Informe: Optimizacion Convexa Taller No. 4

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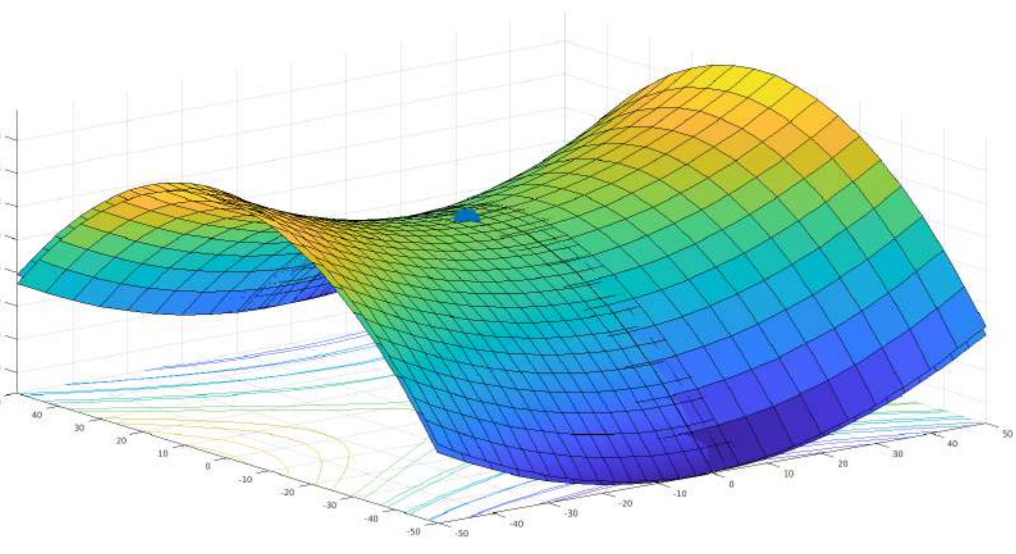
1. Desarrollo

1. Show that the function $f(x) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$, has only one stationary point, and that it is neither a maximum or minimum, but a saddle point. Sketch the contour lines of f.



$$\partial F_x = 8 + 2x_1$$
$$8 + 2x_1 = 0$$
$$x_1 = -4$$
$$\partial F_y = 12 + 4x_2$$
$$12 + 4x_2 = 0$$
$$x_2 = 3$$
$$f(-4, 3) = 2$$
$$P(-4, 3, 2) \text{ Saddle Point.}$$

2. Program the steepest descent and Newton algorithms using the backtracking line search. Use them to minimize the Rosenbrock function. Set the initial step length $\alpha_0 = 1$ and print the step length used by each method at each iteration. First try the initial point $x_0 = (1{,}2, 1{,}2)^T$ and then the more difficult starting point $x_0 = (1{,}2, 1)^T$.

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f = @(x1,x2) 100*(x2 - x1.^2).^2 + (1 - x1).^2;

gradiente1 = @(x1,x2) -400*(x2 - x1.^2) - 2*(1 - x1);
gradiente2 = @(x1,x2) 200*(x2 - x1.^2);

newton1 = @(x1,x2) (-x1+1)/(200*x1^2-200*x2+1);
newton2 = @(x1,x2)
    (200*x1^4-x1^2*x1+200*x2^2-400*x1^2-x2)/(200*x1^2-200*x2+1);

x1 = -1.2;
x2 = 1;
fprintf("Steepest\n")
for i=1:4000
  grad1 = gradiente1(x1,x2);
  grad2 = gradiente2(x1,x2);

  pGrad1 = -grad1;
  pGrad2 = -grad2;

  if i == 2001
    fprintf("Newton\n")
    x1 = -1.2;
    x2 = 1;
  end
  if i >= 2001
    pGrad1 = newton1(x1,x2);
    pGrad2 = newton2(x1,x2);
  end
```

```
alpha = 1;
c1 = 0.01;

condicionIzq = f((x1+(alpha*pGrad1)),(x2+(alpha*pGrad2)));
condicionDer = f(x1,x2) + (alpha*c1*((grad1*pGrad1)+(grad2*pGrad2)));

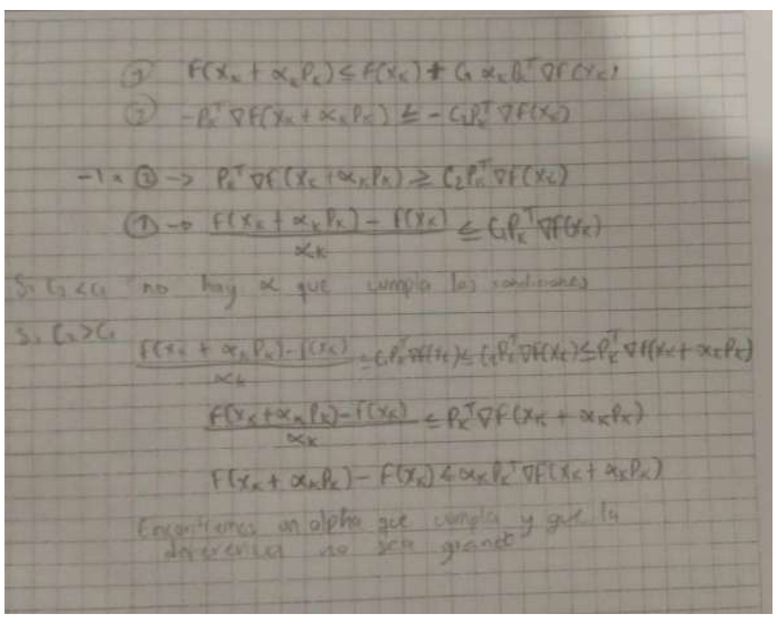
while condicionIzq > condicionDer
  alpha = 0.9*alpha;

  condicionIzq = f((x1 + (alpha.*pGrad1)) ,(x2+(alpha.*pGrad2)));
  condicionDer = f(x1,x2) + (alpha.*c1.*((grad1.*pGrad1)+(grad2.*pGrad2)));
end

x1 = x1 + alpha.*pGrad1;
x2 = x2 + alpha.*pGrad2;

if mod(i,200) == 0
  fprintf("f(x1,x2):%f alpha:%d x1,x2:%f,%f \n",f(x1,x2),alpha,x1,x2)
end
end
```

3. Show that if $0 < c_2 < c_1 < 1$ there may be no step lengths that satisfy the Wolfe conditions.



(1) $f(x_k + \alpha_k p_k) \leq f(x_k) + c_1 \alpha_k p_k^T \nabla f(x_k)$

(2) $-p_k^T \nabla f(x_k + \alpha_k p_k) \leq -c_2 p_k^T \nabla f(x_k)$

$-1 \times$ (2) $\rightarrow$ $p_k^T \nabla f(x_k + \alpha_k p_k) \geq c_2 p_k^T \nabla f(x_k)$

(1) $\rightarrow$ $\dfrac{f(x_k + \alpha_k p_k) - f(x_k)}{\alpha_k} \leq c_1 p_k^T \nabla f(x_k)$

Si $c_2 < c_1$ no hay α que cumpla las condiciones

Si $c_2 > c_1$ $\dfrac{f(x_k + \alpha_k p_k) - f(x_k)}{\alpha_k} \leq c_1 p_k^T \nabla f(x_k) \leq c_2 p_k^T \nabla f(x_k) \leq p_k^T \nabla f(x_k + \alpha_k p_k)$

$\dfrac{f(x_k + \alpha_k p_k) - f(x_k)}{\alpha_k} \leq p_k^T \nabla f(x_k + \alpha_k p_k)$

$f(x_k + \alpha_k p_k) - f(x_k) \leq \alpha_k p_k^T \nabla f(x_k + \alpha_k p_k)$

Encontremos un alpha que cumpla y que la diferencia no sea grande