Numerical Experiments of Glacial Inceptions in Northern Europe

Research Project

Presented in partial fulfillment of the requirements for the degree of

Master Applied Mechanics

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Abstract

Contents

1 Introduction

In today's climate , the CO_2 largely exceeds its natural variability recorded by ice cores over the last million years. Its concentrat highest since the Pliocene period, about 3 millions y sheet on Greenland, the one on Antarctic was significantly smaller and the sea level was 15 to 30 meters higher. (Haywood *et al.* (2011))

So what? What do you want to imply? Each pa

The global warming provokes the melting of ice caps, which contains more than 70% of Earth's freshwater.If all the ice were to melt completely, which is not expected any time soon, the sea level would rise by an estimated 65m and force populations to emigrate, their land being submerged by water. That is without taking into account the thermal expansion of water.(Fretwell *et al.* (2013) and Morlighem *et al.* (2017))

The melting of ice sheets have several impacts on the climate: It produces freshwater fluxes released into the ocean, which change locally the temperature and disrupts large scale ocean circulations. This can have an impact on region far from the polar zones, such as the Greenland meltwater altering the monsoon regime in West Africa (Defrance *et al.* (2017)). It also disrupts the positive feedback loop of ice's albedo. This is non negligible, for example, the decline of Arctic sea ice between 1979 and 2011 caused a forcing of the same order of magnitude as CO_2 emissions during this period (Pistone *et al.* (2014))

If these are indications of the future of today's ice sheets, then models of their evolution would be a valuable resources to try to predict what is to come, both on the climate and geography of the world. Therefore, my project will be to perform numerical simulations, using the Elmer/Ice. During this project, we will perform simulations, using the ElmerIce model, to study glacial inception at a continental scale, starting by an idealised model, to a realistic topography of Northern Europe.

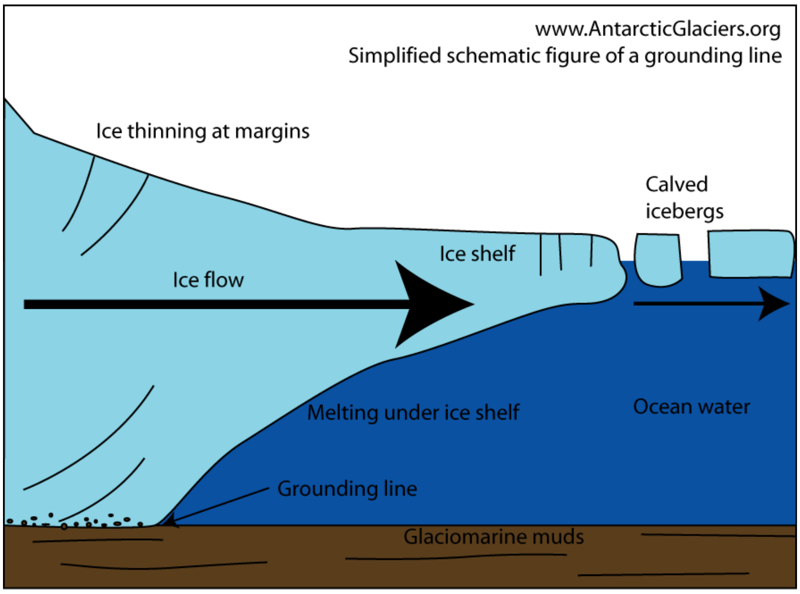

Figure 1: Ice Sheet Diagram from Davis (????)

Nice figure... But,

2 Physical Properties of Ice

An ice sheet, also called continental glacier, is a continuous sheet of land ice that covers a very large area. The difference with an ice cap is the scale, we talk about ice sheets on continental scales. It is formed by an accumulation of snow which will density under its weight, until it becomes ice. This ice will then flow downhill under its weight, and can eventually reach the sea. If it does, and the ice propagates above the sea, this part of the ice sheet will be called an ice shelf. The thickness of an ice sheet is comprised between 0 to a few thousands meters, like Greenland's, which is 3300m thick at most. A typical horizontal lengths of an ice sheet is of the order of magnitude of a 1000km, which makes a ratio of horizontal length over vertical length of more than 300. It is why we assume later that the flow is mostly horizontal.

As we have just seen, the ice is considered as a very viscous fluid. That is for very large time scales. We then start from the Stokes equation to get the Ice flow equation:

$$div\boldsymbol{\sigma} + \rho\boldsymbol{g} = div\boldsymbol{\tau} - gradp + \rho\boldsymbol{g} = 0 \tag{1}$$

and the mass conservation

$$div\boldsymbol{u} = tr\dot{\boldsymbol{\varepsilon}} = 0 \tag{2}$$

with $\boldsymbol{\sigma}$ the stress tensor, ρ the density of the ice, $\boldsymbol{g}$ the gravity vector, $\boldsymbol{\tau}$ the deviatoric stress tensor, with $\boldsymbol{\sigma} = \boldsymbol{\tau} - p\boldsymbol{I}$ and $p = tr\boldsymbol{\sigma}/3$

You write

and Glen's law :

$$\tau = 2\eta\dot{\varepsilon} \qquad \text{where} \qquad \eta = \frac{1}{2}(EA)^{\frac{-1}{n}}\dot{\varepsilon}_e^{\frac{(1-n)}{n}} \tag{3}$$

with A a rheological parameter depending on T', the ice temperature relative to the pressure melting point. E an enhancement factor used to account for an anisotropic effect, which is expected to be larger than 1 for grounded ice sheets, and lower than 1 for floating ice shelves. Gagliardini *et al.* (2013)

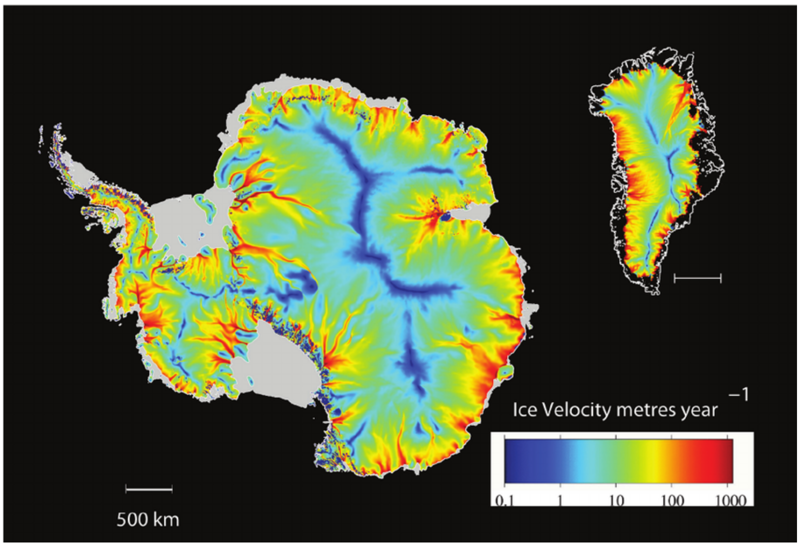

Figure 2: Ice velocities in Antarctica and Greenland from Allison *et al.* (2009)

3 Numerical Model

In order to model ice sheets, finite difference method(FDM) and finite element method(FEM) are commonly used. They are numerical methods, since most often analytical solutions are very difficult, or even impossible to obtain. Finite Difference Method consists in converting ordinary and partial differential equations into a system of linear equations by approximating derivatives as finite differences. Elmer/Ice, on the other hand, uses the Finite Element Method. It considers a continuum as an assembly of non-overlapping elements forming the same geometry, which makes the modelling of complex geometries possible. Each element is made of at least two points, on which are applied the forces and computed the displacements.

Some approximations are then made, depending on where in the flow we are located. One of them is the Shallow Ice Approximation. It assumes a large ratio of horizontal to vertical length, that the basal shear stress is balanced by the gravitational driving stress, and a large vertical to horizontal stress ratio. That represents a slow flow in the interior of an ice sheet. (see fig. 2])The approximation makes the method constitutionally cheap, and works well over long simulations. (Adhikari & Marshall (2012))

In fig. 2, where the flow rate is the highest, we cannot approximate that the basal shear stress is balanced by the gravity. We instead take the basal shear stress as zero and the longitudinal stress dominates. This is the Shallow Shelf approximation, initially developed for ice shelves, but which has been extended to dragging ice streams. It is a 2D vertically integrated model, the ice velocity being depth-averaged, which makes it inefficient where vertical variations of speed are important.

The most precise is the Full Stokes model, it accounts for all nine stress components. They are very useful around the parts that are at the limits of the others models, where vertical variations are important for example, but not needed for the interior of ice sheets, where the improvement would be minimal but the computational cost way higher. (Larour *et al.* (2012))

Finally, the mesh may also need to be adapted according to the position in the ice sheet we are interested in. One solution is an Adaptive mesh(Cornford *et al.* (2013)). In that regard, it is easier to implement into a finite element model than a finite difference. It is a complex process, and we will no need it for this project. The elements should have a size of about the ice sheet thickness, but are in use bigger, to save computational resource.(e.g. Golledge *et al.* (2014) uses 15km)

Elmer/Ice then uses solvers to solve the final equations. To each equation corresponds one solver to be referenced in the input file, together with the different parameters of the problem.

4 Numerical Setup

First, we will model two mountains, which will look like domes, separated by a valley. That will be done using different equations, such as sinusoidal or gaussian (fig. 3 and fig. 4). These domes will sit on a flat ground. Above this ground will later be added a sea level, that can be adjusted, to see how the ice cap will behave when it will come in contact to the water. On top of each mountain, a small volume of ice will be added at initial time, the simulation will then start from there, showing the evolution of the ice cap with the passing years. The first thing to model is the topography of the bedrock. In our idealised simulation, we want two different mountains, separated by a valley. They can be defined by several equations: A gaussian, as in figure 3, of equation

Several? You're

$$y(x) = H_1 * exp(-\frac{(x-x_0)^2}{\sigma^2}) + H_2 * exp(-\frac{(x-x_0-x_d)^2}{\sigma^2}) \tag{4}$$

(with H_1 and H_2 the height of mountains 1 and 2, x_0 the position of the first mountain and σ the mean deviation) And a sinusoidal, as in figure 4, of equation

$$y(x) = H_1 * sin(\frac{3\pi x}{L}) * H(-x + \frac{L}{3}) + H_2 * sin(\frac{3\pi x}{L}) * H(x - \frac{2L}{3}) \tag{5}$$

(with H(x) the heaviside function and L the domain length). Eventually, after we are done with the idealised models, we will use real world topography of Northern Europe instead of these functions.

In order to do these simulations, several parameters have to be known. First the viscosity, determined by Glen's law(equation (3)) with E=2.0, n=3 and A=4.9E-7s, the ice density $\rho = 0.92 g/cm^3$, the gravity $g = 9.8 m.s^{-2}$ and the mass accumulation of 10cm per year above 250m, and 0 below. Finally, a linear basalt friction $\boldsymbol{\tau} = \beta \boldsymbol{u}$ will be applied, with β the friction parameter and $\boldsymbol{u}$ the ice velocity. The first thing to do will be to vary these parameters, and especially the topography, the mass balance, the friction and the sea level, and analyse the results.

We will also have different parameters from the simulation to choose, depending on the precision of the results wanted and the resources allocated to it. The time step Δt will vary from 1 day to $\frac{1}{4}$ years, and the resolution $\Delta x = \Delta y$ from 2km to 100m. In our first idealised cases, the domain will be a squa L=75km.

No. You can have a PDE with all the boudary conditions and no solu d boundary conditions.

Not only. You won't model

then assume that the bedrock impermeable.

5 Timetable

Write this section using bullets.

From the end of January to the beginning of February, I will run the first idealised simulations, in order to get used to the software and input files. Then during the second part of February, I will hopefully know how make simple simulations, and proceed to run and analyse sensitivity experiments of the idealised cases, to understand the impact of each parameter on the flow. This will go on until the

No y-label assotiated? Simplify your figures. Put the same height, we are going to deal v

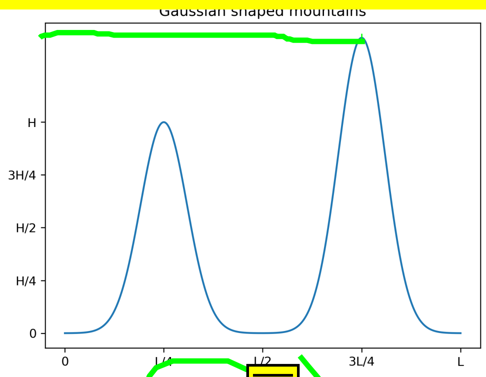

Figure 3

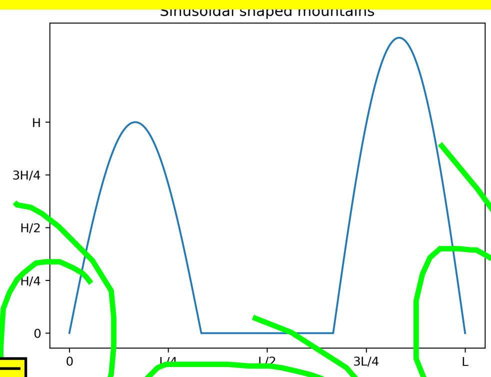

Figure 4

mid to end of March, when I will start applying what I have learned on idealised cases to a real topography, with parameters getting closer and closer to reality. This should end approximately during the week of the 10th of May, to spare some time to finalise the report, before handing it on the 20th of May

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