6. PRIMITIVAÇÃO (SOLUÇÕES)

6.1.

a) $\frac{3}{2}x^2 + 2x$;

b) $\frac{\left(x^3-3\right)^3}{9}$;

c) $\frac{x^5}{5} - \frac{2}{3}x^3 + x$;

d) $\frac{5}{7}\sqrt[5]{x^7}$;

e) $\frac{e^{x^3}}{3}$;

f) $\frac{3}{16}\sqrt[3]{(1+2x^2)^4}$;

g) $-\frac{5}{12}\sqrt[5]{(2-3x)^4}$;

h) $\frac{5^x}{\ln 5}$;

i) $\frac{1}{2}\ln\left(1+x^2\right)$;

j) $-\ln|\cos x|$;

k) $\frac{1}{3}arc\ tg\left(x^3\right)$;

l) $\frac{1}{2}\ln|2tgx+1|$;

m) $\frac{\ln^2 x}{2}$;

n) $\frac{2}{1-senx}$;

o) $sen\left(\ln x\right)$;

p) $-arc\ tg\left(\cos x\right)$;

q) $-e^{\frac{1}{x}}$;

r) $\frac{1}{2}\left(tg\ x + x\right)$;

s) $\frac{1}{2}sen\left(e^{2x}\right)$;

t) $\frac{1}{\sqrt{5}}arc\ tg\left(\frac{x}{\sqrt{5}}\right)$;

u) $\frac{1}{3}arc\ tg^2\left(\frac{x}{3}\right)$;

v) $arc\ tg\left(e^x\right)$;

w) $\frac{1}{2}arc\ sen\left(x^2\right)$;

x) $\ln\left|tg\left(\frac{x}{2}\right)\right|$;

y) $-x + tg\ x$;

z) $\frac{1}{4}\arg\ senh\left(x^4\right)$

6.5.

a) $-x\cos x + senx$, em $\mathbb{R}$;

b) $\frac{e^x}{2}\left(\cos x + senx\right)$, em $\mathbb{R}$;

c) $e^{x+2}\left(x-1\right)$, em $\mathbb{R}$;

d) $e^x\left(x^2-2x+2\right)$, em $\mathbb{R}$;

e) $x\left[\ln\left(2x\right)-1\right]$ em $\mathbb{R}^+$;

f) $\frac{senx\cos x + x}{2}$, em $\mathbb{R}$;

g) $\frac{1}{2}\left[sen\left(4x-1\right)+\frac{1}{4}\cos\left(4x-1\right)\right]$, em $\mathbb{R}$;

h) $xarc\ tg\left(2x\right) - \frac{1}{4}\ln\left(1+4x^2\right)$, em $\mathbb{R}$;

i) $xarc\ sen\ x + \sqrt{1-x^2}$, em $[-1,1]$;

j) $\frac{x^2}{2}\left(\ln x^2 - 1\right)$ em $\mathbb{R}\setminus\{0\}$;

k) $\frac{1}{2}\left(xsen^2x + \frac{\cos x senx}{2} - \frac{x}{2}\right)$, em $\mathbb{R}$; *l)* $\frac{1}{4}\left(\frac{3}{2}x - \frac{3}{2}senx\cos x - sen^3x\cos x\right)$, em $\mathbb{R}$;

m) $x\left(\ln^2 x - 2\ln x + 2\right)$ em $\mathbb{R}^+$; *n)* $\frac{2}{3}\sqrt{2+x^3} - \frac{4}{9}\sqrt{(2+x^3)^3}$, em $\left]-\sqrt[3]{2}, +\infty\right[$;

o) $\frac{x}{2}\left[sen\left(\ln x\right) - \cos\left(\ln x\right)\right]$, em $\mathbb{R}^+$. *p)* $\frac{1}{2}\left(x^2 arctgx - x + arctgx\right)$, em $\mathbb{R}$;

q) $x\arccos\ x - \sqrt{1-x^2}$, em $[-1,1]$;

6.6.

a) $-2\cos\left(\sqrt{x}\right) + c,\ c \in \mathbb{R}$; *b)* $\frac{2}{9}\left(\frac{\sqrt{(1+3x)^5}}{5} - \frac{\sqrt{(1+3x)^3}}{3}\right) + c\ c \in \mathbb{R}$;

c) $\frac{arc\ sen\ x}{2} - \frac{x}{2}\sqrt{1-x^2} + c,\ c \in \mathbb{R}$; *d)* $2arctg\left(\sqrt{e^x - 1}\right) + c,\ c \in \mathbb{R}$;

e) $-\frac{1}{\sqrt{1+x^2}} + c,\ c \in \mathbb{R}$; *f)* $\frac{\sqrt{3}}{3}\operatorname{arcsec}\frac{\sqrt{3}x}{3} + c,\ c \in \mathbb{R}$;

g) $\ln(2x) - \ln 2\ln\left|\ln(4x)\right| + c,\ c \in \mathbb{R}$; *h)* $\frac{2}{3}\sqrt{(1+\ln x)^3} - 2\sqrt{1+\ln x} + c,\ c \in \mathbb{R}$;

i) $\frac{4}{3}\sqrt{(1+\sqrt{x})^3} + c,\ c \in \mathbb{R}$.

6.7.

a) $f(x) = x^4 + \frac{x^3}{3} - 3x^2 + x + 1$;

b) $f(x) = \frac{1}{2}\ln\left(1+x^2\right) + 2$;

c) $f(x) = xarc\ tg\ x - \frac{1}{2}\ln\left(1+x^2\right) + 2x - 1$.

6.8. $3m/s^2$. **6.9.***a)* $8m/s$. *b)* $18m$.

6.10.

a) $\ln|x+1| + c,\ c \in \mathbb{R}$, em $\mathbb{R}\setminus\{-1\}$;

b) $\frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1| + c,\ c \in \mathbb{R}$, em $\mathbb{R}\setminus\{-1\}$;

c) $x + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + c,\ c \in \mathbb{R}$, em $\mathbb{R}\setminus\{-1,1\}$;

d) $2\ln|x-1| - \ln|x| - \ln|x+1| + c,\ c \in \mathbb{R},\ \text{em } \mathbb{R}\setminus\{-1,0,1\}$;

e) $2\ln\left|\frac{x+2}{x+1}\right| - \frac{4}{x+2} + c,\ c \in \mathbb{R},\ \text{em } \mathbb{R}\setminus\{-2,-1\}$;

f) $\frac{1}{2}\ln|x-1| - \frac{1}{4}\ln\left(x^2+1\right) - \frac{1}{2}arc\ tg\ x + c,\ c \in \mathbb{R},\ \text{em } \mathbb{R}\setminus\{1\}$;

g) $\frac{x^2}{2} - 2x + \frac{16}{3}\ln|x+2| + \frac{1}{6}\ln|x-1| - \frac{1}{2}\ln|x+1| + c,\ c \in \mathbb{R},\ \text{em } \mathbb{R}\setminus\{-2,-1,1\}$;

h) $\frac{1}{2}\ln\left(x^2+2x+3\right) - \frac{1}{\sqrt{2}}arc\ tg\left(\frac{x+1}{\sqrt{2}}\right) + c,\ c \in \mathbb{R},\ \text{em } \mathbb{R}$;

i) $2arc\ tg\ x + \frac{2\sqrt{3}}{3}arc\ tg\left(\frac{2}{\sqrt{3}}x + \frac{1}{\sqrt{3}}\right) + c,\ c \in \mathbb{R},\ \text{em } \mathbb{R}$.

6.11.

a) $\ln|3\sqrt[3]{x}+1| + c,\ c \in \mathbb{R}$;

b) $-\frac{1}{2}\frac{1}{\sqrt{x^4-1}} + c,\ c \in \mathbb{R}$;

c) $-\frac{\sqrt{4-x^2}}{x} - arc\ sen\left(\frac{x}{2}\right) + c,\ c \in \mathbb{R}$;

d) $-\ln|1+\cos x| + c,\ c \in \mathbb{R}$;

e) $\ln|senx + \cos x| + c,\ c \in \mathbb{R}$;

f) $\frac{\sqrt{3}}{3}\ln\left|\frac{3e^x-\sqrt{3}}{3e^x+\sqrt{3}}\right| + c,\ c \in \mathbb{R}$;

g) $\ln\left|\frac{\ln x - 1}{\ln x}\right| - \frac{1}{\ln x - 1} + c,\ c \in \mathbb{R}$;

h) $\ln\left|\frac{e^x-1}{e^x}\right| + c,\ c \in \mathbb{R}$;

i) $\frac{6}{7}\sqrt[6]{(x-1)^7} + \frac{6}{5}\sqrt[6]{(x-1)^5} + 2\sqrt{x-1} + 6\sqrt[6]{x-1} + 3\ln\left|\frac{\sqrt[6]{x-1}-1}{\sqrt[6]{x-1}+1}\right| + c$;

k) $\frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right| + c,\ c \in \mathbb{R}$;

l) $\ln\left|\dfrac{tg\left(\frac{x}{2}\right)-1}{tg\left(\frac{x}{2}\right)}\right|+c,\ c\in\mathbb{R};$

m) $2\sqrt{x}-4\sqrt[4]{x}+4\ln|\sqrt[4]{x}+1|+c,\ c\in\mathbb{R};$

n) $\dfrac{6}{7}\sqrt[6]{x^7}-\dfrac{6}{5}\sqrt[6]{x^5}-\dfrac{3}{2}\sqrt[3]{x^2}+2\sqrt{x}+3\sqrt[3]{x}-6\sqrt[6]{x}+6arctg\left(\sqrt[6]{x}\right)-3\ln|\sqrt[3]{x}+1|+c,\ c\in\mathbb{R};$

o) $\dfrac{arctg^5x}{5}+c,\ c\in\mathbb{R};$

p) $-\dfrac{2}{e^x}+\dfrac{1}{2}\ln\left(e^{2x}+3\right)+\dfrac{\sqrt{3}}{3}arctg\left(\dfrac{e^x}{\sqrt{3}}\right)+c,\ c\in\mathbb{R};$

q) $\dfrac{2}{5}\sqrt{(x-1)^5}+\dfrac{2}{3}\sqrt{(x-1)^3}+c,\ c\in\mathbb{R};$

r) $2\sqrt{x}-x-\ln|1+2\sqrt{x}|+c,\ c\in\mathbb{R};$

s) $x+c,\ c\in\mathbb{R};$

t) $\ln|\ln x|+c,\ c\in\mathbb{R};$

u) $\ln|x|\ln(\ln x)+c,\ c\in\mathbb{R}.$

6.12.

a) $f(x)=x^3+\ln x+1,\ \ em\ \ (0,+\infty);$

b) $f(x)=\dfrac{x^4}{4}+x\ln x-x-e^3x+e^3+\dfrac{11}{4},\ em\ (0,+\infty).$

6.13. $g(x)=\ln\left(1+\dfrac{1}{e^x}\right)+\dfrac{\pi}{2};$