Variables, Parameters, Equations - Tarne & Bezemer & Theobald (2020)

Ruben Tarne

December 9, 2020

1 Variables

Indices

i	index for household, unique identifier; attributes of i include y, b, Ξ, k...
j	index for each individual housing and rental transaction; attributes for j include i of seller and buyer, $k, p_{k,t}, r_{k,t}$
k	index for house, from 1 to total number of houses, attributes of k include Q, owner$_i$, resident$_i$, days on the market,...
t	index for the current month
Q	index quality value of house, value between 0 and 34

Variables

c_i	non-essential consumption	(1)
y_i	annual gross total income	(3)
y_i^m	monthly gross total income	(4)
$y_i^{m,emp}$	monthly employment income, dependent on age and income percentile of hh. Calibrated against UK data	(5)
$y_i^{m,rent}$	monthly rental income, dependent on rented-out property	(6)
$y_i^{m,div}$	monthly dividend income, paid out of commercial bank's interest income, dependant on household's deposits	(7)
I_t	aggregate monthly interest payments	(7)
$\sum_{i=1}^{i=n_{p,t}} b_{i,t}$	total financial wealth	(7)
y_i^{disp}	monthly disposable income	(8)
$y_i^{m,net}$	monthly net total income	(9)
$T(y_i^{emp})$	monthly tax expenditure, dependent on annual gross employment income	(9)
$Insurance(y_i^{emp})$	monthly insurance expenditure, dependent on annual gross employment income	(9)
s_i^m	monthly saving of hh i	(10)

b_i	current bank balance	(11)
W^h	total housing wealth	(12)
SH	set of hhs in social housing. This includes BTL investors when they enter the simulation and bid for their home.	(14)
R	set of hhs renting a house.	(15)
OO	set of hhs living in their bought home without having investment property.	(16)
BTL	set of hhs that live in their bought home and bought at least one more house. If a household can buy more than its home is set exogenously.	(17)
$action$	types of actions agents perform involving a probability	(19)
$Prob$ (placing a bid)$_{t,k}^{SH\to OO}$	probability of i $\in$ SH to bid for home	(20)
$p_{t,k}^{SH\to OO}$	desired expenditure for buying a home	(21)
$q_t^{SH\to OO}$	maximum mortgage principal for $i \in$ SH	(22)
$d_t^{SH\to OO}$	downpayment for $i \in$ SH	(23)
$d_{\min,t}^{SH\to OO}$	downpayment for $i \in$ SH set by bank	(24)
int_t	mortgage interest rate	(25)
int_t^{spread}	interest rate spread, set by the bank	(25)
g_t	expected monthly house price appreciation, same for all HHs	(26)
h_t	house price index	(27)
p_Q^{ref}	reference price for quality Q	(27)
$n_t^{hmsales}$	total number of housing transactions in t	(27)
$m_{k,i,t}$	mortgage payment	(28)
$q_{i,t}$	principal, either calculated as $q^{SH\to OO}$ or $q_t^{\substack{BTL \\ OO}\to BTL}$, depending on the agents class	(28)
$\overline{r_{Q,t}}$	expected average rental price for Q	(29)
r_Q^{ref}	rental reference price	(30)
$Prob$ (placing a bid)$_t^{SH\to R}$	probability to place bid on rental market	(31)
$r_t^{SH\to R}$	bid price rental market $i \in$ SH	(32)
$Prob$(to SH)$_t^{R\to SH}$	probability to go to social housing for agent $i \in$ R	(33)
$Prob$ (offering home)$_t^{OO\to SH}$	probability of i $\in$ OO to place an offer for its home on the housing market	(34)
$p_{i,t,k}$	initial offer price for home ($p_{i,t,k}^{OO\to SH}$) or investment property ($p_{i,t,k}^{\substack{BTL \\ OO}\to BTL}$)	(35)
if k unsold in $t-1$: $p_{t,k}$	monthly price reduction of unsold houses already on the market	(36)
$n_{p,t}$	total number of households	(37)

$\overline{p_{Q,t}}$	expected average sale price for house with quality Q	(38)
$Prob$ (placing a bid)$_t^{OO^{BTL}\to BTL}$	probability to invest	(39)
$p_t^{OO^{BTL}\to BTL}$	bid price for investment property	(40)
Ω_i	expected yield of capital investment	(41)
$d_t^{OO^{BTL}\to BTL}$	downpayment for $i \in$ BTL	(42)
$d_{\min,t}^{OO^{BTL}\to BTL}$	downpayment for $i \in$ BTL set by bank	(43)
$q_t^{OO^{BTL}\to BTL}$	maximum mortgage principal for $i \in$ BTL	(44)
$\overline{r_t^{yield}}$	overall rental yield	(45)
$\overline{o_{Q,t}}$	average occupancy for a house of quality Q	(46)
$\overline{D_t^{rm}}$	average days on rental market	(47)
$D_{k,t}^{rm}$	days on the rental market of house k	(47)
$r_{k,t}^{BTL\to BTL}$	initial offer rental price	(48)
if k not rented out in $t-1$: $r_{k,t}^{BTL\to BTL}$	price reduction rental offer	(49)
$Prob$(placing offer)$_{t,k}^{BTL\to BTL}$	probability of placing offer for investment property	(50)
$\Psi_{k,i,t}$	expected effective yield	(51)
$u_{k,t}$	equity of house k	(52)
$r_{k,t}^{yield}$	rental yield of house k	(53)
$p_{k,t}$	sale price of house k realised in auction	(54)
$p_{t,k}^{offer}$	set of offer prices in housing market	(55)
p_t^{bid}	set of bid prices in housing market	(56)
$r_{k,t}^{exp\ yield}$	expected rental gross yield	(57)
o_Q^{exp}	expected occupancy	(58)
$p_k^{\text{bid up}}$	when offered house receives more than one bid, the price is 'bid up'	(59)
l	bid-up price variable, chosen at random from a geometric distribution	(60)
B	number of bids received in time stamp	(60)
$r_{k,t}$	rental price of house k realised in auction	(61)

2 Parameters

Recurring parameters, like α or ϵ are numbered according to the number of the equation they appear in.

Parameters

(1)	$c_0 = y_{m,min}^{param}$	essential consumption, set to minimum monthly earnings	

(1)	$\alpha_i = 0.99, 0.96, 0.93, 0.9, 0.85, 0.6$; $\beta_i = 0.0075, 0.006, 0.005, 0.004, 0.002, 0.0002$; $\gamma = 0.25$	marginal propensities to consume due to income (α_i) and wealth (β_i) set according to households income percentile Ξ_i (1st quarter, 2nd quarter, 3rd quarter, 4th quarter, top 10%, top 1%)	Calibrated to match the UK wealth distribution and financial wealth to mortgage debt relation (Wealth and Asset Survey (2020))
(2)	$\zeta = 2$, $\delta = 0.8$	liquidity preference and consumption response when households experience negative equity.	
(5)	Ξ_i, value between 0 and 1	income percentile of hh i, set at birth of hh, according to calibrated distribution	
(8)	$y^{param}_{m,min} = 492.7$	monthly essential consumption by every hh. $y^{param}_{m,min}$ = monthly minimum earnings (£492.7)	minimum earnings for a married couple from income support
(20)	$\beta_{(20)} = \frac{1}{3500}$	sensitivity parameter	
(21)	α_{21}=4.5, β_{21}=0.08, ε_{21} = N(0,0.5)	equation can be understood as setting the desired expenditure so that the long-term cost of the house (which takes into account the expected house capital appreciation) is a noisy fraction of income	
(22)	$\chi = 0.8$	LTV ratio (can be specified differently for for first-time buyers, owner-occupiers, buy-to-let investors. Set exogenously.	
(22)	$\psi = 6$	LTI ratio set exogenously by the bank and central bank, and dependent on characteristic of hh.	
(22)	$\nu = 0.5$	Maximum fraction of the hh's income to be spent on mortgage repayments under stressed conditions	
(22)	$\alpha_{22} = 300$	25-year mortgage contract with monthly payments	
(23)	$\alpha_{23} = 10.35$ (11.15), $\beta_{23} = 0.898$ (0.958)	scale parameter α_{23} and the shape parameter β_{23} for first-time buyers (owner-occupiers)	Calibrated against mortgage approval/housing transaction ratio, Bank of England core indicators average 1987-2006

(25)	$i_{base}^{param} = 0.005$, $i_{t=0}^{spread} = 0.03$	base and interest rate is set exogenously, fixed	
(26)	α_{26}=0.44, β_{26}=-0.007	represents the house price trend households estimate	estimated from NMG Survey and Land Registry data for 2014/2018
(27)	$\ln(p_Q^{ref}) = N(12.1186, 0.6414)$	Distribution of house reference prices for each quality band	Input calibrated from Land Registry Price Paid Data for 2011
(29)	$\rho_{(29)} = 0.84$	Decay factor to adjust for lower absolute market turnover in the model opposed to the real economy	
(29)	$\ln(r_Q^{ref}) = N(6.0708, 0.4796)$	Distribution of rental reference prices for each quality band	calibrated from English Housing Survey data for 2012-2013
(34)	$\zeta_{34} = \frac{1}{17}$	long-term selling probability once every 17 years	English Housing Survey (EHS) data for 2011
(35)	$\theta(0,1)^p$	Distribution of initial house sale price mark-ups over average price of same quality houses; defined between 0 and 1	based on back-projecting Zoopla data using HPI data
(35)	$\epsilon_{35} = U(0,1)$	random number between 0 and 1	
(36)	$\epsilon_{36} = N(1.603, 0.617)$	Monthly probability of reducing the price of a house on the market	calibrated against Zoopla data
(39)	$\alpha_{39} = 0.5$, $\beta_{39} = 50.0$	α is the maximum share of post-tax income that investors want to spend on mortgage payments, β is intensity of choice on effective yield	
(41)	$\delta_i = 0.5$ or 0.9	weight of BTL hh i on capital yield (as opposed to rental yield)	
(42)	$\epsilon_{42} = RN \sim U[0,1]$	Random number between 0 and 1	
(44)	$\xi = 1.25, i_{BTL}^{param} = 0.05$	Interest Cover Ratio (ICR) value and "stress interest rate" for BTL-investors exogenously set by Central Bank	

(45)	$\rho_{45} = 0.82$	Decay factor to adjust for lower absolute market turnover in the model's rental market than in the real economy	
(46)	$\rho_{46} = 0.995$	Decay factor to adjust for lower absolute market turnover in the model opposed to the real economy	
(46)	$\alpha_{(46)} = 18$	average tenancy length in months	ARLA - Members survey of the Private Rented Sector Q4 2013
(48)	$\theta(0,1)^r$	Distribution of initial rental price mark-ups over average price of same quality houses; defined between 0 and 1	based on back-projecting Zoopla data using HPI data
(58)	$547 = 18$ months in days	expected occupancy based on 18 month rental contract followed by a number of days waiting	

Initialisation

variable	value	comments

3 Agent states, probabilities and prices

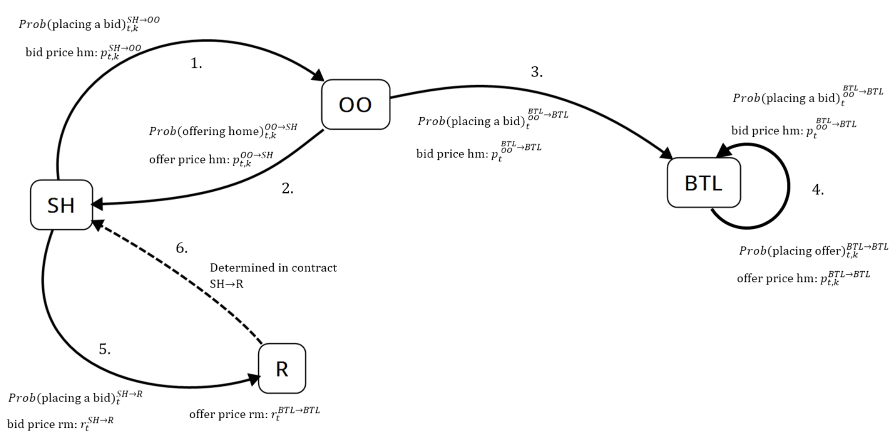

Figure 1: Agent states, probabilities to change states and respective bid and ask prices

4 Equations

desired consumption:

$$c_{i,t}^{desired} = c_0 + \alpha_i y_{i,t}^{m,disp} + \beta_i \left(b_{i,t} + \gamma(w_{i,t}^h - q_{i,t})\right) \tag{1}$$

consumption:

$$c_{i,t} = \begin{cases} \text{if } b_{i,t} - (c_{i,t}^{desired} - y_{i,t}^{m,disp}) < \zeta y_{i,t}^{m,disp} & , c_{i,t} = \alpha_i y_{i,t}^{m,disp} \\ \text{if } c_{i,t}^{desired} < 0 & , c_{i,t} = 0 \\ \text{if equity position negative} & , c_{i,t} = \delta c_{i,t}^{desired} \\ \text{else} & , c_{i,t} = c_{i,t}^{desired} \end{cases} \tag{2}$$

gross income yearly:

$$y_{i,t} = 12 y_{i,t}^m \tag{3}$$

gross income monthly:

$$y_{i,t}^m = y_{i,t}^{m,emp} + y_{i,t}^{m,rent} + y_{i,t}^{m,div} \tag{4}$$

employment income:

$$y_{i,t}^{m,emp} = f(age_{i,t}, \Xi_i) \tag{5}$$

rent income:

$$y_{i,t}^{m,rent} = r_{k,i,t}^{BTL \to BTL} \tag{6}$$

dividend income:

$$y_{i,t}^{m,div} = I_t \cdot \frac{b_{i,t}}{\sum_{i=1}^{i=n_{p,t}} b_{i,t}} \tag{7}$$

disposable income:

$$y_{i,t}^{m,disp} = y_{i,t}^{m,net} - y_{m,min}^{param} - m_{k,i} - r_{k,i}^{SH \to R} \tag{8}$$

net income:

$$y_{i,t}^{m,net} = y_{i,t}^m - T(y_{i,t}^{emp}) - Insurance(y_{i,t}^{emp}) \tag{9}$$

saving:

$$s_{i,t}^m = y_{i,t}^{m,disp} - c_{i,t} \tag{10}$$

bank balance:

$$b_{i,t} = b_{i,t-1} + s_{i,t}^m \tag{11}$$

total housing wealth:

$$W^h = \sum_{i=0}^{k} \overline{p_{Q,k,t}} \tag{12}$$

states agents can be in:

$$i \in SH \cup R \cup OO \cup BTL \tag{13}$$

agents in social housing:

$$\begin{aligned} SH = \{&\text{households who enter the simulation,} \\ &\text{renters whose contracts just ended,} \\ &\text{homeowners who sold their home in } t-1\} \end{aligned} \tag{14}$$

renters:

$$R = \{\text{households who rented a home}\} \tag{15}$$

owner-occupiers:

$$OO = \{\text{households who bought a home}\} \tag{16}$$

buy-to-let investors:

$$BTL = \{\text{households who bought more than one house}\} \tag{17}$$

general rule for bids and offers to be made:

$$i \begin{cases} \text{bids or offers if } RN \sim U[0,1] < Prob(\text{action}) \\ \text{does not act if } RN \sim U[0,1] \geq Prob(\text{action}) \end{cases} \tag{18}$$

types of actions:

$$\begin{aligned} action = \{&\text{SH bids on housing market,} \\ &\text{SH bids on rental market,} \\ &\text{OO bids on housing market,} \\ &\text{BTL bids on housing market,} \\ &\text{R becomes SH,} \\ &\text{OO places offer on housing market,} \\ &\text{BTL places offer on housing market,} \\ &\text{BTL places offer on rental market}\} \end{aligned} \tag{19}$$

probability of i ∈ SH to bid for home (see arrow 1 in Figure 1):

$$Prob(\text{placing a bid})^{SH\to OO}_{i,t,k} = \frac{1}{1+exp(-\beta_{20}[12\overline{r_{Q,t}} - (12\cdot m^{SH\to OO}_{i,t} - p^{SH\to OO}_{i,t,k}\cdot g_t)])}, \tag{20}$$

if action is taken is decided by using equation (18).

desired expenditure for buying a house[1]:

$$p^{SH\to OO}_{t,k} = \min\left(q^{SH\to OO}_{i,t} + b_{i,t}, \quad \frac{\alpha_{21} 12 y^{m,emp}_i \exp(\varepsilon_{21})}{1-\beta_{21} g_t}\right) \tag{21}$$

maximum mortgage principal for $i \in$ SH[2]:

$$q^{SH\to OO}_{i,t} = \min(\text{LTV, LTI, Affordability constraint})$$

$$q^{SH\to OO}_{i,t} = \min\left(\frac{b_i \chi_{i,t}}{1-\chi_{i,t}}, \quad y_i\psi, \quad y^{m,d}_i \nu \frac{1-(1+\frac{int}{12})^{-\alpha_{22}}}{\frac{int}{12}}\right) \tag{22}$$

downpayment for $i \in$ SH:

$$d^{SH\to OO}_{i,t} = \begin{cases} 0, & \text{if } \Xi \le 0.3 \\ \max\left(d^{SH\to OO}_{\min,i,t}, \quad h_t F^{-1}((\Xi_i - 0.3)/0.7)\right), & \text{if } \Xi > 0.3 \end{cases} \tag{23}$$

$$F^{-1}(\alpha_{23}, \beta_{23}) : \text{inverse cumulative log-normal distribution function}$$

downpayment for $i \in$ SH set by bank:

$$d^{SH\to OO}_{\min,i,t} = p^{SH\to OO}_{i,t,k} - q^{SH\to OO}_{i,t} \tag{24}$$

mortgage interest rate:

$$int_t = int^{param}_{base} + int^{spread}_t \tag{25}$$

expected HPI appreciation:

$$g_t = \alpha_{(26)}\left[\left(\frac{h_{t-1}+h_{t-2}+h_{t-3}}{h_{t-13}+h_{t-14}+h_{t-15}}\right)^{\frac{1}{24}} - 1\right] - \beta_{(26)} \tag{26}$$

[1] $SH \to OO$ can be read as: "for changing from the social housing to owner occupier state."

[2] LTI and Affordability constraint can be deactivated. LTV ratios can be agent-type and time dependent.

house price index:

$$h_t = \frac{\sum_{j=1}^{n_{t-1}^{hm\ sales}} p_{k,t-1,j}}{\sum_{j=1}^{n_{t-1}^{hm\ sales}} p_{Q,t-1,j}^{ref}} \tag{27}$$

mortgage payment:

$$m_{k,i,t} = q_{i,t} \frac{\frac{int_{t=s}}{12}}{1-(1+\frac{int_{t=s}}{12})^{-300}} \tag{28}$$

$$s = \text{mortgage contract starting period}$$

expected average rental price for Q:

$$\overline{r_{Q,t}} = 0.25 \cdot \left(\rho_{(29)} \overline{r_{Q,t-1}} + (1-\rho_{(29)}) \frac{\left(\sum_{j=1}^{n_{Q,t-1}^{rm\ sales}} r_{k,t-1,j} \right)}{n_{Q,t-1}^{rm\ sales}} \right) + 0.75 r_Q^{ref} \cdot \mathrm{RPI}_t \tag{29}$$

rental price index:

$$\mathrm{RPI}_t = \frac{\sum_{j=1}^{n_{t-1}^{rm\ sales}} r_{k,t-1,j}}{\sum_{j=1}^{n_{t-1}^{rm\ sales}} r_{Q,t-1,j}^{ref}} \tag{30}$$

probability to place bid on rental market (see arrow 5 in Figure 1):

$$Prob(\text{placing a bid})_t^{SH \to R} = \begin{cases} 0, & \text{if } i \in \text{SH already placed bid on housing market} \\ 1, & \text{if } i \in \text{SH did not place bid on housing market} \end{cases} \tag{31}$$

bid price rental market $i \in$ SH:

$$r_{i,t}^{SH \to R} = 0.33 y_{t,i}^{m,emp} \tag{32}$$

probability to go to social housing for agent $i \in$ R (see arrow 6 in Figure 1):

$$Prob(\text{to SH})_t^{R \to SH} = \begin{cases} 0, & \text{if rental contract valid} \\ 1, & \text{if rental contract expired} \end{cases} \tag{33}$$

probability of i $\in$ OO to place an offer for its home on the housing market (see arrow 2 in Figure 1):

$$Prob(\text{offering home})_t^{OO \to SH} = \tfrac{1}{12} \zeta_{34} \tag{34}$$

if action is taken is decided by using equation (18)*.*

initial offer price for home or investment property:

$$p_{t,k} = \max\left(q_{k,t}, \quad \theta(\epsilon_{35})^{p} \cdot \overline{p_{Q,t}}\right) \tag{35}$$

monthly price reduction of unsold homes or investment properties already on the market:

$$\text{if } k \text{ unsold in } t-1\text{: } p_{t,k} = \begin{cases} p_{t-1,k}(1-\exp(\varepsilon_{36})), & \text{with probability } 0.06 \\ p_{t-1,k}, & \text{with probability } 0.94 \end{cases} \tag{36}$$

total number of households:

$$n_{p,t} = \#SH_t + \#BTL_t + \#OO_t + \#R_t \tag{37}$$

expected average sale price for house with quality Q:

$$\overline{p_{Q,t}} = 0.25 \cdot \frac{\sum_{j=1}^{n_{Q,t-1}^{hm\ sales}} p_{k,Q,t-1,j}}{n_{Q,t-1}^{hm\ sales}} + 0.75 h_t p_Q^{ref} \tag{38}$$

probability to invest (see arrow 3 and 4 in Figure 1):

$$Prob(\text{placing a bid})_t^{\substack{BTL\\OO} \to BTL} = \begin{cases} 0, & \text{if } \sum m_{i,k} > \alpha_{39} y_{i,t}^{m,net} \\ 1 - \left(1 - \frac{1}{1+e^{(-\beta_{39}\Omega_{i,t})}}\right)^{\frac{1}{12}}, & \text{if else} \end{cases} \tag{39}$$

bid price for investment property[3]:

$$p_{i,t}^{\substack{BTL\\OO} \to BTL} = q_{i,t}^{\substack{BTL\\OO} \to BTL} + b_{t,i} \tag{40}$$

expected yield of capital investment:

$$\Omega_{i,t} = \frac{p_t^{\substack{BTL\\OO} \to BTL}}{d_t^{\substack{BTL\\OO} \to BTL}} (\delta_i g_t + (1-\delta_i)\overline{r_t^{yield}}) - \frac{m_{k,i,t}}{d_t^{\substack{BTL\\OO} \to BTL}} \tag{41}$$

downpayment for $i \in$ BTL:

$$d_t^{\substack{BTL\\OO} \to BTL} = \max\left(d_{\min,t}^{\substack{BTL\\OO} \to BTL}, \quad p_{t,k}^{\substack{BTL\\OO} \to BTL} \cdot (0.3 + 0.1 \cdot \epsilon_{35})\right) \tag{42}$$

[3] $p^{\substack{BTL\\OO} \to BTL}$ can be read as: "price for a house for BTL agents and Owner-Occupiers that would then become BTL agents."

downpayment for $i \in$ BTL set by bank:

$$d^{BTL \to BTL}_{\substack{OO \\ \min,t}} = p^{BTL \to BTL}_{\substack{OO \\ t,k}} - q^{BTL \to BTL}_{\substack{OO \\ t}} \tag{43}$$

maximum mortgage principal for $i \in$ BTL:

$$q^{BTL \to BTL}_{\substack{OO \\ t}} = \min(\text{LTV, interest cover contstraint (ICR)})$$

$$q^{BTL \to BTL}_{\substack{OO \\ t}} = \min\left(\frac{b_{i,t}\chi}{1-\chi}, \frac{b_{i,t}}{1 - \frac{\overline{r^{yield}_t}}{\xi \cdot i^{param}_{BTL}}} \right) \tag{44}$$

mortgage payment for $i \in$ BTL: overall rental yield[4]:

$$\overline{r^{yield}_t} = \rho_{45}\overline{r^{yield}_{t-1}} + (1-\rho_{45}) \cdot \frac{\sum_{Q=1}^{N_Q} \left(\frac{12 n^{rm\ sales}_{Q,t-1} \overline{r_{Q,t-1}} \cdot \overline{o_{Q,t-1}}}{\overline{p_{Q,t-1}}} \right)}{n^{rm\ sales}_{t-1}} \tag{45}$$

average occupancy for a house of quality Q:

$$\overline{o_{Q,t}} = \frac{\alpha_{(46)}}{\alpha_{(46)} + \rho_{(46)}\overline{o_{Q,t-1}} + (1-\rho_{(46)})\overline{D^{rm}_{t-1}}} \tag{46}$$

average days on rental market:

$$\overline{D^{rm}_{t-1}} = \frac{\sum_{j=1}^{n^{rm\ sales}_{t-1}} D^{rm}_{k,t-1,j}}{n^{rm\ sales}_{t-1}} \tag{47}$$

initial offer rental price:

$$r^{BTL \to BTL_{k,t}} = \theta(\epsilon_{35})^r \cdot \overline{r_{Q,t}} \tag{48}$$

price reduction rental offer

$$\text{if } k \text{ not rented out in } t-1\text{: } r^{BTL \to BTL}_{k,t} = 0.95 r^{BTL \to BTL}_{k,t-1} \tag{49}$$

probability of placing offer for investment property (see arrow 4 in Figure 1):

$$Prob(\text{placing offer})^{BTL \to BTL}_{t,k} = \begin{cases} 0, & \text{if } i \text{ has only 2 houses} \\ 1 - \left(\frac{1}{1+e^{(-\beta_{39}\Psi_{k,i,t})}} \right)^{\frac{1}{12}}, & \text{else} \end{cases} \tag{50}$$

[4]the sum is the average flow yield, which I could put in a separate formula

expected effective yield:

$$\Psi_{k,i,t} = \frac{\overline{p_{Q,t}}}{u_{k,t}}(\delta_i g_t + (1-\delta_i) r^{yield}_{k,t}) - \frac{m_{k,i,t}}{u_{k,t}} \tag{51}$$

equity of house k:

$$u_{k,t} = \overline{p_{Q,t}} - q_t^{{}^{BTL}_{OO}\rightarrow BTL} \tag{52}$$

rental yield of house k:

$$r^{yield}_{k,t} = \frac{12 r_{k,t}}{\overline{p_{Q,t}}} \tag{53}$$

housing market clearing process
round 1, step 1:

$$\forall\ BTL \max_{k \leq \max\left(r^{exp\ yield}_{k,t}\right)} \left(p^{offer}_{t,k} \leq p_t^{{}^{BTL}_{OO}\rightarrow BTL}\right)$$

$$\forall\ SH \max_{k \leq \max(Q)} \left(p^{offer}_{t,k} \leq p_t^{SH\rightarrow OO}\right)$$

round 1, step2:

$$\begin{aligned} &offer \text{ is machted, if } \exists!\ BTL \vee SH : p^{offer}_{t,k} < p^{bid}_t \ \rightarrow\ p_{k,t} = p^{offer}_{t,k} \\ &\text{else } p^{offer}_{t,k} = p^{bid-up}_{k,i}, \text{ then pick randomly one bid} \\ &\qquad \text{with } p^{bid-up}_{k,i} \leq p^{bid}_t \ \rightarrow\ p_{k,t} = p^{bid-up}_{k,i} \end{aligned} \tag{54}$$

round 1, step 3:

return unmatched offers and bids into the pool

round 2 and following:
repeat until:

- $bid = 0$
- $offer = 0$

set of offer prices

$$p^{offer}_{t,k} = \{p^{BTL\rightarrow BTL}_{k,t}, p^{OO\rightarrow SH}_{t,k}\} \tag{55}$$

set of bid prices:

$$p^{bid}_t = \{p_t^{{}^{BTL}_{OO}\rightarrow BTL}, p_t^{SH\rightarrow OO}\} \tag{56}$$

expected rental gross yield

$$r_{k,t}^{exp\ yield} = \frac{12\overline{r_{Q,t}} \cdot o_Q^{exp}}{p_{k,t}^{offer}} \tag{57}$$

expected occupancy:

$$o_Q^{exp} = \frac{547}{547 + \overline{D_{t-1}^{rm}}} \tag{58}$$

bid-up price in auction:

$$p_{k,i}^{\text{bid up}} = 1.0075^l \tag{59}$$

choosing bidding-up variable l from a geometric distribution:

$$P(l) = (1 - e^{-7B/30})^{l-1} e^{-7B/30} \tag{60}$$

rental market clearing process
round 1, step 1:

$$\forall\ SH \max_{k \leq \max(Q)} \left(r_{t,k}^{BTL \to BTL} \leq r_t^{SH \to R} \right)$$

round 1, step2:

$$\begin{aligned} &offer \text{ is machted, if } \exists!\ SH : r_{t,k}^{BTL \to BTL} < r_t^{SH \to R} \ \rightarrow \ r_{k,t} = r_{t,k}^{BTL \to BTL} \\ &\quad \text{else } r_{t,k}^{BTL \to BTL} = p_{k,i}^{bid-up}, \text{ then pick randomly one bid} \\ &\qquad \text{with } p_{k,i}^{bid-up} \leq r_t^{SH \to R} \rightarrow \ r_{k,t} = p_{k,i}^{bid-up} \end{aligned} \tag{61}$$

round 1, step 3:

return unmatched offers and bids into the pool

round 2 and following:
repeat until:

- $bid = 0$
- $offer = 0$