5. CÁLCULO DIFERENCIAL EM $\mathbb{R}$ (SOLUÇÕES)

5.1.

a) $f'(x) = nx^{n-1}, n \in \mathbb{R};$ *b)* $f'(x) = \dfrac{1}{2\sqrt{x}};$ *c)* $f'(x) = e^x;$

d) $f'(x) = \cos x;$ *e)* $f'(x) = -senx;$ *f)* $f'(x) = \dfrac{1}{x}.$

5.2. *a)* Os pontos são $(-1,-1)$ e $\left(\dfrac{1}{2}, -\dfrac{31}{4}\right);$ *b)* Os pontos são $(0,-6)$ e $\left(-\dfrac{1}{2}, -\dfrac{11}{4}\right);$

c) O ponto de tangência é $(1,-5)$.

5.3. *a)* $g'(2) = -\dfrac{1}{9}.$ *b)* $y = -\dfrac{1}{9}x + \dfrac{5}{9}.$

5.4. $f'(1) = 1.$

5.5. $a = -2$ e $b = 4.$

5.6. $a = 1, b = 0$ e $c = -1.$

5.7. *a)* f não é diferenciável em $x = 0;$ *b)* f não é diferenciável em $x = 2;$

c) $f'(1) = +\infty.$

5.8.

a) $f'(x) = \dfrac{2}{\sqrt[3]{x}};$

b) $f'(x) = 3x^2 - 2x;$

c) $f'(x) = -\dfrac{2}{x^2} + \dfrac{2}{\sqrt[3]{x^2}};$

d) $f'(x) = \dfrac{x^2 - 6x + 1}{(x-3)^2};$

e) $f'(x) = (x-4)\left(\dfrac{5}{3}x^3 - 4x^2 + \dfrac{6}{5}\sqrt[5]{x} - \dfrac{4}{5\sqrt[5]{x^4}}\right);$

f) $f'(x) = 3\dfrac{(2-3x)^2}{(x-1)^4};$

g) $f'(x) = \dfrac{2\sqrt{x^5} + 3x^2 + 3x + 2\sqrt{x} + 1}{2\sqrt{x}\,(x + \sqrt{x})^2};$

h) $f'(x) = -3\ sen\ x \cos^2 x;$

i) $f'(x) = 2xe^{x^2+1};$

j) $f'(x) = \dfrac{1}{x};$

k) $f'(x) = \sqrt{\ln x} + \dfrac{1}{2\sqrt{\ln x}};$

l) $f'(x) = -\dfrac{2\cos x}{sen^3 x};$

m) $f'(x) = \dfrac{2}{x^2+1} - \dfrac{1}{x^2}\ln(x^2+1);$

n) $f'(x) = \dfrac{e^x}{\cos^2(e^x)};$

o) $f'(x) = 2;$

p) $f'(x) = \dfrac{2x}{\cos^2(x^2-1)};$

q) $f'(x) = \dfrac{1}{x\ sen x} - \dfrac{\cos x \ln(2x)}{sen^2 x};$

r) $f'(x) = e^{\cos^2 x}\left[1 - x\ sen(2x)\right];$

s) $f'(x) = \dfrac{3\,(2x+1)\cos(3x+5) - 2sen(3x+5)}{(2x+1)^2};$

t) $f'(x) = \dfrac{2e^{2x+1}\left[sen\,(2x+1) + \cos(2x+1)\right]}{\cos^2(2x+1)};$

u) $f'(x) = \left[2 - \dfrac{1}{(x-1)^2}\right] e^{(x-1)^2};$

v) $f'(x) = 2x\ arctg\ x + \dfrac{x^2}{x^2+1};$

x) $f'(x) = \dfrac{1}{x \ln x};$

z) $f'(x) = -\dfrac{x}{\sqrt{1-x^2}}.$

5.10. $a = 4,\ \ b = -4$ e $f'(x) = \begin{cases} 2x & \text{se } x < 2, \\ 4 & \text{se } x \geq 2, \end{cases}$

5.11. $f'(x) = \begin{cases} 2xsen\left(\dfrac{1}{x}\right) - \cos\left(\dfrac{1}{x}\right) & \text{se } x \neq 0, \\ 0 & \text{se } x = 0. \end{cases}$

5.13.

a) $-f'(-x);$

b) $e^x f'(e^x);$

c) $\dfrac{2x}{x^2+1} f'(\ln(x^2+1));$

d) $f'(x) f'\left[f(x)\right].$

5.14. $(arcsen\ x)' = \dfrac{1}{\sqrt{1-x^2}},$ $\quad (\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$ e $(arc\cot g\ x)' = -\dfrac{1}{1+x^2}.$

5.15. $\dfrac{f'(x)}{1+f^2(x)} + \dfrac{1}{1+x^2} f'(arctg\ x).$

5.18. $c = 0.$

5.22. $10 + \dfrac{5}{22} < \sqrt{105} < 10 + \dfrac{1}{4}.$

5.23.

a) 1; *b)* 4; *c)* $-\frac{1}{6}$; *d)* $\frac{4}{3}$; *e)* $-\sqrt{3}$;

f) 1; *g)* 0; *h)* 0; *i)* $\frac{1}{2}$; *j)* 1;

k) 1; *l)* 0; *m)* 1; *n)* $sen(5)$; *o)* $\frac{1}{2}$;

p) 0; *q)* 1; *r)* $+\infty$.

5.24. $f(x)$ é monótona decrescente em $\left]-\infty, \frac{3}{2}\right[$ e monótona crescente em $\left]\frac{3}{2}, +\infty\right[$. Além disso, tem a concavidade voltada para cima em $]-\infty, 0[\, \cup \left]\frac{1}{2}, +\infty\right[$ e a concavidade voltada para baixo em $\left]0, \frac{1}{2}\right[$.

5.25.

a) $f^{(n)}(x) = sen\left(x + \frac{n\pi}{2}\right)$;

b) $f^{(n)}(x) = 2^n \cos\left(2x + \frac{n\pi}{2}\right)$;

c) $f^{(n)}(x) = (-1)^n \frac{n!}{(1+x)^{n+1}}$;

d) $f^{(n)}(x) = (-1)^{n+1} \frac{(n-1)!}{(1+x)^n}$;

e) $f^{(n)}(x) = (-1)^n (x-n)e^{-x}$;

f) $f^{(n)}(x) = 0$.

5.26. $p_n(x) = 1 - \frac{1}{2!}\left(x - \frac{\pi}{2}\right)^2 + \frac{1}{4!}\left(x - \frac{\pi}{2}\right)^4 - \frac{1}{6!}\left(x - \frac{\pi}{2}\right)^6$.

5.27.

a) $p_n(x) = -1 + x^3$;

b) $p_n(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$;

c) $p_n(x) = 1 - x + x^2 + \cdots + (-1)^n x^n$;

d) $p_n(x) = 1 - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n+1}\frac{x^n}{n}$;

e) $p_n(x) = \frac{1}{e} + \frac{5}{e}x + \frac{5^2}{2!e}x^2 + \cdots + \frac{5^n}{n!e}x^n$;

f) $p_n(x) = sen(3) + 2sen\left(3 + \frac{\pi}{2}\right)x + 2^2 sen\left(3 + \pi\right)\frac{x^2}{2} + \cdots + 2^n sen\left(3 + n\frac{\pi}{2}\right)\frac{x^n}{n!}$.