$$

$$= \sum_{n,x,y} 2 \left\{ \hat{I}_n(x,y) - I_n(x,y) \right\} \sum_{z} \underbrace{\frac{\partial}{\partial A(\vec{r}_s')} A(\vec{r}_s)}_{\delta^3_{\vec{r}_s, \vec{r}_s'}}$$

Kronicker Dollc

$$= \sum_{n,x,y} \sum_{z} 2 \left\{ \hat{I}_n(x,y) - I_n(x,y) \right\} \delta^3_{\vec{r}_s, \vec{r}_s'}$$

$$= 2\sum_n \sum_{x,y,z} \{ \hat{I}_n(x,y) - I_n(x,y)\} \, \delta^3_{\vec{r}_s,\vec{r}_s'}$$

$$= 2\sum_n \sum_{x,y,z} \{ \hat{I}_n(x_s\cos\alpha - z_s\sin\alpha, y_s) - I_n(x_s\cos\alpha - z_s\sin\alpha, y_s)\} \, \delta^3_{\vec{r}_s,\vec{r}_s'}$$

In Sample Coords

$$= 2\sum_n \sum_{x_s,y_s,z_s} \{ \hat{I}_n(x_s\cos\alpha - z_s\sin\alpha, y_s) - I_n(x_s\cos\alpha - z_s\sin\alpha, y_s)\} \times \delta_{\vec{r}_s,\vec{r}_s'}$$

$$= 2\sum_n \{ \hat{I}_n(x_s'\cos\alpha - z_s'\sin\alpha, y_s') - I_n(x_s'\cos\alpha - z_s'\sin\alpha, y_s')\}$$

Only one nonzero coord

$$= 2\sum_n \{ \hat{I}_n(x',y') - I_n(x',y')\}$$

$$\frac{\partial \varepsilon}{\partial A(\vec{r}_s)} = \sum_n \{ \hat{I}_n(x,y) - I_n(x,y)\}$$

Test

$$= \sum_n \left\{ \sum_z A(r_s) - I_n(x,y) \right\} \; ?$$

one projection first. Then use arb-projection