Variables, Parameters, Equations - INET housing market model, Baptista et al. 2016

Ruben Tarne

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1 Variables

Indices

i	index for household, unique identifier; attributes of i include y, b, Ξ, k...
t	index for the current month
Q	index quality value of house, value between 0 and 47
k	index for house, from 1 to total number of houses, attributes of k include Q, owner$_i$, resident$_i$, days on the market,...

Variables

c_i	non-essential consumption	(1)
b_i^d	desired bank balance of household	(4)
y_i	annual gross total income	(5)
y_i^m	monthly gross total income	(6)
$y_i^{m,emp}$	monthly employment income, dependent on age and income percentile of hh. Calibrated against UK data	(7)
$y_i^{m,rent}$	monthly rental income, dependent on rented-out property	(8)
y_i^{disp}	monthly disposable income	(9)
$y_i^{m,net}$	monthly net total income	(10)
$T(y_i^{emp})$	monthly tax expenditure, dependent on annual gross employment income	(10)
$Insurance(y_i^{emp})$	monthly insurance expenditure, dependent on annual gross employment income	(10)
s_i^m	monthly saving of hh i	(11)
b_i	current bank balance	(12)
W^h	total housing wealth	(13)
SH	set of hhs in social housing. This includes BTL investors when they enter the simulation and bid for their home.	(15)
R	set of hhs renting a house.	(16)

OO	set of hhs living in their bought home without having investment property.	(17)
BTL	set of hhs that live in their bought home and bought at least one more house. If a household can buy more than its home is set exogenously.	(18)
$action$	types of actions agents perform involving a probability	(20)
$Prob$ (placing a bid)$_{t,k}^{SH\to OO}$	probability of i $\in$ SH to bid for home	(21)
$p_{t,k}^{SH\to OO}$	desired expenditure for buying a home	(22)
$q_t^{SH\to OO}$	maximum mortgage principal for $i \in$ SH	(23)
$d_t^{SH\to OO}$	downpayment for $i \in$ SH	(24)
${}_{\min,t}^{SH\to OO}$	downpayment for $i \in$ SH set by bank	(25)
int_t	mortgage interest rate	(26)
int_t^{spread}	interest rate spread, set by the bank	(27)
M_t	target credit supply	(28)
g_t	expected monthly house price appreciation, same for all HHs	(29)
h_t	house price index	(30)
p_Q^{ref}	reference price for quality. Calibrated to 2013 UK house price data Q	(31)
$m_{k,i,t}^{SH\to OO}$	mortgage payment for $i \in$ SH	
$\overline{r_{Q,t}}$	expected average rental price for Q	(33)
r_Q^{ref}	rental reference price	(35)
$Prob$ (placing a bid)$_t^{SH\to R}$	probability to place bid on rental market	(36)
$r_t^{SH\to R}$	bid price rental market $i \in$ SH	(37)
$Prob$(to SH)$_t^{R\to SH}$	probability to go to social housing for agent $i \in$ R	(38)
$Prob$ (offering home)$_t^{OO\to SH}$	probability of i $\in$ OO to place an offer for its home on the housing market	(39)
$p_{t,k}^{OO\to SH}$	initial offer price for home	(40)
if k unsold in $t-1$: $p_{t,k}^{OO\to SH}$	monthly price reduction of unsold houses already on the market	(41)
$n_{h,t}$	houses on the market per household	(42)
$\overline{n_{p,t}}$	total number of households	(43)
$\overline{f_t^{hm}}$	expected average days on the housing market	(44)
$\overline{D_{t-1}^{hm}}$	average days houses are on the housing market	(45)
$\overline{p_{Q,t}}$	expected average sale price for house with quality Q	(46)
$Prob$ (placing a bid)$_t^{{}^{BTL}_{OO}\to BTL}$	probability to invest	(47)
$p_t^{{}^{BTL}_{OO}\to BTL}$	bid price for investment property	(48)
Ω_i	expected yield of capital investment	(49)

$d_t^{BTL_{OO}\rightarrow BTL}$	downpayment for $i \in$ BTL	(50)
$d_{\min,t}^{BTL_{OO}\rightarrow BTL}$	downpayment for $i \in$ BTL set by bank	(51)
$q_t^{BTL_{OO}\rightarrow BTL}$	maximum mortgage principal for $i \in$ BTL	(52)
$m_{k,i,t}^{BTL_{OO}\rightarrow BTL}$	mortgage payment for $i \in$ BTL	(53)
$\overline{r_t^{yield}}$	overall rental yield	(54)
$\overline{o_{Q,t}}$	average occupancy for a house of quality Q	(55)
$\overline{D_t^{rm}}$	average days on rental market	(56)
$r_{k,t}^{BTL\rightarrow BTL}$	initial offer rental price	(57)
if k not rented out in $t-1$: $r_{k,t}^{BTL\rightarrow BTL}$	price reduction rental offer. Minimum rental price is set to 4.8% of the house price.	(58)
$\overline{f_t^{rm}}$	expected average days on the rental market	(59)
$Prob(\text{placing offer})_{t,k}^{BTL\rightarrow BTL}$	probability of placing offer for investment property	(60)
$p_{k,t}^{BTL\rightarrow BTL}$	initial sale price investment property	(61)
$\Psi_{k,i,t}$	expected effective yield	(62)
$u_{k,t}$	equity of house k	(63)
$r_{k,t}^{yield}$	rental yield of house k	(64)
$p_{k,t}$	sale price of house k realised in auction	(65)
$p_{t,k}^{offer}$	set of offer prices in housing market	(67)
p_t^{bid}	set of bid prices in housing market	(68)
$r_{k,t}^{exp\ yield}$	expected rental gross yield	(69)
o_Q^{exp}	expected occupancy	(70)
$p_k^{\text{bid up}}$	when offered house receives more than one bid, the price is 'bid up'	(71)
l	bid-up price variable, chosen at random from a geometric distribution	(72)
B	number of bids received in time stamp	(72)

2 Parameters

Recurring parameters, like α or ϵ are numbered according to the number of the equation they appear in.

Parameters

(1)	$\alpha_1 = 0.5$	consumption fraction of monthly budget used for non-essential consumption	
(4)	α_4=-32.00, β_4=4.07, $\varepsilon_{4,i}$ = N(0,0.1)	ε = Gaussian noise (heterogeneity) and constant for every HH	Wealth and Asset Survey (Office for National Statistics (2014))
(6)	$i_{deposit}^{param} = 0.02$	fixed interest rate on deposits	

(7)	Ξ_i, value between 0 and 1	income percentile of hh i, set at birth of hh, according to calibrated distribution	
(9)	$\alpha_9 = 0.8$, $y^{param}_{m,min} = 492.7$	monthly essential consumption by every hh. α_9 = essential consumption fraction $y^{param}_{m,min}$ = monthly minimum earnings	minimum earnings for a married couple from income support
(21)	$\tau = \frac{1.1}{12.0}$, $\beta_{(21)} = \frac{1}{3500}$	psychological cost of renting and sensitivity parameter	
(22)	α_{22}=4.5, β_{22}=0.08, ε_{22} = N(0,0.5)	equation can be understood as setting the desired expenditure so that the long-term cost of the house (which takes into account the expected house capital appreciation) is a noisy fraction of income	
(23)	$\chi = 0.95, 0.9, 0.8$	LTV ratio for first-time buyers, owner-occupiers, buy-to-let investors. Set exogenously.	
(23)	ψ =LTI	LTI set exogenously by the bank and central bank, and dependent on characteristic of hh.	
(23)	$\nu = 0.5$	Maximum fraction of the hh's income to be spent on mortgage repayments under stressed conditions	
(23)	$\alpha_{23} = 300$	25-year mortgage contract with monthly payments	
(24)	α_{24} = 10.30 (11.155), β_{24} = 0.9093 (0.7538)	scale parameter α_{24} and the shape parameter β_{24} for first-time buyers (owner-occupiers)	PSD data
(26)	i^{param}_{base} = 0.005 $i^{spread}_{t=0} = 0.02$	base rate is set exogenously, fixed; interest spread is initialised at 0.02	
(27)	$\alpha_{27} = 0.5 * 1E + 11$	aim is to halve the difference between current demand and target supply of credit. $1E + 11$ is credit demand elasticity with respects to interest rates	
(27)	$T^{param} = 380tp^{param}$	target supply of credit is 380 pounds per household per month	
(29)	α_{29}=0.5	α_{29} represents the estimation of the trend. α_{29}=1 implies that the same trend is predicted.	Value suggested by John Muellenbauer (reference?)

(31)	$N_Q = 48$, scale= $\alpha_{(31)} = \ln(195000.0)$, shape= $\beta_{(31)} = 0.555$	Inverse cumulative distribution function to adjust for the small number of transactions in the model. 48= number of quality bands of houses	Shape parameter for the log-normal distribution of housing prices, taken from the ONS (2013). 195000 = median house price
(33)	$\rho_{(33)} = e^{\left(\frac{-N_Q}{0.02 * tp^{param}}\right)}$	Decay factor	
(33)	$tp^{param} = 10000$	target population, set exogenously	
(35)	$h_{t=0} = 0.8$, 250	HPI initialisation value. 250 = Maximum period of time BTL investors are ready to wait to get back their investment through rents, this determines the minimum rent they are ready to accept	why HPI is used, or where the 250 come from is not entirely clear.
(39)	$\alpha_{39} = 4.0$; $\beta_{39} = 5.0$	α_{39} is a penalty when an excessive number of houses is for sale, β_{39} is a penalty when interest rates are excessively high	
(39)	$\zeta_{39} = \frac{1}{11}$	long-term selling probability once every 11 years	English Housing Survey (Department for Communities and Local Government (2013))
(39)	$\bar{i}^{param} = 0.03$	exponential moving average,in the model right now set static	
(39)	$\bar{n}_h^{param} = 0.05$	exponential moving average, in the model right now set static	
(40)	$\alpha_{40} = 0.04$, $\beta_{40} = 0.011$,	α_{40} = sale mark-up	
(40)	$\epsilon_{40} = N(0, 0.5)$		
(41)	$\epsilon_{41} = N(1.603, 0.617)$		
(44)	$\rho_{44} = e^{\left(\frac{-1.0}{0.02 * tp^{param}}\right)}$	constant. No explanation for this formula is provided	
(49)	$\delta_i = 0.5$ or 0.9	weight of BTL hh i on capital yield (as opposed to rental yield)	

(47)	$\alpha_{47} = 0.2,\ \beta_{47} = 50.0$	α is the share of post-tax income that investors want to spend on mortgage payments, β is intensity of choice on effective yield	
(50)	$\epsilon_{50} = N(0.3, 0.1)$	parameters set, so that the LTV ratios are within the expected range	
(52)	$\xi = 1.25, i_{BTL}^{param} = 0.05$	Interest Cover Ratio (ICR) value and "stress interest rate" for BTL-investors exogenously set by Central Bank	
(54)	$\rho_{54} = e^{\left(\frac{-10000}{tp^{param}*50.0}\right)}$	Decay factor. Values not justified nor mentioned in the paper.	
(55)	18	average tenancy length in months	ARLA - Members survey of the Private Rented Sector Q4 2013
(57)	$\alpha_{57} = 0,\ \beta_{57} = 0011$		
(70)	547 = 18 months in days	expected occupancy based on 18 month rental contract followed by a number of days waiting	

Initialisation

variable	value	comments

3 Agent states, probabilities and prices

4 Equations

desired non-essential consumption:

$$c_{i,t} = \max(\alpha_1(b_{i,t-1} - b_{i,t}^d), 0) \tag{1}$$

alternative non-essential consumption function:

$$c_{i,t} = \begin{cases} \text{if } b_{i,t} - c_{i,t}^{desired} < \zeta y_{i,t}^{m,disp} & , c_{i,t} = \alpha_i y_{i,t}^{m,disp} \\ \text{if } c_{i,t}^{desired} > b_{i,t} & , c_{i,t} = b_{i,t} \\ \text{if } c_{i,t}^{desired} < 0 & , c_{i,t} = 0 \\ \text{if equity position negative} & , c_{i,t} = \delta c_{i,t}^{desired} \\ \text{else} & , c_{i,t} = c_{i,t}^{desired} \end{cases} \tag{2}$$

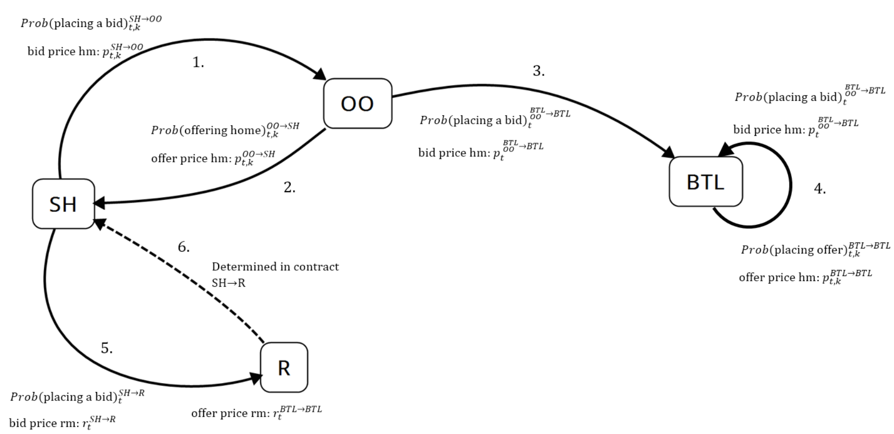

Figure 1: Agent states, probabilities to change states and respective bid and ask prices

alternative desired non-essential consumption:

$$c_{i,t}^{desired} = \alpha_i y_{i,t}^{m,disp} + \beta_i \left(b_{i,t} + \gamma (w_{i,t}^h - q_{i,t}) \right) \tag{3}$$

desired bank balance:

$$\ln(b_{i,t}^d) = \alpha_4 + \beta_4 \ln(y_{i,t}) + \varepsilon_{4,i} \tag{4}$$

gross income yearly:

$$y_{i,t} = 12 y_{i,t}^m \tag{5}$$

gross income monthly:

$$y_{i,t}^m = y_{i,t}^{m,emp} + y_{i,t}^{m,rent} + b_{i,t-1} \cdot i_{deposit}^{param} \tag{6}$$

employment income:

$$y_{i,t}^{m,emp} = f(age_{i,t}, \Xi_i) \tag{7}$$

rent income:

$$y_{i,t}^{m,rent} = r_{k,i,t}^{BTL\to BTL} \tag{8}$$

disposable income:

$$y_{i,t}^{m,disp} = y_{i,t}^{m,net} - \alpha_9 y_{m,min}^{param} - m_{k,i} - r_{k,i}^{SH\to R} \tag{9}$$

net income:

$$y_{i,t}^{m,net} = y_{i,t}^{m} - T(y_{i,t}^{emp}) - Insurance(y_{i,t}^{emp}) \tag{10}$$

saving:

$$s_{i,t}^{m} = y_{i,t}^{m,disp} - c_{i,t} \tag{11}$$

bank balance:

$$b_{i,t} = b_{i,t-1} + s_{i,t}^{m} \tag{12}$$

total housing wealth:

$$W^{h} = \sum_{i=0}^{k} \overline{p_{Q,k,t}} \tag{13}$$

states agents can be in:

$$i \in SH \cup R \cup OO \cup BTL \tag{14}$$

agents in social housing:

$$\begin{aligned} SH = \{&\text{households who enter the simulation,} \\ &\text{renters whose contracts just ended,} \\ &\text{homeowners who sold their home in } t-1\} \end{aligned} \tag{15}$$

renters:

$$R = \{\text{households who rented a house}\} \tag{16}$$

owner-occupiers:

$$OO = \{\text{households who bought a house}\} \tag{17}$$

buy-to-let investors:

$$BTL = \{\text{households who bought more than one house}\} \tag{18}$$

general rule for bids and offers to be made:

$$i \begin{cases} \text{bids or offers if } RN \sim U[0,1] < Prob(\text{action}) \\ \text{does not act if } RN \sim U[0,1] \geq Prob(\text{action}) \end{cases} \tag{19}$$

types of actions:

$$
\begin{aligned}
\mathit{action} = \{&\text{SH bids on housing market},\\
&\text{SH bids on rental market},\\
&\text{OO bids on housing market},\\
&\text{BTL bids on housing market},\\
&\text{R becomes SH},\\
&\text{OO places offer on housing market},\\
&\text{BTL places offer on housing market},\\
&\text{BTL places offer on rental market}\}
\end{aligned}
\tag{20}
$$

probability of i ∈ SH to bid for home (see arrow 1 in Figure 1):

$$
\begin{aligned}
&Prob(\text{placing a bid})^{SH\to OO}_{t,k}\\
&\quad = \frac{1}{1+exp(-\beta_{21}[12\overline{r_{Q,t}}(1+\tau)-(12\cdot m^{SH\to OO}_t - p^{SH\to OO}_{t,k}\cdot g_t)])},
\end{aligned}
\tag{21}
$$

$$\textit{if action is taken is decided by using equation } (19).$$

desired expenditure for buying a house[1]:

$$
p^{SH\to OO}_{t,k} = \min\left(q^{SH\to OO}_t + b_{i,t}, \quad \frac{\alpha_{22} 12 y^{m,emp}_i \exp(\varepsilon_{22})}{1-\beta_{22} g_t}\right) \tag{22}
$$

maximum mortgage principal for $i \in$ SH:

$$
q^{SH\to OO}_t = \min(\text{LTV, LTI, Affordability constraint})
$$

$$
q^{SH\to OO}_t = \min\left(\frac{b_i\chi}{1-\chi}, \quad y_i\psi, \quad y^{m,d}_i \nu \frac{1-(1+\frac{int}{12})^{-\alpha_{23}}}{\frac{int}{12}}\right) \tag{23}
$$

downpayment for $i \in$ SH:

$$
d^{SH\to OO}_t = \begin{cases} 0, & \text{if } \Xi \le 0.3\\ \max\left(d^{SH\to OO}_{\min,t}, \quad h_t F^{-1}((\Xi_i - 0.3)/0.7)\right), & \text{if } \Xi > 0.3 \end{cases} \tag{24}
$$

$$F^{-1}(\alpha_{24},\beta_{24}) : \text{inverse cumulative log-normal distribution function}$$

downpayment for $i \in$ SH set by bank:

$$
d^{SH\to OO}_{\min,t} = p^{SH\to OO}_{t,k} - q^{SH\to OO}_t \tag{25}
$$

[1] $SH \to OO$ can be read as: "for changing from the social housing to owner occupier state"

mortgage interest rate:

$$int_t = int_{base}^{param} + int_t^{spread} \tag{26}$$

interest rate spread:

$$int_{t+1}^{spread} = int_t^{spread} + \alpha_{27}(M_t - T^{param}) \tag{27}$$

target credit supply:

$$M_t = \sum_{j=1}^{n_t^{hm\ sales}} q_t^{SH \to OO, {}_{OO}^{BTL} \to BTL} \tag{28}$$

expected HPI appreciation:

$$g_t = \alpha_{(29)} \left(\frac{h_{t-1} + h_{t-2} + h_{t-3}}{h_{t-13} + h_{t-14} + h_{t-15}} - 1 \right) \tag{29}$$

house price index:

$$h_t = \frac{\sum_{j=1}^{n_{t-1}^{hm\ sales}} p_{k,t-1,j}}{\sum_{j=1}^{n_{t-1}^{hm\ sales}} p_{Q,t-1,j}^{ref}} \tag{30}$$

reference price for quality Q:

$$p_Q^{ref} = h_{t=0}^{param} F^{-1} \left(\frac{Q + 0.5}{N_Q} \right) \tag{31}$$

$$F^{-1}(\alpha_{31}, \beta_{31}) : \text{inverse cumulative log-normal distribution function}$$

mortgage payment for $i \in$ SH:

$$m_{k,i,t}^{SH \to OO} = q_t^{SH \to OO} \frac{\frac{int_{t=s}}{12}}{1 - (1 + \frac{int_{t=s}}{12})^{-300}} \tag{32}$$

$$s = \text{mortgage contract starting period}$$

expected average rental price for Q:

$$\overline{r_{Q,t}} = 0.25 \cdot \left(\rho_{(33)} \overline{r_{Q,t-1}} + (1 - \rho_{(33)}) \frac{\left(\sum_{j=1}^{n_{Q,t-1}^{rm\ sales}} r_{k,t-1,j} \right)}{n_{Q,t-1}^{rm\ sales}} \right) + 0.75 r_Q^{ref} \cdot \mathrm{RPI}_t \tag{33}$$

rental price index:

$$\mathrm{RPI}_t = \frac{\sum_{j=1}^{n_{t-1}^{rm\ sales}} r_{k,t-1,j}}{\sum_{j=1}^{n_{t-1}^{rm\ sales}} r_{Q,t-1,j}^{ref}} \tag{34}$$

rental reference price:

$$r_Q^{ref} = \frac{h_{t=0}^{param} F^{-1}\left(\frac{Q+0.5}{N_Q}\right)}{250} \tag{35}$$

$F^{-1}(\alpha_{31}, \beta_{31})$: inverse cumulative log-normal distribution function

probability to place bid on rental market (see arrow 5 in Figure 1):

$$Prob(\text{placing a bid})_t^{SH\to R} = \begin{cases} 0, & \text{if } i \in \text{SH already placed bid on housing market} \\ 1, & \text{if } i \in \text{SH did not place bid on housing market} \end{cases} \tag{36}$$

bid price rental market $i \in$ SH:

$$r_t^{SH\to R} = 0.33 y_{t,i}^{m,emp} \tag{37}$$

probability to go to social housing for agent $i \in$ R (see arrow 6 in Figure 1):

$$Prob(\text{to SH})_t^{R\to SH} = \begin{cases} 0, & \text{if rental contract valid} \\ 1, & \text{if rental contract expired} \end{cases} \tag{38}$$

probability of i $\in$ OO to place an offer for its home on the housing market (see arrow 2 in Figure 1):

$$Prob(\text{offering home})_t^{OO\to SH} = \tfrac{1}{12}\zeta_{39} \tag{39}$$

if action is taken is decided by using equation (19).

initial offer price for home:

$$\ln(p_{t,k}^{OO\to SH}) = \max\left(\ln(q_{k,t}^{OO\to SH}), \quad \alpha_{40} + \ln(\overline{p_{Q,t}}) - \beta_{40}\ln(1 + \overline{f_t^{hm}}) + \varepsilon_{40}\right) \tag{40}$$

monthly price reduction of unsold houses already on the market:

$$\text{if } k \text{ unsold in } t-1\text{: } p_{t,k}^{OO\to SH} = \begin{cases} p_{t-1,k}^{OO\to SH}(1 - \exp(\varepsilon_{41})), & \text{with probability 0.06} \\ p_{t-1,k}^{OO\to SH}, & \text{with probability 0.94} \end{cases} \tag{41}$$

houses on the market per household:

$$n_{h,t} = \frac{n_{k,t}^{hm}}{n_t^p} \tag{42}$$

$n_{k,t}^{hm}$: number of houses on the market at this moment

total number of households:

$$n_{p,t} = \#SH_t + \#BTL_t + \#OO_t + \#R_t \tag{43}$$

expected average days on the housing market:

$$\overline{f_t^{hm}} = \rho_{44}\overline{f_{t-1}^{hm}} + (1-\rho_{44})\overline{D_{t-1}^{hm}} \tag{44}$$

average days houses are on the housing market:

$$\overline{D_{t-1}^{hm}} = \frac{\sum_{j=1}^{n_{t-1}^{hm\ sales}} D_{k,t-1,j}^{hm}}{n_{t-1}^{hm\ sales}} \tag{45}$$

$D_{k,t}^{hm}$: days on the market of house k

expected average sale price for house with quality Q:

$$\overline{p_{Q,t}} = 0.25 \cdot \left(\rho_{(33)}\overline{p_{Q,t-1}} + (1-\rho_{(33)}) \frac{\sum_{j=1}^{n_{Q,t-1}^{hm\ sales}} p_{k,Q,t-1,j}}{n_{Q,t-1}^{hm\ sales}} \right) + 0.75 h_t p_Q^{ref} \tag{46}$$

probability to invest (see arrow 3 and 4 in Figure 1):

$$Prob(\text{placing a bid})_t^{{}^{BTL}_{OO}\to BTL} = \begin{cases} 1, & \text{if } i \text{ has only 1 house} \\ 0, & \text{if } \sum m_{i,k} > \alpha_{47} y_{i,t}^{m,net} \\ 0, & \text{if } p_i < \overline{p_{Q=0}} \\ \frac{1}{1+e^{(-\beta_{47}\Omega_{i,t})^{\frac{1}{12}}}}, & \text{if else} \end{cases} \tag{47}$$

bid price for investment property[2]:

$$p_t^{{}^{BTL}_{OO}\to BTL} = q_t^{{}^{BTL}_{OO}\to BTL} + b_{t,i} \tag{48}$$

[2] $p_{OO}^{BTL}{}^{\to BTL}$ can be read as: "price for a house for BTL agents and Owner-Occupiers that would then become BTL agents."

expected yield of capital investment:

$$\Omega_{i,t} = \frac{p_t^{OO^{BTL}\to BTL}}{d_t^{OO^{BTL}\to BTL}}(\delta_i g_t + (1-\delta_i)\overline{r_t^{yield}}) - \frac{m_{k,i,t}^{OO^{BTL}\to BTL}}{d_t^{OO^{BTL}\to BTL}} \tag{49}$$

downpayment for $i \in$ BTL:

$$d_t^{OO^{BTL}\to BTL} = \max\left(d_{\min,t}^{OO^{BTL}\to BTL}, \quad p_{t,k}^{OO^{BTL}\to BTL} \cdot \epsilon_{50}\right) \tag{50}$$

downpayment for $i \in$ BTL set by bank:

$$d_{\min,t}^{OO^{BTL}\to BTL} = p_{t,k}^{OO^{BTL}\to BTL} - q_t^{OO^{BTL}\to BTL} \tag{51}$$

maximum mortgage principal for $i \in$ BTL:

$$q_t^{OO^{BTL}\to BTL} = \min(\text{LTV, interest cover contstraint (ICR)})$$

$$q_t^{OO^{BTL}\to BTL} = \min\left(\frac{b_{i,t}\chi}{1-\chi}, \frac{b_{i,t}}{1 - \frac{\overline{r_t^{yield}}}{\xi \cdot i_{BTL}^{param}}}\right) \tag{52}$$

mortgage payment for $i \in$ BTL:

$$m_{k,i,t}^{OO^{BTL}\to BTL} = q^{OO^{BTL}\to BTL}\frac{int_{t=s}}{12} \tag{53}$$

overall rental yield[3]:

$$\overline{r_t^{yield}} = \rho_{54}\overline{r_{t-1}^{yield}} + (1-\rho_{54}) \cdot \frac{\sum_{Q=1}^{N_Q}\left(\frac{12 n_{Q,t-1}^{rm\ sales}\overline{r_{Q,t-1}} \cdot \overline{o_{Q,t-1}}}{\overline{p_{Q,t-1}}}\right)}{n_{t-1}^{rm\ sales}} \tag{54}$$

average occupancy for a house of quality Q:

$$\overline{o_{Q,t}} = \frac{18}{18 + \rho_{44}\overline{o_{Q,t-1}} + (1-\rho_{44})\overline{\overline{D_{t-1}^{rm}}}} \tag{55}$$

average days on rental market:

$$\overline{D_{t-1}^{rm}} = \frac{\sum_{j=1}^{n_{t-1}^{rm\ sales}} D_{k,t-1,j}^{rm}}{n_{t-1}^{sales}} \tag{56}$$

[3]the sum is the average flow yield, which I could put in a separate formula

$$D^{rm}_{k,t} : \text{days on the rental market of house } k$$

initial offer rental price:

$$\ln(r^{BTL\to BTL}_{k,t}) = \alpha_{57} + \ln(\overline{r_{Q,t}}) - \beta_{57}\ln(1 + \overline{f^{rm}_t}) + \varepsilon_{40} \tag{57}$$

price reduction rental offer[4]:

$$\text{if } k \text{ not rented out in } t-1\text{: } r^{BTL\to BTL}_{k,t} = 0.95 r^{BTL\to BTL}_{k,t-1} \tag{58}$$

expected average days on the rental market:

$$\overline{f^{rm}_t} = \rho_{44}\overline{f^{rm}_{t-1}} + (1-\rho_{44})\overline{D^{rm}_{t-1}} \tag{59}$$

probability of placing offer for investment property (see arrow 4 in Figure 1):

$$Prob(\text{placing offer})^{BTL\to BTL}_{t,k} = \begin{cases} 0, & \text{if } i \text{ has only 2 houses} \\ 0, & \text{if } k \text{ has a tenant} \\ 1 - \left(\dfrac{1}{1+e^{(-\beta_{47}\Psi_{k,i,t})^{\frac{1}{12}}}}\right), & \text{else} \end{cases} \tag{60}$$

initial sale price investment property:

$$\ln(p^{BTL\to BTL}_{k,t}) = \max\left(\ln(q^{\overset{BTL}{OO}\to BTL}_{k,t}), \quad \alpha_{40} + \ln(\overline{p_{Q,t}}) - \beta_{40}\ln(\zeta_{40}(1+\overline{f^{hm}_t})) + \varepsilon_{40}\right) \tag{61}$$

expected effective yield:

$$\Psi_{k,i,t} = \frac{\overline{p_{Q,t}}}{u_{k,t}}(\delta_i g_t + (1-\delta_i) r^{yield}_{k,t}) - \frac{m^{\overset{BTL}{OO}\to BTL}_{k,i,t}}{u_{k,t}} \tag{62}$$

equity of house k:

$$u_{k,t} = \overline{p_{Q,t}} - q^{\overset{BTL}{OO}\to BTL}_t \tag{63}$$

rental yield of house k:

$$r^{yield}_{k,t} = \frac{12 r_{k,t}}{\overline{p_{Q,t}}} \tag{64}$$

[4]Here I could/should include that the minimum rental price is set to 4.8% of the house price.

housing market clearing process
round 1, step 1:

$$\forall\ BTL \max_{k \leq \max\left(r_{k,t}^{exp\ yield}\right)} \left(p_{t,k}^{offer} \leq p_t^{{}^{BTL}_{OO} \to BTL}\right)$$

$$\forall\ SH \max_{k \leq \max(Q)} \left(p_{t,k}^{offer} \leq p_t^{SH \to OO}\right)$$

round 1, step2:

$$\begin{aligned} &offer \text{ is machted, if } \exists!\ BTL \vee SH : p_{t,k}^{offer} < p_t^{bid} \ \rightarrow \ p_{k,t} = p_{t,k}^{offer} \\ &\text{else } p_{t,k}^{offer} = p_{k,i}^{bid-up}, \text{ then pick randomly one bid} \\ &\qquad \text{with } p_{k,i}^{bid-up} \leq p_t^{bid} \ \rightarrow \ p_{k,t} = p_{k,i}^{bid-up} \end{aligned} \tag{65}$$

round 1, step 3:

return unmatched offers and bids into the pool

round 2 and following:
repeat until:

- $bid = 0$
- $offer = 0$
- #rounds $= X$

maximum number of rounds:

$$X = \max(10, \frac{n_t^p}{50}) \tag{66}$$

set of offer prices

$$p_{t,k}^{offer} = \{p_{k,t}^{BTL \to BTL}, p_{t,k}^{OO \to SH}\} \tag{67}$$

set of bid prices:

$$p_t^{bid} = \{p_t^{{}^{BTL}_{OO} \to BTL}, p_t^{SH \to OO}\} \tag{68}$$

expected rental gross yield

$$r_{k,t}^{exp\ yield} = \frac{12\overline{r_{Q,t}} \cdot o_Q^{exp}}{p_{k,t}^{offer}} \tag{69}$$

expected occupancy:

$$o_Q^{exp} = \frac{547}{547 + \overline{\overline{D_{t-1}^{rm}}}} \tag{70}$$

bid-up price in auction:

$$p_{k,i}^{\text{bid up}} = 1.0075^{l} \tag{71}$$

choosing bidding-up variable l from a geometric distribution:

$$P(l) = (1 - e^{-7B/30})^{l-1} e^{-7B/30} \tag{72}$$

rental market clearing process
round 1, step 1:

$$\forall\ SH \max_{k \leq \max(Q)} \left(r_{t,k}^{BTL \to BTL} \leq r_{t}^{SH \to R} \right)$$

round 1, step2:

$$\begin{aligned} &offer \text{ is machted, if } \exists!\ SH : r_{t,k}^{BTL \to BTL} < r_{t}^{SH \to R} \ \rightarrow \ r_{k,t} = r_{t,k}^{BTL \to BTL} \\ &\quad \text{else } r_{t,k}^{BTL \to BTL} = p_{k,i}^{bid-up}, \text{ then pick randomly one bid} \\ &\qquad \text{with } p_{k,i}^{bid-up} \leq r_{t}^{SH \to R} \rightarrow \ r_{k,t} = p_{k,i}^{bid-up} \end{aligned} \tag{73}$$

round 1, step 3:

return unmatched offers and bids into the pool

round 2 and following:
repeat until:

- $bid = 0$
- $offer = 0$
- #rounds $= X$