Harmonic Signals and Elementary Operations

Exercise 1:

A harmonic signal has the form:

$$y(t) = A\cos(\omega t + \phi)$$

In this, A, ω and ϕ are parameters.

Note: *Plot the graphs of exercises A) and E) in one figure using the subplot(2,1,1) and subplot(2,1,2) commands respectively.*

For $A = 1$, $\omega = 2\pi f$ with $f = 5\,Hz$ and $\phi = 0\,rad$,

A) Plot the signal *y(t)* using a time vector ranging from 0 to 1 second that has 200 time steps per period of *y(t)*.
B) What is the effect of A in *y(t)*? (Vary this parameter in the code to see the effect)
C) What is the effect of ω in *y(t)*? (Vary this parameter in the code to see the effect)
D) What is the effect of ϕ in *y(t)*? (Vary this parameter in the code to see the effect)

In essence, our answer in A) is a discretization of the signal *y(t)*. However, it is plotted versus nT_s instead of versus n. Moreover, the sample time T_s is taken so small that it looks as if it's a continuous-time signal. You can see this by plotting markers using "*plot(t,y,'-x')*". The marker *'-x'* can be replaced by a variety of other markers: *'-o'*, *'-.'*, *'-s'*, etc.

For $A = 1$, $\omega = 2\pi f$ with $f = 5\,Hz$ and $\phi = 0\,rad$,

E) Plot $y[n]$, the discrete version of *y(t)*, versus n with $\Omega = \frac{1}{10} * 2\pi$ using the *stem(x,y)* command. Make sure you choose n such that the domain in continuous time is $0 \le t \le 1$ s.
F) What is the sample time T_s in E), ?

Using:

$$x_1(t) = 0.5\cos\left(2\pi 5t + \frac{1}{4}\pi\right)$$
$$x_2(t) = 0.3\cos(2\pi 10t)$$

G) What is A in x_1?
H) What is ϕ in x_2?
I) What is ω in x_1?
J) What is the period of x_2?

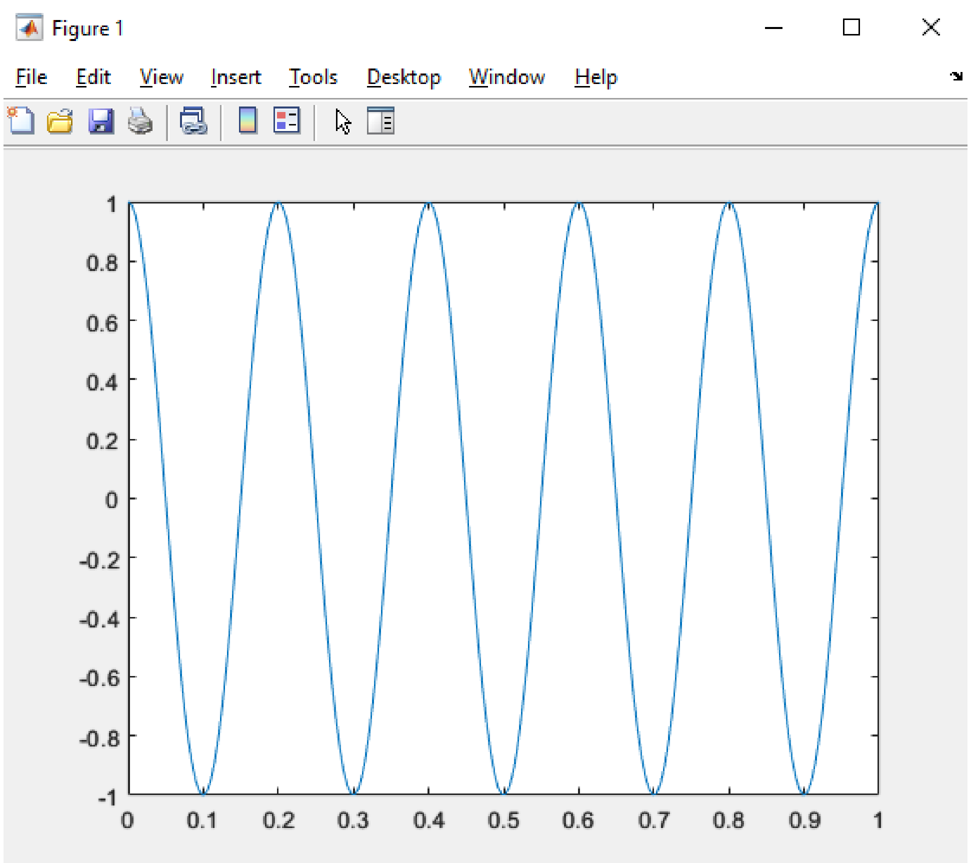
Assume now that we sample with a sample frequency of 7 Hz, which leaves us with $x_1[n]$ and $x_2[n]$.

$$x_1[n] = 0.5\cos\left(\frac{10\pi}{7}n + \frac{1}{4}\pi\right)$$
$$x_2[n] = 0.3\cos\left(\frac{20\pi}{7}n\right)$$

Exercise 1

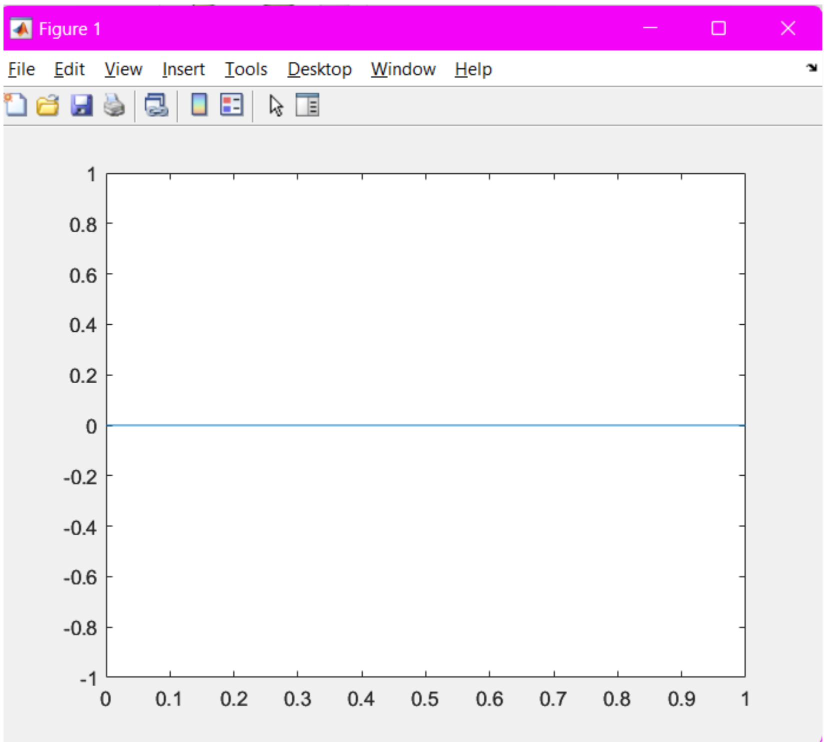
Task A

```
A=1;

f=5;

fi=0;

w=2*pi()*f;


t=linspace(0,1,200);

for ii=1:length(t)

 y(ii)=A*cos(w*t(ii)+fi);

 end

%y=A*cos(w*t+fi); (Another solution to make a graph)


plot(t,y)
```

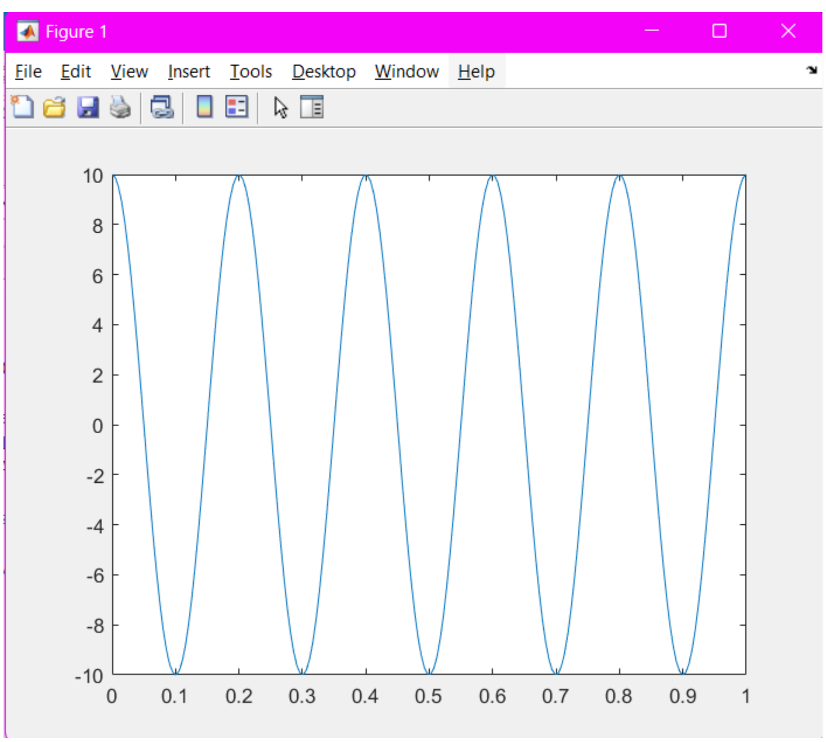


Task B

A= 0

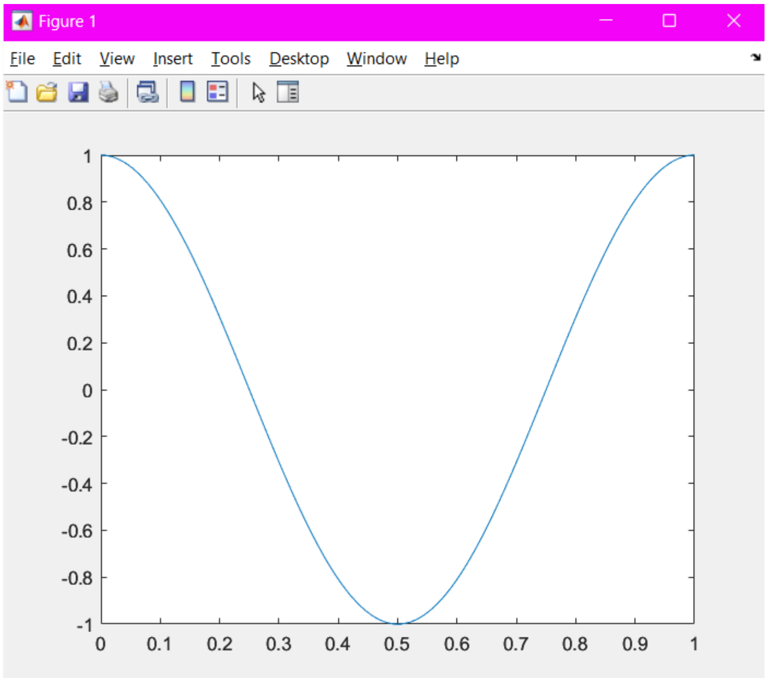


A = 10



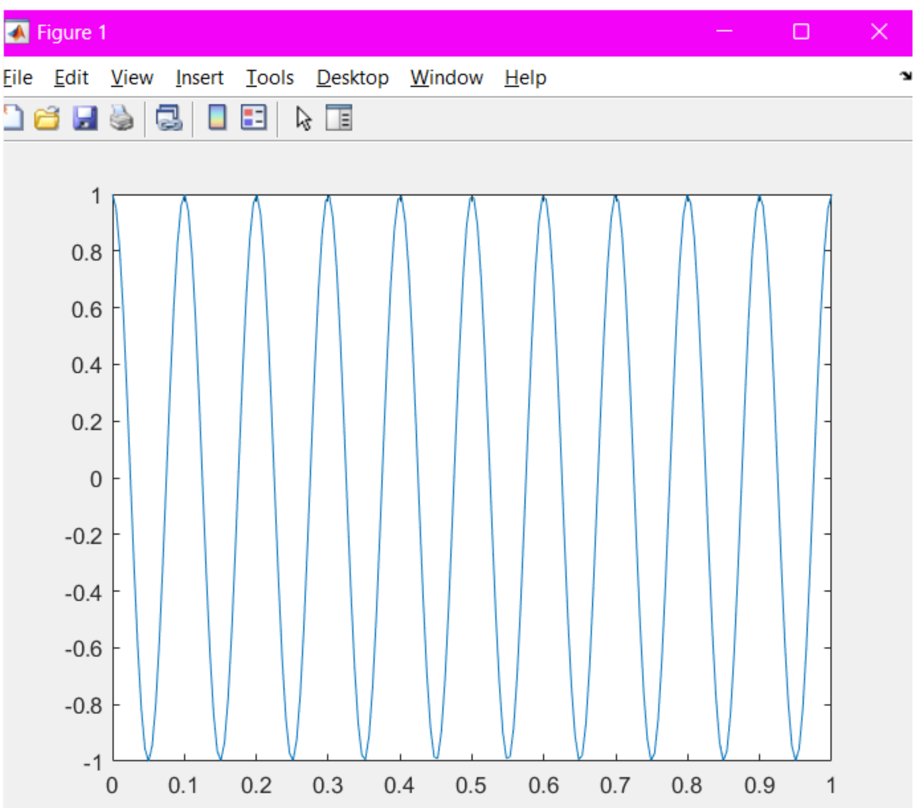
If you change A , then Y axis increases, amplitude becomes higher.

Task C



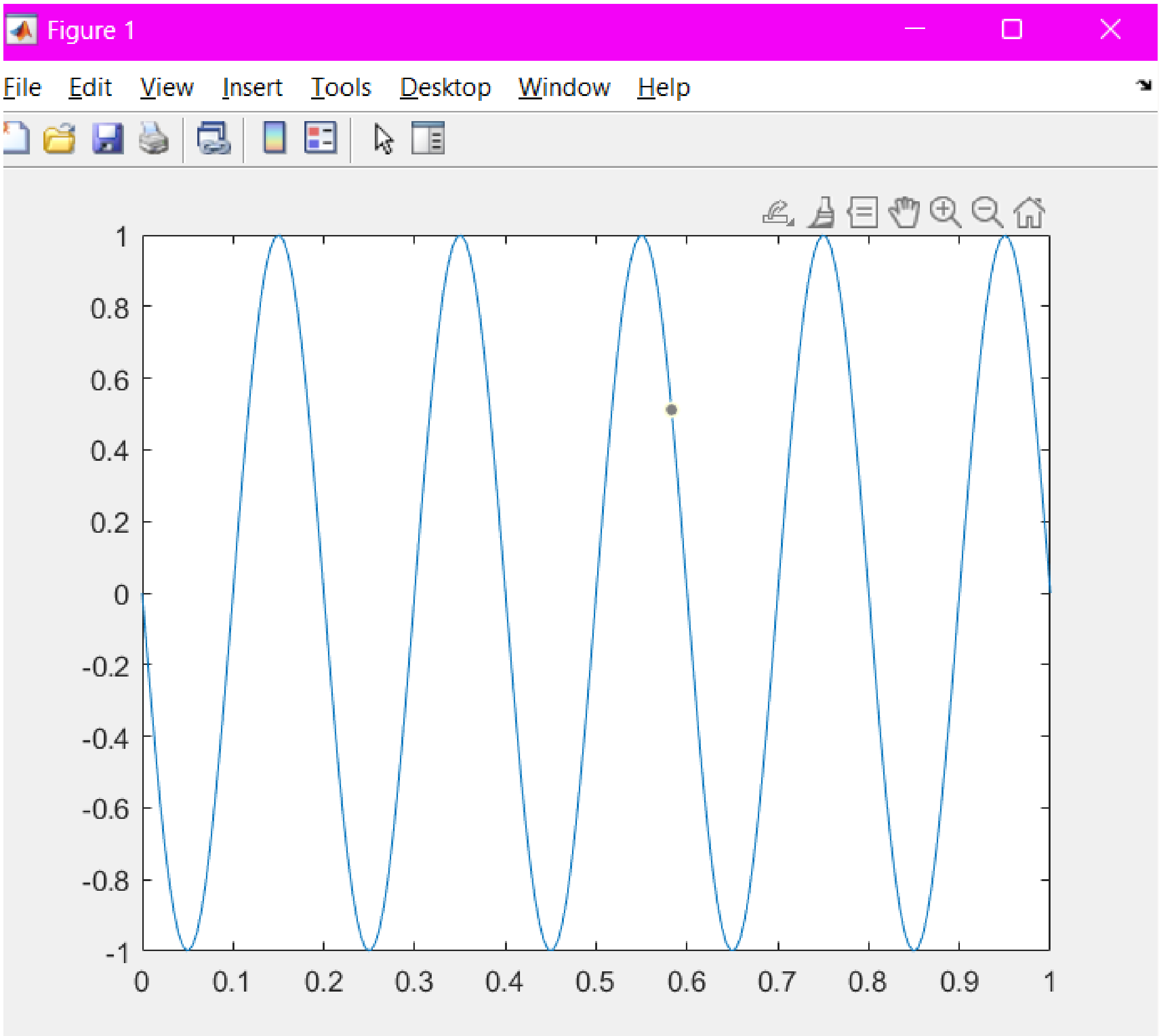
W=6.28 if f = 1

If you change f, then omega also changes and if omega is changed, then period of the graph decreases (is decreasing). Check the graph under, in this case w(omega) = 62.83, f = 10 Hz



Task D

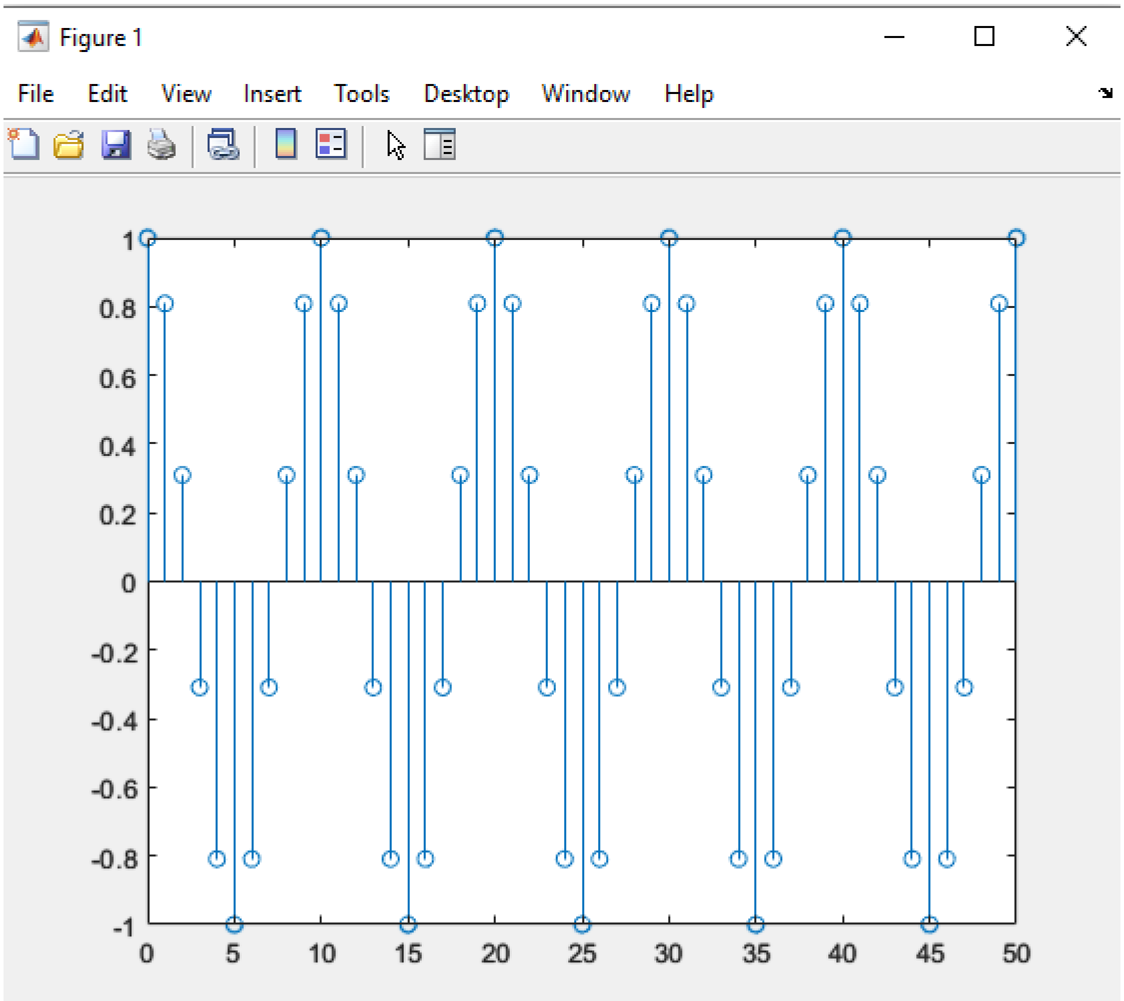
If you change ϕ, then graph shifts on horizontal axis making phase shifts.



In this case graph fi is pi/2, so graph shifts for 90 degrees from Task A graph.

Task E

```
t=linspace(0,1,201);
%t=0:1;
A=1;
f=5;
fi=0;
w=2*pi()*f;
W=((1/10)*2*pi());
Ts=(W/w);
n=(t./Ts);
n1=round(n);
N=unique(n1);
y=A*cos(W*N+fi);
stem(N,y)
```



Task F

Sample time Ts = 0.02 s

Tasks G, H, I, J

G) A = 0,5

H) fi = 0 degrees

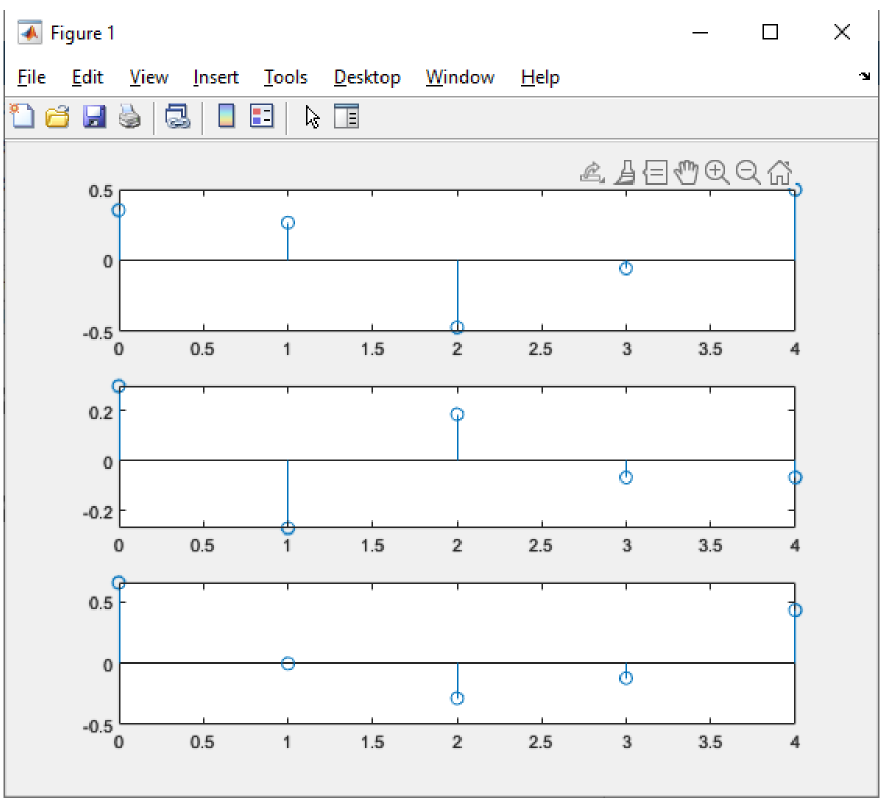
I) w = 2*pi*5

J) w=2*pi/T ➔ 2*pi*10 = 2*pi/T ➔ T = 1/10 = 0,1 s

Exercise 2

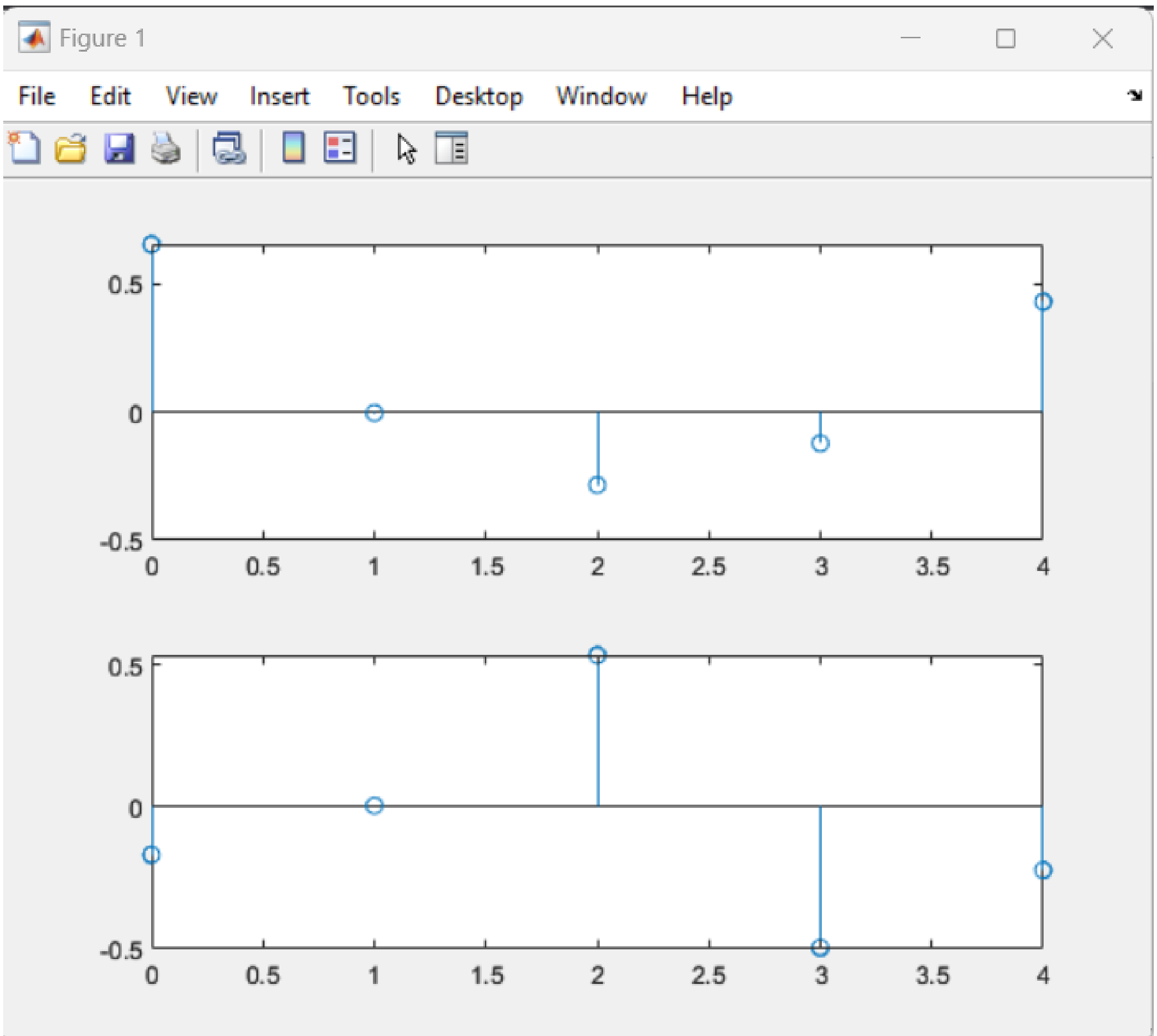
Tasks K and L

```
n=0:4;
subplot(3,1,1);
x1=0.5*cos(((10*pi)/7)*n+(1/4)*pi);
stem(n,x1)

subplot(3,1,2);
x2=0.3*cos(((20*pi)/7)*n);
stem(n,x2)

subplot(3,1,3); %sum
x3=x1+x2;
stem(n,x3)
```



Task M

```
subplot(2,1,1); %mirror
n=-10:10;
x1=0.5*cos(((10*pi)/7)*n+(1/4)*pi);
x2=0.3*cos(((20*pi)/7)*n);
x4=x1+x2;
stem(n(11:15),x4(11:15))

subplot(2,1,2); %delay
x1=0.5*cos(((10*pi)/7)*n+2+(1/4)*pi);
x2=0.3*cos(((20*pi)/7)*n);
x5=x1+x2;
stem(n(11:15),x5(11:15))
```



Exercise 3

Exercise 3

2nd order form $\Rightarrow \frac{\omega_0^2}{(j\omega)^2 + 2\beta\omega_0 j\omega + \omega_0^2}$

a) $H(j\omega) = \frac{1}{j\omega^2 + 0{,}01 j\omega + 1}$

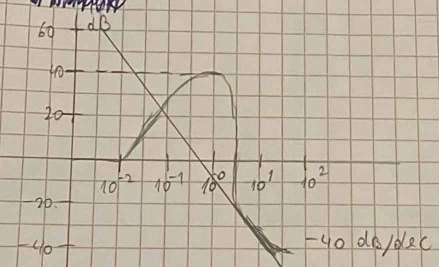
$\frac{1}{|j\omega^2 + 0{,}01 j\omega + 1|} = |H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + (0{,}01\omega)^2}}$

$\omega \to 0: |H(0)| = \frac{1}{\sqrt{(1-0)^2 + (0{,}01 \cdot 0)^2}} = \frac{1}{\sqrt{1}} = 1 \longrightarrow 0\,dB$

$\omega \to \infty: |H(\infty)| = \frac{1}{\sqrt{(1-\infty)^2 + (0{,}01 \cdot \infty)^2}} = \frac{1}{\sqrt{\infty}} \approx 0 \to -\infty\,dB$

$\omega_0^2 = 1 \Rightarrow \omega_0 = \sqrt{1} = 1$

$\omega \to \omega_0 \Rightarrow |H(\omega_0)| = \frac{1}{\sqrt{(1-1)^2 + (0{,}01 \cdot 1)^2}} = \frac{1}{\sqrt{0^2 + 0{,}01^2}} = \frac{1}{0{,}01} = 100 \to 40\,dB$



B) $\omega_n^2 = 1 \Rightarrow \omega_n = \sqrt{1} = 1$

Magnitude of ω_n is 40 dB, that can be seen above.