

Taller 6 - Optimización Convexa - 2022-1

Ruben Dario Rodriguez Moreno - 2181969

1, 2, 3

$$\min_{x,y} F(x,y) := (x-1)^2 + (y-2)^2 \quad \text{st } (x-1)^2 = 5y$$

$$(x,y) = (1,2)$$

$$\nabla F = \begin{bmatrix} 2(x-1) \\ -2(y-2) \end{bmatrix} \quad \nabla F(1,2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\nabla c = \begin{bmatrix} 2(x-1) \\ \frac{2}{5}(x-1) \end{bmatrix} \quad \nabla c(1,2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\min_{x,y,\lambda} (x-1)^2 + (y-2)^2 - \lambda((x-1)^2 - 5y)$$

$$\nabla \mathcal{L} = \begin{bmatrix} 2(x-1) - 2\lambda_1(x-1) \\ 2(y-2) + 5\lambda_1 \\ 5y - (x-1)^2 \end{bmatrix} = 0 \quad \nabla \mathcal{L} = \begin{bmatrix} 1 \\ 0 \\ 4/5 \end{bmatrix} \quad \text{KKT point}$$

$$2(x-1) - 2\lambda_1(x-1) = 0 \rightarrow x=1$$

$$5y - (x-1)^2 = 0 \rightarrow y=0$$

$$2(y-2) + 5\lambda_1 = 0 \rightarrow \lambda_1 = 4/5$$

$$F(1,0) = 4$$

$$(1,0,4) \text{ solution}$$

$$(4) (x-1)^2 = 5y \rightarrow \min_y 5y + (y-2)^2$$

$$\nabla f = 5 + 2(y-2)$$

$$5 + 2(y-2) = 0$$

$$y = -\frac{5}{2} + 2 = -\frac{1}{2}$$

$$(x-1)^2 = -\frac{1}{2}$$

$$x = \pm i\sqrt{\frac{1}{2}} + 1$$

no pueden ser solución
del primer problema
por su dominio

$$(5) F(x) = x_1 x_2 \quad \max_x x_1 x_2 \quad \text{st} \quad 1 - x_1^2 - x_2^2 \geq 0$$

$$\nabla f = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} \quad \nabla c = \begin{bmatrix} -2x_1 \\ -2x_2 \end{bmatrix}$$

$$\max_{x, \lambda} x_1 x_2 - \lambda(1 - x_1^2 - x_2^2)$$

$$\nabla \mathcal{L} = \begin{bmatrix} x_2 - \lambda(-2x_1) \\ x_1 - \lambda(-2x_2) \\ 1 - x_1^2 - x_2^2 \end{bmatrix} = 0$$

$$x_2 + 2\lambda x_1 = 0$$

$$x_1 + 2\lambda x_2 = 0$$

$$1 - x_1^2 - x_2^2 = 0$$

$$x_1 = x_2 = \pm \sqrt{\frac{1}{2}}$$

$$\lambda = \frac{-x_2}{2x_1} = -\frac{1}{2}$$

$$\rightarrow \max \left(\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}, \frac{1}{2} \right)$$

$$f\left(\begin{bmatrix} \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \end{bmatrix}\right) = \frac{1}{2}$$

