1) $\lim_{(x,y)\to(5,-1)} (x^2+y^2)$

Solución

$= 5^2+(-1)^2$ $\quad \lim_{(x,y)\to(5,-1)} = 26$

$= 25+1$

$= 26$

3) $\lim_{(x,y)\to(0,0)} \dfrac{5x^2+y^2}{x^2+y^2}$

$\Rightarrow \dfrac{5(0)^2+(0)^2}{(0)^2+(0)^2}$ $\quad x\Rightarrow 0 \quad \dfrac{5(0)^2+y^2}{0^2+y^2} \Rightarrow \dfrac{y^2}{y^2} = 1$

$\Rightarrow \dfrac{0}{0}$ $\quad y\Rightarrow 0 \quad \dfrac{5x^2+0^2}{x^2+0^2} = \dfrac{5\cancel{x}}{\cancel{x}} \Rightarrow 5$

No existe

5) $\lim_{(x,y)\to(1,1)} \dfrac{4-x^2-y^2}{x^2+y^2} = \dfrac{4-1^2-1^2}{1^2+1^2}$

$\Rightarrow \dfrac{2}{2}$ $1 \Rightarrow \lim_{(x,y)\to(1,1)} = 1$

2) $$\lim_{(x,y)\to(2,1)} \frac{x^2-y}{x-y} = \frac{2^2-1}{2-1} = \frac{3}{1} = 3$$

$$\lim_{(x,y)\to(2,1)} = 3$$

4) $$\lim_{(x,y)\to(1,2)} = \frac{4x^2+y^2}{16x^4+y^4} = \frac{4(1)^2+2^2}{16(1)^4+2^4} = \frac{8}{20}$$

$$\lim_{(x,y)\to(1,2)} = \frac{8}{20}.$$

6) $$\lim_{(x,y)\to(0,0)} = \frac{2x^2-y}{x^2+2y^2} \quad \frac{2(0)^2-(0)}{(0)^2+2(0)^2} = \frac{0}{0}$$

$$x \Rightarrow 0 \quad \frac{2(0)^2-y}{(0)+2y^2} = \frac{-\cancel{y}}{2y^{\cancel{2}}} = \frac{-1}{2y}$$

$$y \Rightarrow 0 \quad \frac{2x^2-0}{x^2+2(0)^2} = \frac{2\cancel{x^2}}{\cancel{x^2}} \Rightarrow 2$$

No existe

1) $z = 7x + 8y^2$

$\frac{\partial z}{\partial x} = ?\ \frac{\partial x}{\partial y} = ?$

$\frac{\partial z}{\partial x} = \lim_{h\to 0} \frac{f(x+h, y) - f(x,y)}{h}$

$\lim_{h\to 0} = \frac{[7(x+h) + 8y^2] - [7x + 8y^2]}{h}$

$\lim_{h\to 0} = \frac{[7x + 7h + 8y^2] - [7x + 8y^2]}{h}$

$\lim_{h\to 0} = \frac{\cancel{7x} + 7h + \cancel{8y^2} - \cancel{7x} - \cancel{8y^2}}{h}$

$\lim_{h\to 0} = \frac{7\cancel{h}}{\cancel{h}} \Rightarrow f_x = 7$

$\frac{\partial z}{\partial y} = \lim_{h\to 0} = \frac{[7x + 8(y+h)^2] - [7x + 8y^2]}{h}$

$\lim_{h\to 0} = \frac{7x + 8(y^2 + 2yh + h^2) - 7x - 8y^2}{h}$

$\lim_{h\to 0} = \frac{\cancel{7x} + \cancel{8y^2} + 16yh + 8h^2 - \cancel{7x} - \cancel{8y^2}}{h}$

$\lim_{h\to 0} = \frac{16yh + 8h^2}{h} \Rightarrow \frac{\cancel{h}(16y + 8h)}{\cancel{h}} \Rightarrow f_y = 16y$

(2) $z = xy$

$$\frac{\partial z}{\partial x} = \lim_{h\to 0} = \frac{[(x+h)\,y] - [xy]}{h}$$

$$= \frac{\cancel{xy} + hy - \cancel{xy}}{h}$$

$$= \frac{\cancel{h}y}{\cancel{h}} = y = f_x$$

(3) $z = 3x^2y + 4xy^2$

$$f_x \Rightarrow \lim_{h\to 0} \frac{[3(x+h)^2y + 4xy^2] - [3x^2y + 4xy^2]}{h}$$

$$\lim_{h\to 0} = \frac{[3(x^2+2xh+h^2)y + 4xy^2 - 3x^2y - 4xy^2}{h}$$

$$\lim_{h\to 0} = \frac{\cancel{3x^2y} + 2xhy + 3h^2y + \cancel{4xy^2} - \cancel{3x^2y} - \cancel{4xy^2}}{h}$$

$$\lim_{h\to 0} = \frac{2xhy + 3h^2y}{h} \Rightarrow \frac{\cancel{h}(2xy + 3hy)}{\cancel{h}} = 2xy \cancel{+3hy}$$

$$= 2xy = f_x$$

$$f_y = \lim \frac{[3x^2(y+h) + 4x(y+h)^2] - [3x^2y + 4xy^2]}{h}$$

$$\lim_{h\to 0} = \frac{[3x^2y + 3x^2h + 4x(y^2 + 2yh + h^2)] - [3x^2y + 4xy^2]}{h}$$

$$\lim_{h\to 0} = \frac{[\cancel{3x^2y} + 3x^2h + \cancel{4xy^2} + 8xyh + 4xh^2 - \cancel{3x^2y} - \cancel{4xy^2}}{h}$$

$$\lim_{h\to 0} = \frac{3x^2h + 8xyh + 4xh^2}{h} \Rightarrow \frac{\cancel{h}(3x^2 + 8xy + 4xh)}{\cancel{h}}$$

$$\lim_{h\to 0} = 3x^2 + 8xy + \cancel{4xh} = 3x^2 + 8xy = f_y$$

(4) $z = \dfrac{x}{x+y}$

$$\lim_{h\to 0} = \frac{\left[\dfrac{x+h}{(x+h)+y}\right] - \left[\dfrac{x}{x+y}\right]}{h}$$

$$\lim_{h\to 0} = \frac{\dfrac{(x+h)(x+y) - (x)(x+h+y)}{(x+h+y)(x+y)}}{h}$$

$$\lim_{h\to 0} = \frac{\dfrac{x^2 + xy + hx + hy - (x^2 - xh - xy)}{(x+h+y)(x+y)}}{\dfrac{h}{1}}$$

$$\lim_{h\to 0} \frac{\cancel{x^2} + \cancel{xy} + hx + hy - \cancel{x^2} - xh - \cancel{xy}}{h(x+h+y)(x+y)}$$

$$\frac{hx + hy - xh}{h(x+h+y)(x+y)}$$

$$\lim_{h\to 0} \frac{\cancel{h}(x+y-x)}{\cancel{h}(x+h+y)(x+y)} \Rightarrow \frac{\cancel{x}+y-\cancel{x}}{(x+h+y)(x+y)}$$

$$\lim_{h\to 0} \frac{y}{(x+0+y)(x+y)} = \frac{y}{(x+y)(x+y)} = \frac{y}{(x+y)^2}$$

(5) $z = x^2 - xy^2 + 4y^5$

$f_x = 2x - y^2$	$f_{xx} = 2$	$f_{xxx} = 0$
$f_y = 2xy + 20y^4$	$f_{yy} = 2x + 80y^3$	$f_{yyy} = 240y$

(6) $z = -x^3 + 6x^2y^3 + 5y^2$

$f_x = -3x^2 + 12xy^3$	$f_{xx} = -6x + 36xy^2$	$f_{xxx} = 72x$
$f_y = 18x^2y^2 + 10y$	$f_{yy} = 36x + 10$	$f_{yyy} = 0$

(7) $z = 5x^4y^3 - x^2y^6 + 6x^5$

$f_x = 20x^3y^3 - 2xy^6 + 30x^4$
$f_{xx} = 60x^2y^3 - 2y^6 + 120x^3$
$f_{xxx} = 120xy^3 + 360x^2$

$f_y = 15x^4y^2 - 6x^2y^5$
$f_{yy} = 30x^4y - 30x^2y^4$
$f_{yyy} = 30x^4 - 120x^2y^3$

8) $z = \tan(x^3y^2)$

$f_x = \sec^2(x^3y^2)\cdot(3x^2y^2)$

$f'(x) = 3x^2y^2\sec^2(x^3y^2)$

$f''(x) = 6x\ 2(\sec x^3y^2)(\sec x^3y^2\cdot\tan x^3y^2)\cdot 3x^2y^2$

$f''(x) = 2(3x^2y^2)(6x)\sec^2 x^3y^2\cdot\tan x^3y^2$

$f''(x) = (6x^2 2y^2)(6x)\ \sec^2 x^3y^2\cdot\tan x^3y^2$

$f''(x) = 36x^2 12xy^2\sec^2 x^3y^2\cdot\tan x^3y^2$

$f''(x) = 36x^2 12xy^2\sec^2 x^3y^2\cdot\tan x^3y^2$

$f'''(x) = (72x\ 12y^2)2(\sec x^3y^2)(\sec x^3y^2\cdot\tan x^3y^2)\cdot(3x^2y^2)$

$f'''(x) = (72x\ 12y^2)\ 2(3x^2y^2)(\sec^2 x^3y^2\cdot\tan x^3y^2\cdot(\sec^2(x^3y^2)\cdot$
$(3x^2y^2))$

$f'''(x) = (72x\ 12y^2)\ 2(3x^2y^2)(3x^2y^2)\ \sec^2 x^3y^2\cdot\tan x^3y^2\cdot$
$\sec^2 x^3y^2$

$f'''(x) = (72x\ 12y^2)\ 2(3x^2y^2)(3x^2y^2)\sec^4 x^3y^2\cdot\tan x^3y^2$

$f'''(x) = (72x\ 12y)\ 2(3x^4y^4)\sec^4 x^3y^2\cdot\tan x^3y^2$

$f'''(x) = (72x\ 12y)(6x^4 2y^4)\sec^4 x^3y^2\tan x^3y^2$

$f'''(x) = (432x^5\ 144xy^4\ 72yx^4\ 24y^5)\sec^4 x^3y^2\tan x^3y^2$

(9) $\dfrac{4\sqrt{x}}{3y^2+1}$

$f(x) = \dfrac{4}{3x^2+1} \cdot \dfrac{1}{2} x^{\frac{1}{2}-1}$

$f(x) = \dfrac{4}{3y^2+1} \cdot \dfrac{1}{2} x^{-1/2}$ $f'(x) = \dfrac{4}{2\sqrt{x}(3y^2+1)}$

$f'(x) = \dfrac{2}{\sqrt{x}(3y^2+1)}$

$f'(x) = \dfrac{2}{\sqrt{x}(3y^2+1)}$

$f''(x) = \dfrac{1}{2} x^{-1/2} \cdot \dfrac{2}{(3y^2+1)} \Rightarrow \dfrac{1}{2\sqrt{x}} \cdot \dfrac{2}{(3y^2+1)}$

$f''(x) = \dfrac{2}{2\sqrt{x}(3y2+1)} \Rightarrow \dfrac{1}{\sqrt{x}(3y^2+1)}$

$f''(x) = \dfrac{1}{\sqrt{x}(3y^2+1)}$

$f'''(x) = \dfrac{1}{2} x^{-1/2} \cdot \dfrac{1}{(3y^2+1)}$ $f'''(x) = \dfrac{1}{2\sqrt{x}} \cdot \dfrac{1}{(3y^2+1)}$

$f'''(x) = \dfrac{1}{2\sqrt{x}(3y^2+1)}$

(10) $z = 4x^3 - 5x^2 + 8x$

$f'(x) = 12x^2 - 10x + 8$

$f''(x) = 24x - 10$

$f'''(x) = 24.$

(11) $z = (x^3 - y^2)^{-1}$

$f'(x) = -\frac{1}{(x^3-y^2)^2} \Rightarrow -\frac{1}{(x^3-y^2)} \cdot 3x^2$

$f'(x) = -\frac{3x^2}{(x^3-y^2)}$

$f'(x) = \frac{3x^2}{(x^3-y^2)} \Rightarrow \left(\frac{u}{v}\right) \frac{u'v - uv'}{v^2}.$

$f''(x) = 3 \frac{x^2}{x^3+y^2}$ $f''(x) = \frac{(2x)(x^3-y^2)-(3x^2)(x^2)}{(x^3-y^2)^2}$

$f''(x) = \frac{2x^4 - 2xy^2 - 3x^4}{(x^3-y^2)^2} = \frac{-x^4 - 2xy^2}{(x^3-y^2)^2}$

$f''(x) = \frac{3(-x^4 - 2xy^2)}{(x^3-y^2)^2}.$

$f''(x) = \frac{3(-x^4 - 2xy^2)}{(x^3-y^2)^2}$ $\left(\frac{u}{v}\right) \frac{u'v - uv'}{v^2}$