Numerical experiments of glacial inceptions in Northern Europe

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Northern Europe

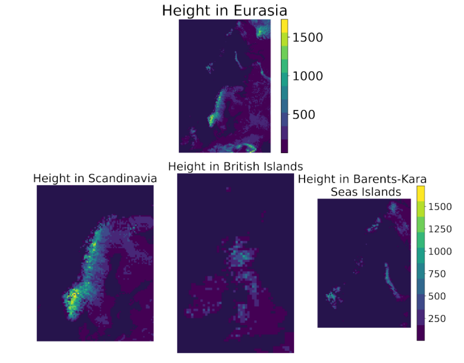


Figure: Elevation of the terrain in Eurasia in meters. The top row shows the topography used by Dainche (2022) for his study. On the second row, zooms have been performed on regions where glacial inceptions started. Plotted from the data from ETOPO1 projected into a cartesian grid at 20km resolution.

Idealised Northern Europe topography by Dainche (2022)

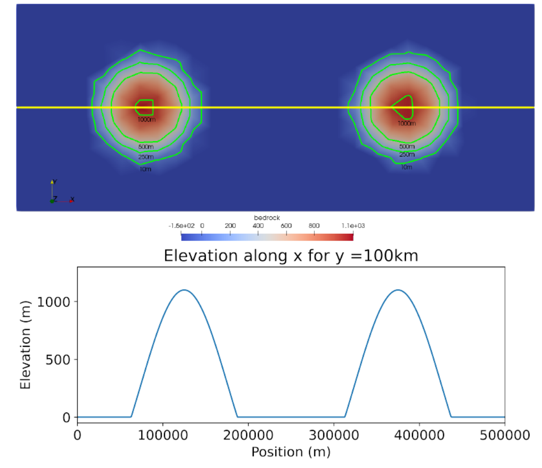


Figure: Topography of idealised system made of two Gaussian shaped mountains carried out by Dainche (2020).

Glaciers dynamics

For large time scales the ice is considered as a very viscous fluid. The ice flow equations are then derived from the Stokes equation:

$$div\sigma + \rho g = div\tau - gradp + \rho g = 0; \tag{1}$$

with σ the stress tensor, ρ the density of the ice, g the gravity vector, τ the deviatoric stress tensor, with $\sigma = \tau - pI$ and $p = \frac{tr\sigma}{3}$. And the mass conservation:

$$\frac{dh}{dt} + div(uH) = M_s + M_b; \tag{2}$$

With u the velocity, H the ice thickness, M_s and M_b the mass balance at the surface and at the bottom respectively. M_s will be defined, and M_b is considered as 0 for convenience purposes.

Glaciers dynamics

The stresses are related to the viscosity and the strain by the Glen's law:

$$\tau = 2\eta\dot{\epsilon}, \tag{3}$$

with the viscosity given by:

$$\eta = \frac{1}{2}(EA)^{\frac{-1}{n}}\dot{\epsilon_e}^{\frac{(1-n)}{n}}. \tag{4}$$

Where:

- The strain rate $\dot{\epsilon}$.
- Glen's constant.
- The enhancement factor to account for an anisotropic effect E.
- The rheological parameter A.

Grounding line

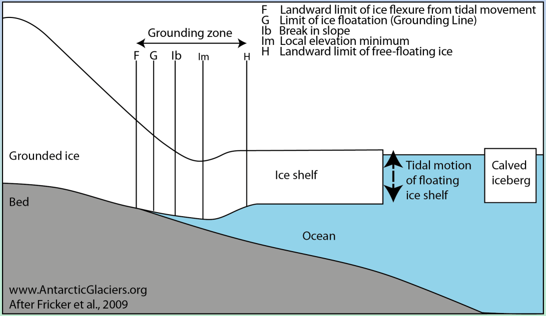


Figure: Schematic of a tributary glacier where we can observe the different parts denoting the grounding zone (Fricker et al., 2009).

Grounding line stability

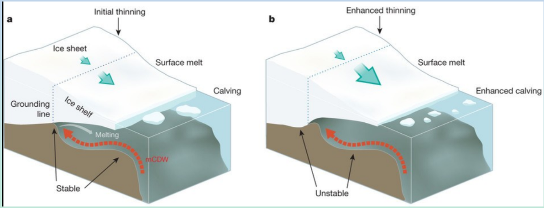


Figure: Schematic representation of the marine ice sheet instability with a) an initial state grounding line position and b) an unstable grounding line position (Hanna et al., 2013).

Objective

- Understand the direct impact of changes in the position of the grounding line on the flow dynamics of ice sheets glaciers.
- We will use the finite element model Elmer/Ice to explore the spatial resolution impact on the grounding line position.
- We will compare the changes in the position of the grounding line for two different idealised topographies. This will let us to understand the behavior of real present and past glaciers such as the Northern European glacier.

Idealised topographies: cone

This numerical experiment uses two different topographic profiles. A cone and a more realistic topography (thule). These idealised topographies are used in the world-wide model intercomparison project CalvinMIP (See `https://github.com/JRowanJordan/CalvingMIP/wiki/` for updates).

- Cone: the idealised model consists of a circular bedrock configuration given by:

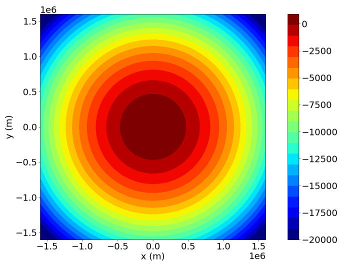
$$\theta = arctan2(y, x); \tag{5}$$

$$l = R - cos(2\theta)\frac{R}{2} \tag{6}$$

$$Bed_0 = Bc - (Bc - Bl)\frac{|x^2 + y^2|}{R2}^{;} \tag{7}$$

Where $R = 800x10^3m$, $Bc = 0.9x10^3m$, and $Bl = -2x10^3m$.

Idealised topographies: cone



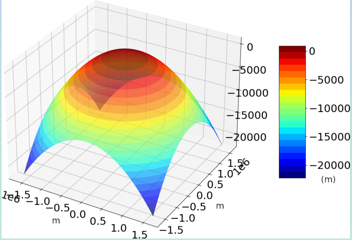


Figure: Circular bedrock topography. On the left side top view and on the right side, lateral view.

Idealised topographies: thule

- Thule: The thule bedrock configuration is given by:

$$\theta = arctan2(y, x); \tag{8}$$

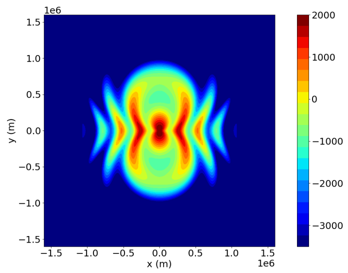
$$I = R - cos(2\theta)\frac{R}{2}; \tag{9}$$

$$Bed_0 = Bc - (Bc - BI)\frac{|x^2 + y^2}{R^2}; \tag{10}$$

$$Bed = Bacos(3\pi\frac{\sqrt[2]{x^2 + y^2}}{I}) + Bed_0; \tag{11}$$

With $R = 800x10^3 m$, $Bc = 0,9x10^3 m$, $BI = -2x10^3 m$, and $Ba = 1,1x10^3$.

Idealised topographies



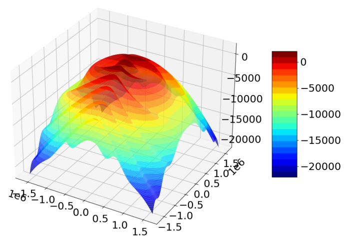


Figure: Thule bedrock topography 3D. On the left side the top view, and on the right side a lateral view.

Parameters

Variable	Description	Units
$g = 9.81$	gravitational acceleration	$\mathrm{m\,s^{-2}}$
$a_s = 0.3$	surface mass balance	$\mathrm{m\,a^{-1}}$
$a_s = 0$	basal mass balance	$\mathrm{m\,a^{-1}}$
$\rho = 917$	ice density	$\mathrm{kg\,m^{-3}}$
$\rho_o = 1030$	ocean density	$\mathrm{kg\,m^{-3}}$
$A = 2.9377 \times 10^{-9}$	ice rate factor	$\mathrm{kPa^{-3}\,a^{-1}}$
$n = 3$	flow law stress exponent	
$C = 0.001$	basal slipperiness	$\mathrm{m\,a^{-1}\,kPa^{-3}}$
$m = 3$	sliding law stress exponent	
$\mathrm{d2a} = 365.2422$	days in a year	days

Figure: Constants parameters based on Hilmar Gudmundsson's experiment for thule's configuration, used in MIP experiments.

Elmer/Ice finite element method

- Open source finite element code, mainly depeloped by the CSC in Finland.
- The ice sheet/ice flow model Elmer/ice is based on Elmer and includes developments related to glaciological problems. Elmer/ice includes a large number of dedicated solvers and users functions.
- Elmer/ice solves the full-stokes equations for various ice rheologies. It includes solvers for the classical asymptotical expansions of the stokes equations, namely the shallow ice approximation (SIA) and the shallow shelf approximation (SSA).

Grid and bedrock

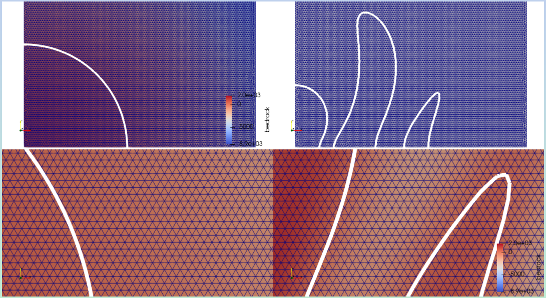


Figure: Elmer/Ice grid used to simulate one of the idealised topographies. On the left side the cone topography and on the right side the mesh for the simulation of the thule topography.

Boundary conditions

- The bedrock is impermeable (The vertical component of the ice flow velocity is 0).
- The flow follows Weertman friction law.
- Mass acumulation is a constant parameter.
- The simulation will be performed on a quarter of the domain, since the geometry of the topography is symmetric, which allows to have free slip boundary condition at the left and down side of the topography, and open boundary condition at the right and top side of the domain.

Work plan

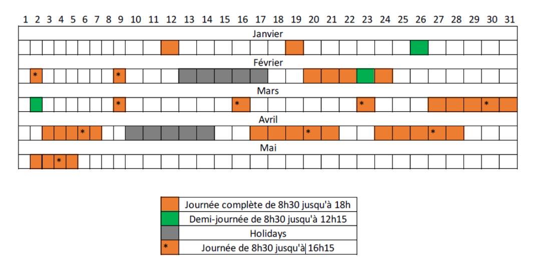


Figure: Tasks schedule of the project, showing the days we will spend working on the different stages of it.

Work plan

January

- Second and third week:Running the first idealised simulations.(10km and 5km resolution. 1 and 2 days, respectively.)

February

- First week: Simulations using a 2km resolution. (3-4 days.).
- Second week: 1km resolution (5 days).
- Fourth week: 500m and 250m resolutions (1-2 weeks resp.)

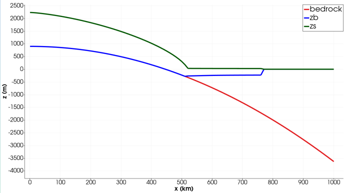
March

- First, second, and third week: Analysis of the simulations results.
- Fourth week: Writing the report.

April

- Writing the results.

Preliminary results



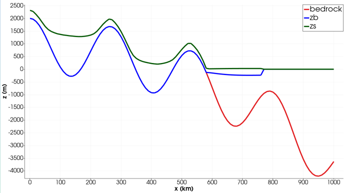


Figure: 10 km resolution simulation for cone (left side) and thule (right side) topographies. In green the upper heigh of the ice sheet, in blue the lower heigh, and in red the bedrock position.