Math 100 Homework 4

1- **Acknowledgment Statement:** Please write the following statement as the answer to Exercise 1 and place your signature right below the statement.

"I acknowledge that it is my responsibility to carefully read the class notes **before** attempting the homework problems. I understand that what is in the class notes is the minimum I should know, and I should not expect to pass this course if I do not fully understand the material covered in the class notes."

2- Which of the following statements describes our precise definition of a vector in $\mathbb{R}^n$?

(a) We defined a vector in $\mathbb{R}^n$ as a "quantity" having magnitude as well as direction.

(b) We defined a vector in $\mathbb{R}^n$ as a directed line segment.

(c) We defined a vector in $\mathbb{R}^n$ as an element of $\mathbb{R}^n$, that is, a vector in $\mathbb{R}^n$ is a matrix with exactly one column and n entries.

3- Let $\mathbf{v} = \begin{bmatrix}-1\\1\end{bmatrix}$. Compute the following scalar multiples of v. In each case, draw a diagram depicting v and the computed scalar multiple of v with arrows originating from the origin.

(a) $2\mathbf{v}$

(b) $-\mathbf{v}$

(c) $\frac{1}{2}\mathbf{v}$

(d) $\frac{-1}{2}\mathbf{v}$

a $\begin{bmatrix}-2\\2\end{bmatrix}$ c $\begin{bmatrix}-1/2\\1/2\end{bmatrix}$

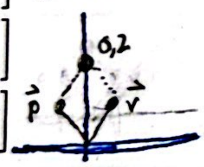
b $\begin{bmatrix}1\\-1\end{bmatrix}$ d $\begin{bmatrix}1/2\\-1/2\end{bmatrix}$

4- Compute the following sums. In each case, draw a diagram to visually verify the parallelogram rule.

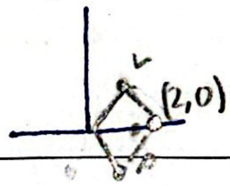
(a) $\underset{\vec{v}}{\begin{bmatrix}1\\1\end{bmatrix}} + \underset{\vec{p}}{\begin{bmatrix}-1\\1\end{bmatrix}} = \begin{bmatrix}0\\2\end{bmatrix}$

(b) $\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}1\\-1\end{bmatrix}$

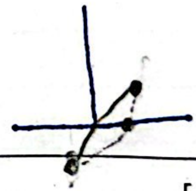
(c) $\begin{bmatrix}-1\\-1\end{bmatrix} + \begin{bmatrix}2\\1\end{bmatrix}$



A $\underset{\vec{v}}{\begin{bmatrix}1\\1\end{bmatrix}} + \begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$



B $\underset{\vec{v}}{\begin{bmatrix}-1\\-1\end{bmatrix}} + \underset{\vec{p}}{\begin{bmatrix}2\\1\end{bmatrix}} = \begin{bmatrix}1\\0\end{bmatrix}$



C

5- What type of geometric object do we obtain when considering the collection of all scalar multiples of the vector $\mathbf{v} = \begin{bmatrix}-1\\-2\end{bmatrix}$?

A line through $\vec{v}$ & origin

6- What type of geometric object do we obtain when considering the collection of all scalar multiples of the vector $\mathbf{v} = \begin{bmatrix}1\\1\\0\end{bmatrix}$?

A line through $\vec{v}$ & origin

7- What type of geometric object do we obtain when considering the collection of all the linear combinations of the vectors $\mathbf{v} = \begin{bmatrix}1\\1\\0\end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix}1\\1\\1\end{bmatrix}$?

$\left\{c\begin{bmatrix}1\\1\\0\end{bmatrix} + d\begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$ so $\left\{\begin{bmatrix}c\\c\\0\end{bmatrix} + \begin{bmatrix}d\\d\\d\end{bmatrix}\right\}$ $\begin{matrix}c\\c\\0\end{matrix}$ or $\begin{matrix}d\\d\\d\end{matrix}$ or all real numbers in x,y,z

A plane in 3D space through origin & $\vec{v}$ & $\vec{w}$

8- Let L be the line $y = 2x$ in the xy-plane. What type of geometric object do we obtain if we add the vector $\mathbf{p} = \begin{bmatrix}-1\\0\end{bmatrix}$ to each point on this line? We will get a new line through point $p = \begin{bmatrix}-1\\0\end{bmatrix}$ parallel to original line

9- Determine if b is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix}1\\0\\1\end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix}-2\\3\\-2\end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix}-6\\7\\5\end{bmatrix}, \mathbf{b} = \begin{bmatrix}11\\-5\\9\end{bmatrix}$$

Yes b is a linear combo of a_1, a_2, a_3 When $x_3 = \frac{-2}{11}$, $x_2 = \frac{-41}{33}$, $x_1 = \frac{245}{33}$

9 Determine if B is a linear Combo of a_1, a_2, a_3

$a_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $a_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$ $a_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$

$\vec{b}$ is linear combo of $\vec{a_1}, \vec{a_2}, \vec{a_3}$ if there are scalars x_1, x_2, x_3 that $x_1 \vec{a_1}, x_2 \vec{a_2}, x_3 \vec{a_3} = \vec{b}$

$$\begin{aligned} x_1 - 2x_2 - 6x_3 &= 11 \\ 0x_1 + 3x_2 + 7x_3 &= -5 \\ x_1 - 2x_2 + 5x_3 &= 9 \end{aligned} \quad \begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix} = \begin{bmatrix} \boxed{1} & 0 & 0 & 245/33 \\ 0 & \boxed{1} & 0 & -41/33 \\ 0 & 0 & \boxed{1} & -2/11 \end{bmatrix}$$

Yes b is a linear combo of a_1, a_2, a_3 when

$x_3 = \frac{-2}{11}$, $x_2 = \frac{-41}{33}$, $x_1 = \frac{245}{33}$

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10- Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, let $\mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 7 \end{bmatrix}$, and let W be the set of all linear combinations of the columns of A.

a. Is b in W? No since RHS = pivot column therefore there doesnt exist numbers that makes b a linear combo of w.

b. Show that the second column of A is in W. $\vec{0A_1} + \vec{1A_2} + \vec{0A_3} = \vec{A_2}$

Bold Problem

11- Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$. For what value(s) of h is y in the plane generated by $\mathbf{v}_1$ and $\mathbf{v}_2$?

10 $\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 7 \end{bmatrix}$ reduced echelon form $\sim \begin{bmatrix} \boxed{1} & 0 & 3 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$

$\frac{-b}{a}$ $\frac{1}{-2}$ $\frac{-b}{c}$

$\begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$

$\frac{1}{2}$ row 1 + row 2 → row 2

$-\frac{1}{2}$ row 1 + row 3 → row 3

1 0 3 | -1 0 -3
-1 8 5 | 1 -2 1
0 8 8 | 0 -2 2

$\begin{bmatrix} 2 & 0 & 6 \\ 0 & \boxed{8} & 8 \\ 0 & -2 & 2 \end{bmatrix}$

$\frac{2}{8}$ $\frac{1}{4}$

$\frac{1}{4}$(row 2) → row 3 → row

0 2 2
0 -2 2
0 0 4 row 3

$\begin{bmatrix} 2 & 0 & 6 \\ 0 & 8 & 8 \\ 0 & 0 & 4 \end{bmatrix}$ $\frac{1}{4}$ row 3 → row 3

$\begin{bmatrix} 2 & 0 & 6 \\ 0 & 8 & 8 \\ 0 & 0 & 1 \end{bmatrix}$ $\frac{-8}{1}$

-8(row 3) → row 2 → row 2

0 8 8
0 0 -8
0 8 0

$\begin{bmatrix} 2 & 0 & 6 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

-6

2 0 6
0 0 -6
2 0 0

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\frac{1}{8}$ row 2 → row 2

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\frac{1}{2}$ row 1 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$