Numerical experiments in idealized glacier topographies

Case study of the impact of the mesh resolution on the prediction of the grounding line position

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Definition

Glaciers can be defined as a mass of ice that accumulates from snow and flows slowly downwards

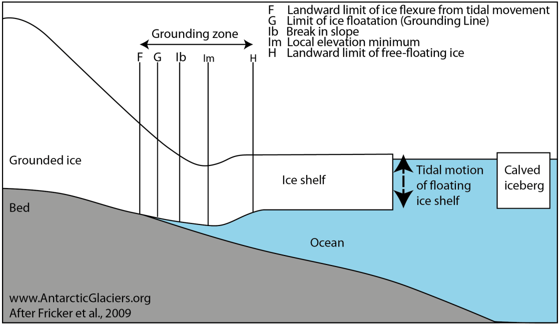

Figure: Schema of a tributary glacier where we can observe the different parts denoting the grounding zone (Fricker et al., 2009). Adapted from AntarticGlaciers.org

Process of modelling

The physical phenomena that impacts the dynamics of the glaciers can be represented using mathematical models that implement partial differential equations, which can then be discretized to be solved using numerical methods.

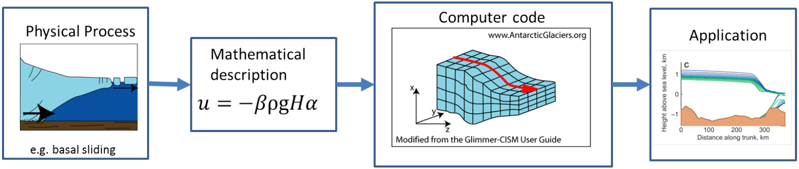

Figure: Process of modelling, starting with a physical phenomena which can be represented mathematically in a physical model than can then be discretized to solve numerically. Adapted from AntarticGlaciers.org

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Importance of understanding the dynamics of glaciers

- The rate of present-day sea-level rise has increased in recent decades and it is expected to continue increasing in coming decades and centuries (Clark et al., 2015).
- Morlighem et al. (2017) and Haywood et al. (2011) mentioned that if all the ice were to melt completely, the sea level would rise by an estimated of 65m.
- The Unites Nations states that around 40% of the world's population lives in coastal regions, within 100km of the coastline (Barbier, 2015; Montgomery, 2007).
- The land area that is less than 10m above sea level is just 2% of the world's total land area, yet it is home to 10% of the world's population and 13% of the world's urban population (Nevermann et al., 2023).

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Mass flux

The velocity out in the x-direction is:

$$U = u + \frac{\delta u}{\delta x} dx; \tag{1}$$

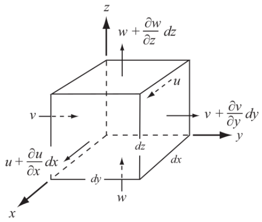

Figure: Derivation of the condition of incompressibility. Adapted from Hooke (2019).

Incompressibility condition

Mass fluxes into and out of a elementary control volume of ice in a glacier in the x-direction are:

$$(\rho u + \frac{\delta \rho u}{\delta x} dx) dydz; \tag{2}$$

Here, ρ is the density of the ice. Similar relations may be written for the mass fluxes into and out of the volume control in the y and z-directions. Summing these fluxes we find that the change in mass with time, $\frac{\delta m}{\delta t}$ in the control volume is:

$$-\frac{1}{dxdydz}\frac{\delta m}{\delta t} = \frac{\delta \rho u}{\delta x} + \frac{\delta \rho v}{\delta y} + \frac{\delta \rho w}{\delta z}; \tag{3}$$

The mass of ice in the control volume can change if the control volume is not full initially. When it is full of incompressible ice, however, $\frac{\delta m}{\delta t} = 0$, and the equation becomes:

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0; \tag{4}$$

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Deviatoric stress

The deviatoric normal stress in the x-direction is:

$$\sigma'_{XX} = \sigma_{XX} - P; \tag{5}$$

Where P is the mean normal stress:

$$P = -\frac{1}{3}(\sigma_{XX} + \sigma_{YY} + \sigma_{ZZ}) \tag{6}$$

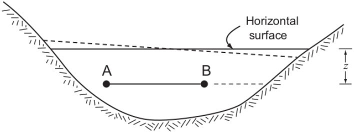

Figure: Sketch to illustrate non-hydrostatic pressure. Adapted from Hooke (2019).

The flow law and governing equations

The most commonly used flow law for ice is Glen's flow law, named after John W. Glen upon whose experiments it is based Glen (1958). This equation was originally written in the form:

$$\dot{\epsilon_e} = (\frac{\sigma_e}{B})^n; \qquad (7)$$

where B is a viscosity parameter that increases as the ice becomes stiffer, and n is an empirically determined constant. Most studies have found that n=3,. An alternative form of the flow law that is commonly used, and that can be used, is:

$$\dot{\epsilon_e} = A{\sigma_e}^n \qquad (8)$$

A is called the rate factor. B is normally given in Mpa yr$^{\frac{1}{n}}$ while A is in MPa^{-n} yr $^{-1}$ or MPa^{-n} s $^{-1}$.

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Governing equations I

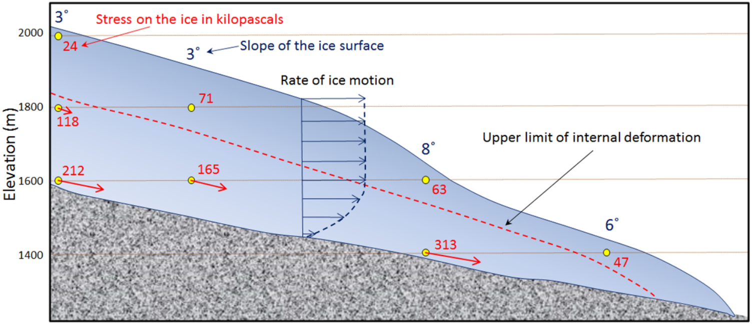

Figure: Stress within a valley glacier (red numbers) and the ice velocity (blue arrows). Figure adapted from Earle (2015)

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The plastic lower ice of the glacier can flow like a very viscous fluid. The incompressibility condition leads to:

$$\nabla u = 0 \tag{9}$$

For ice flow, the acceleration term can be neglected in the Navier-Stokes equations (Hutter, 1982). Therefore:

$$-\nabla p + \nabla(\eta(\nabla u + (\nabla u)^T)) + \rho g = 0; \tag{10}$$

Where η is the viscosity and g is the gravity. Letting σ denote the stress tensor and pressure p is the mean normal stress denoted previously, and the strain rate tensor ϵ_e, related by:

$$\sigma = 2\eta\epsilon_e - pI = \eta(\nabla u + (\nabla u)^T) - pI; \tag{11}$$

Where I is the identity tensor. Together, these two last mathematical equations are called the full-stokes model.

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