Universidad Industrial de Santander
Professor: Jorge Bacca
2021- I
Deadline: 21/08/2022

Workshop No. 7 - ADMM
Convex Optimization
Date: August 16, 2022

Name: _____ **ID:** _____

1 Lasso Problems

An important special case of signal processing problems is the $||\cdot||_1$ regularized linear regression, also called the lasso. This involves solving

$$\min_{\mathbf{x}} ||\mathbf{y} - \mathbf{H}\mathbf{x}||_2^2 + \lambda||\mathbf{x}||_1 \tag{1}$$

where $\lambda > 0$ is a scalar regularization parameter that is usually chosen by cross-validation.

1. (50 points) Implement the ADMM algorithm to solve the lasso problem.
2. (50 points) Load a sky image (64×64) with a lot of zero or near zero values and evaluate the Lasso method under the following scenarios.

Figure 1: dark image example, notice that a lot of pixels are zeros

Noiseless-case

In this scenario, the system of equations has not noise, i.e.,

$$\mathbf{y}=\mathbf{H}\mathbf{x}$$

where $\mathbf{H} \in \mathbb{R}^{m \times n}$ is a random uniform matrix. You can normalize it by columns.

Under this, we try to employ the ADMM for the following cases:

- $\mathbf{H} \in \mathbb{R}^{m \times n}$ where $m < n$
- $\mathbf{H} \in \mathbb{R}^{m \times n}$ where $m > n$
- $\mathbf{H} \in \mathbb{R}^{m \times n}$ where $m = n$

Noise case with 25 dB of SNR:

In this scenario, the measurements are corrupted by noise as

$$\mathbf{y} = \mathbf{H}\mathbf{x}+\boldsymbol{\epsilon}$$

where $\boldsymbol{\epsilon} \in \mathbf{R}^m$ Gaussian noise with mean 0, an standard deviation to satisfy the 25 dB of SNR (signaal-to-noise-radio).

Under this, we try to employ the ADMM for the following cases:

- $\mathbf{H} \in \mathbb{R}^{m \times n}$ where $m < n$
- $\mathbf{H} \in \mathbb{R}^{m \times n}$ where $m > n$
- $\mathbf{H} \in \mathbb{R}^{m \times n}$ where $m = n$