Math 100 Homework 6

1- **Acknowledgment Statement:** Please write the following statement as the answer to Exercise 1 and place your signature right below the statement.

"I acknowledge that it is my responsibility to carefully read the class notes before attempting the homework problems. I understand that what is in the class notes is the minimum I should know, and I should not expect to pass this course if I do not fully understand the material covered in the class notes."

2- Restate each problem using a matrix equation, and then answer the problem using the procedure discussed in class for addressing questions of Type 2.

2-1) Is Span$\left\{\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}3\\4\end{bmatrix}\right\}=\mathbb{R}^2$? $\begin{bmatrix}1&3\\2&4\end{bmatrix}\overset{\text{matlab}}{\sim}\begin{bmatrix}1&0\\0&-2\end{bmatrix}$ Pivot column in every row so true so $\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}3\\4\end{bmatrix}$ span of $\mathbb{R}^2$

2-2) Is Span$\left\{\begin{bmatrix}1\\2\\0\end{bmatrix},\begin{bmatrix}1\\0\\5\end{bmatrix}\right\}=\mathbb{R}^3$? $\begin{bmatrix}1&1\\2&0\\0&5\end{bmatrix}\overset{\text{matlab}}{\sim}\begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix}$ Not pivot column in every row so Not true $\begin{bmatrix}1\\2\\0\end{bmatrix},\begin{bmatrix}1\\0\\5\end{bmatrix}$ Not span of $\mathbb{R}^3$

2-3) Is Span$\left\{\begin{bmatrix}1\\2\\1\end{bmatrix},\begin{bmatrix}2\\3\\-1\end{bmatrix},\begin{bmatrix}3\\5\\0\end{bmatrix}\right\}=\mathbb{R}^3$? $\begin{bmatrix}1&2&3\\2&3&5\\1&-1&0\end{bmatrix}\overset{\text{matlab}}{\sim}\begin{bmatrix}1&0&1\\0&1&1\\0&0&0\end{bmatrix}$ Not pivot column in every row so $\begin{bmatrix}1\\2\\1\end{bmatrix},\begin{bmatrix}2\\3\\1\end{bmatrix},\begin{bmatrix}3\\5\\0\end{bmatrix}$ not span of $\mathbb{R}^3$

2-4) Is Span$\left\{\begin{bmatrix}1\\0\\0\\2\end{bmatrix},\begin{bmatrix}4\\1\\2\\9\end{bmatrix},\begin{bmatrix}1\\3\\6\\5\end{bmatrix},\begin{bmatrix}2\\-4\\7\\-7\end{bmatrix}\right\}=\mathbb{R}^4$? $\begin{bmatrix}1&4&1&2\\0&1&3&-4\\0&2&6&7\\2&9&5&7\end{bmatrix}\overset{\text{matlab}}{\sim}\begin{bmatrix}1&0&-11&0\\0&1&3&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$ No pivot column in every row so $\begin{bmatrix}1\\0\\0\\2\end{bmatrix},\begin{bmatrix}4\\1\\2\\9\end{bmatrix},\begin{bmatrix}1\\3\\6\\5\end{bmatrix},\begin{bmatrix}2\\-4\\7\\-7\end{bmatrix}$ Not span of $\mathbb{R}^4$

> *Hint:* Recall that the question "Is Span$\{\mathbf{a}_1,\cdots,\mathbf{a}_n\}=\mathbb{R}^m$?" can be restated using a matrix equation as follows:
> Let $A=[\mathbf{a}_1 \ \cdots \ \mathbf{a}_n]$. Does $A\mathbf{x}=\mathbf{b}$ have a solution for EVERY vector $\mathbf{b}$ in $\mathbb{R}^m$?

3- Suppose

free $= x_2, x_5$
basic $= x_1, x_3, x_4$

$$x_1 = 2x_2 - 3x_5 + 5,\quad x_2 \text{ is free},\quad x_3 = x_2 - 6x_5 - 1,\quad x_4 = -3x_2 + 5,\quad x_5 \text{ is free}$$

$$\vec{x}=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}2x_2-3x_5+5\\x_2\\x_2-6x_5-1\\-3x_2+5\\x_5\end{bmatrix}=\underbrace{\begin{bmatrix}5\\0\\-1\\5\\0\end{bmatrix}}_{\text{isolated constants}}+\underbrace{\begin{bmatrix}2x_2\\x_2\\x_2\\-3x_2\\0\end{bmatrix}}_{\text{terms } x_2}+\underbrace{\begin{bmatrix}-3x_5\\0\\-6x_5\\0\\x_5\end{bmatrix}}_{\text{terms } x_5}$$

$$\left\{\underbrace{\begin{bmatrix}5\\0\\-1\\5\\0\end{bmatrix}}_{\vec{p}}+x_2\begin{bmatrix}2\\1\\1\\-3\\0\end{bmatrix}+x_5\begin{bmatrix}-3\\0\\-6\\0\\1\end{bmatrix}\right\}$$

x_2 & x_5 are free

is the general solution of a linear system. Write the solution in *parametric vector form.*

4- Suppose

$$x_1 = 2+2x_2,\quad x_2 \text{ is free},\quad x_3 = -2$$

$$\vec{x}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}2+2x_2\\x_2\\-2\end{bmatrix}=\underbrace{\begin{bmatrix}2\\0\\-2\end{bmatrix}}_{\text{isolated constants}}+\underbrace{\begin{bmatrix}2x_2\\x_2\\0\end{bmatrix}}_{\text{terms } x_2}$$

$$\left\{\underbrace{\begin{bmatrix}2\\0\\-2\end{bmatrix}}_{\vec{p}}+x_2\begin{bmatrix}2\\1\\0\end{bmatrix}\right\}$$

is the general solution of a linear system. Write the solution in *parametric vector form.*

5- Suppose

$$x_1 = -6x_2 - 3x_4,\quad x_2 \text{ is free},\quad x_3 = 5+4x_4,\quad x_4 \text{ is free},\quad x_5 = 7$$

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}-6x_2-3x_4\\x_2\\5+4x_4\\x_4\\7\end{bmatrix}=\underbrace{\begin{bmatrix}0\\0\\5\\0\\7\end{bmatrix}}_{\text{isolated constants}}+\underbrace{\begin{bmatrix}-6x_2\\x_2\\0\\0\\0\end{bmatrix}}_{\text{terms in } x_2}+\underbrace{\begin{bmatrix}-3x_4\\0\\4x_4\\x_4\\0\end{bmatrix}}_{\text{terms in } x_4}$$

$$\left\{\underbrace{\begin{bmatrix}0\\0\\5\\0\\7\end{bmatrix}}_{\vec{p}}+x^2\begin{bmatrix}-6\\1\\0\\0\\0\end{bmatrix}+x_4\begin{bmatrix}-3\\0\\4\\1\\0\end{bmatrix}\right\}$$

is the general solution of a linear system. Write the solution in *parametric vector form.*

6- Suppose

$x_1 = -24 + 2x_3 - 3x_4$
$x_2 = -7 + 2x_3 - 2x_4$
x_3 is free
x_4 is free
$x_5 = 4$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -24+2x_3-3x_4 \\ -7+2x_3-2x_4 \\ x_3 \\ x_4 \\ 4 \end{bmatrix} \quad \underbrace{\begin{bmatrix} -24 \\ -7 \\ 0 \\ 0 \\ 4 \end{bmatrix}}_{\text{iso constant}} \quad \underbrace{\begin{bmatrix} 2x_3 \\ 2x_3 \\ x_3 \\ 0 \\ 0 \end{bmatrix}}_{x_3 \text{ terms}} \quad \underbrace{\begin{bmatrix} -3x_4 \\ -2x_4 \\ 0 \\ x_4 \\ 0 \end{bmatrix}}_{x_4 \text{ terms}} \quad \left\{ \underbrace{\begin{bmatrix} -24 \\ -7 \\ 0 \\ 0 \\ 4 \end{bmatrix}}_{\vec{p}} + x_3 \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right.$$

is the general solution of a linear system. Write the solution in *parametric vector form.*

7- Write the solution set of the given homogeneous system in parametric vector form.

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 0 \\ 2x_1 + x_2 - 3x_3 &= 0 \\ -1x_1 + x_2 \quad &= 0 \end{aligned}$$

8- Describe the solutions of the following system in parametric vector form, and provide a geometric comparison with the solution set in Exercise 7.

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 5 \\ 2x_1 + x_2 - 3x_3 &= 13 \\ -x_1 + x_2 \quad &= -8 \end{aligned}$$

9-1) Write the solution of $x_1 + 9x_2 - 4x_3 = 0$ in *parametric vector form.*

9-2) Write the solution of $x_1 + 9x_2 - 4x_3 = -2$ in *parametric vector form,* and provide a geometric comparison with the solution set in part (1).

9-1 $\quad x_1 + 9x_2 - 4x_3 = 0 \quad \begin{array}{cccc} x_1 & x_2 & x_3 & RHS \\ \end{array}$ $\begin{bmatrix} 1 & 9 & -4 & 0 \end{bmatrix}$

reduced echelon form

basic variable $= x_1$
Free variable $= x_2, x_3$

RHS ≠ pivot column so system is consistent
One free variable so infinite solution

$x_1 + 9x_2 - 4x_3 = 0$

$x_1 = -9x_2 + 4x_3$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -9x_2 + 4x_3 \\ x_2 \\ x_3 \end{bmatrix} \quad \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\text{iso constants}} \quad \underbrace{\begin{bmatrix} -9x_2 \\ x_2 \\ 0 \end{bmatrix}}_{\text{terms involve } x_2} \quad \underbrace{\begin{bmatrix} 4x_3 \\ 0 \\ x_3 \end{bmatrix}}_{\text{terms involve } x_3} = \left\{ x_2 \begin{bmatrix} -9 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} \right\}$$

9-2 $\quad x_1 + 9x_2 - 4x_3 = -2 \quad$ ~~$x_1\ x_2\ x_3$~~ RHS $\begin{bmatrix} 1 & 9 & -4 & -2 \end{bmatrix}$

basic variable $= x_1$
Free variable $= x_2, x_3$

RHS ≠ pivot column so system consistent
at lest one free variable so infinite sol

$x_1 = -2 - 9x_2 + 4x_3$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 - 9x_2 + 4x_3 \\ x_2 \\ x_3 \end{bmatrix} \quad \underbrace{\begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}}_{\text{is constants}} \quad \underbrace{\begin{bmatrix} -9x_2 \\ x_2 \\ 0 \end{bmatrix}}_{\text{terms involve } x_2} \quad \underbrace{\begin{bmatrix} 4x_3 \\ 0 \\ x_3 \end{bmatrix}}_{\text{terms involve } x_3} = \left\{ \underbrace{\begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}}_{\vec{p}} + x_2 \begin{bmatrix} 9 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Geometric Description

9-1 is a homogenous equation & we obtain 9-2 by adding vector $\vec{p}$, by adding vector $\vec{p}$ we shift the plane by vector $\vec{p}$ & new shifted plane will be parrall to original plane.