Math 100 Homework 5

1- **Acknowledgment Statement:** Please write the following statement as the answer to Exercise 1 and place your signature right below the statement.

"I acknowledge that it is my responsibility to carefully read the class notes before attempting the homework problems. I understand that what is in the class notes is the minimum I should know, and I should not expect to pass this course if I do not fully understand the material covered in the class notes."

2- What does it mean to say that v is a vector in $\mathbb{R}^5$? a vector with 5 entries or in 5th dimensional space

3- Suppose u and v are two vectors in $\mathbb{R}^3$. What do we mean by Span{u, v}? Span{u,v} means all linear combinations of $\vec{u}$ & $\vec{v}$

4- For each of the following linear systems, write the corresponding vector equation and state the *question of existence* in the language of vectors. (You do not need to answer the *question of existence*; just state it.)

4-1) $\begin{cases} 2x_1 \quad\quad -6x_3 = -8 \\ x_2 + 2x_3 = 3 \\ 3x_1 + 6x_2 - 2x_3 = -4 \end{cases}$

$x_1\begin{bmatrix}2\\0\\3\end{bmatrix} + x_2\begin{bmatrix}0\\1\\6\end{bmatrix} + x_3\begin{bmatrix}-6\\2\\-2\end{bmatrix} = \begin{bmatrix}-8\\3\\-4\end{bmatrix}$ is $\begin{bmatrix}-8\\3\\-4\end{bmatrix}$ a linear combo of $\begin{bmatrix}2\\0\\3\end{bmatrix},\begin{bmatrix}0\\1\\6\end{bmatrix},\begin{bmatrix}-6\\2\\-2\end{bmatrix}$

4-2) $\begin{cases} 2x_1 + 4x_2 + 4x_3 = 4 \\ x_2 - 2x_3 = -2 \\ 2x_1 + 3x_2 \quad\quad = 0 \end{cases}$

$x_1\begin{bmatrix}2\\0\\2\end{bmatrix} + x_2\begin{bmatrix}4\\1\\3\end{bmatrix} + x_3\begin{bmatrix}4\\-2\\0\end{bmatrix} = \begin{bmatrix}4\\-2\\0\end{bmatrix}$ is $\begin{bmatrix}4\\-2\\0\end{bmatrix}$ a linear combo of $\begin{bmatrix}2\\0\\2\end{bmatrix},\begin{bmatrix}4\\1\\3\end{bmatrix},\begin{bmatrix}4\\-2\\0\end{bmatrix}$

4-3) $\begin{cases} x_1 - 2x_2 - x_3 = 4 \\ -2x_1 + 4x_2 - 5x_3 = 6 \end{cases}$

$x_1\begin{bmatrix}1\\-2\end{bmatrix} + x_2\begin{bmatrix}-2\\4\end{bmatrix} + x_3\begin{bmatrix}-1\\-5\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix}$ is $\begin{bmatrix}4\\6\end{bmatrix}$ a linear combo of $\begin{bmatrix}1\\-2\end{bmatrix},\begin{bmatrix}-2\\4\end{bmatrix},\begin{bmatrix}-1\\-5\end{bmatrix}$

4-4) $\begin{cases} x_1 - 2x_2 - x_3 + 3x_4 = 0 \\ -2x_1 + 4x_2 + 5x_3 - 5x_4 = 3 \\ 3x_1 - 6x_2 - 6x_3 + 8x_4 = 2 \end{cases}$

$x_1\begin{bmatrix}1\\-2\\3\end{bmatrix} + x_2\begin{bmatrix}-2\\4\\-6\end{bmatrix} + x_3\begin{bmatrix}-1\\5\\-6\end{bmatrix} + \begin{bmatrix}3\\-5\\8\end{bmatrix} = \begin{bmatrix}0\\3\\2\end{bmatrix}$ is $\begin{bmatrix}0\\3\\2\end{bmatrix}$ a linear combo of $\begin{bmatrix}1\\-2\\3\end{bmatrix},\begin{bmatrix}-2\\4\\-6\end{bmatrix},\begin{bmatrix}-1\\5\\-6\end{bmatrix},\begin{bmatrix}3\\-5\\8\end{bmatrix}$

4-5) $\begin{cases} 3x_2 - 6x_3 + 6x_4 + 4x_5 = -5 \\ 3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9 \\ 3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15 \end{cases}$

$x_1\begin{bmatrix}0\\3\\3\end{bmatrix} + x_2\begin{bmatrix}3\\-7\\-9\end{bmatrix} + x_3\begin{bmatrix}-6\\8\\12\end{bmatrix} + x_4\begin{bmatrix}6\\-5\\-9\end{bmatrix} = \begin{bmatrix}-5\\9\\15\end{bmatrix}$ is $\begin{bmatrix}-5\\9\\15\end{bmatrix}$ a linear combo of $\begin{bmatrix}0\\3\\3\end{bmatrix},\begin{bmatrix}3\\-7\\-9\end{bmatrix},\begin{bmatrix}-6\\8\\12\end{bmatrix},\begin{bmatrix}6\\-5\\-9\end{bmatrix}$

4-6) $\begin{cases} x_1 + 6x_2 + 2x_3 - 5x_4 - 2x_5 = -4 \\ 2x_3 - 8x_4 - x_5 = 3 \\ x_5 = 7 \end{cases}$

$x_1\begin{bmatrix}1\\0\\0\end{bmatrix} + x_2\begin{bmatrix}6\\0\\0\end{bmatrix} + x_3\begin{bmatrix}2\\2\\0\end{bmatrix} + x_4\begin{bmatrix}-5\\-8\\0\end{bmatrix} + x_5\begin{bmatrix}-2\\-1\\1\end{bmatrix} = \begin{bmatrix}-4\\3\\7\end{bmatrix}$ is $\begin{bmatrix}-4\\3\\7\end{bmatrix}$ a linear combo $\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}6\\0\\0\end{bmatrix},\begin{bmatrix}2\\2\\0\end{bmatrix},\begin{bmatrix}-5\\-8\\0\end{bmatrix},\begin{bmatrix}-2\\-1\\1\end{bmatrix}$

Hint: The *question of existence* for the vector equation $x_1\mathbf{a}_1 + \cdots + x_n\mathbf{a}_n = \mathbf{b}$ is as follows:

Is $\mathbf{b}$ in Span$\{\mathbf{a}_1, \cdots, \mathbf{a}_n\}$?

Or, equivalently,

Is $\mathbf{b}$ a linear combination of $\mathbf{a}_1, \cdots, \mathbf{a}_n$?

5- For each linear system in Exercise 4, provide the corresponding matrix equation. (Solving the equation is not required.)

6- Complete the following equation.

$$\begin{bmatrix}1 & 0\\2 & -3\\5 & -1\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix} = (3)\begin{bmatrix}1\\2\\5\end{bmatrix} + (1)\begin{bmatrix}0\\-3\\-1\end{bmatrix}$$

$A\vec{x}=b$

5.

4.1. $\underbrace{\begin{bmatrix} 2 & 0 & -6 \\ 0 & 1 & 2 \\ 3 & 6 & -2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -8 \\ 3 \\ -4 \end{bmatrix}}_{\vec{b}}$

4.2 $\underbrace{\begin{bmatrix} 2 & 4 & 4 \\ 0 & 1 & -2 \\ 2 & 3 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}}_{\vec{b}}$

4.3 $\underbrace{\begin{bmatrix} 1 & -2 & -1 \\ -2 & 4 & -5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 4 \\ 6 \end{bmatrix}}_{\vec{b}}$

4.4 $\underbrace{\begin{bmatrix} 1 & -2 & -1 & 3 \\ -2 & 4 & 5 & -5 \\ 3 & -6 & -6 & 8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}}_{\vec{b}}$

4.5 $\underbrace{\begin{bmatrix} 0 & 3 & -6 & 6 & 4 \\ 3 & -7 & 8 & -5 & 8 \\ 3 & -9 & 12 & -9 & 6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -5 \\ 9 \\ 15 \end{bmatrix}}_{\vec{b}}$

4.6 $\underbrace{\begin{bmatrix} 1 & 6 & 2 & -5 & -2 \\ 0 & 0 & 2 & -8 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -4 \\ 3 \\ 7 \end{bmatrix}}_{\vec{b}}$

ompute the product in two different ways using Method I and Method II as discussed in class. If the product is not defined, write 'Not defined' as your solution.

(a) $\begin{bmatrix}1 & 2 & -1 & 0 & 3\\ 0 & 0 & 1 & 0 & 2\end{bmatrix}\begin{bmatrix}-1\\2\\0\\3\\1\end{bmatrix}$

Method 1

$-1\begin{bmatrix}1\\0\end{bmatrix}+2\begin{bmatrix}2\\0\end{bmatrix}+0\begin{bmatrix}-1\\1\end{bmatrix}+3\begin{bmatrix}0\\0\end{bmatrix}+1\begin{bmatrix}3\\2\end{bmatrix}$

$\begin{bmatrix}-1\\0\end{bmatrix}+\begin{bmatrix}4\\0\end{bmatrix}+\begin{bmatrix}0\\0\end{bmatrix}+\begin{bmatrix}0\\0\end{bmatrix}+\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}6\\2\end{bmatrix}$

Method 2

$-1 + 4 + 0 + 0 + 3$

$(1\cdot -1)+(2\cdot 2)+(-1\cdot 0)+(0\cdot 3)+(3\cdot 1)=6$

$(0\cdot -1)+(0\cdot 2)+(1\cdot 0)+(0\cdot 3)+(2\cdot 1)=2$

$0 + 0 + 0 + 0 + 2$ $\begin{bmatrix}6\\2\end{bmatrix}$

(b) $\begin{bmatrix}1 & 2\\ 0 & 0\\ 1 & 0\\ 2 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix}$ $-1\begin{bmatrix}1\\0\\1\\2\\1\end{bmatrix}+3\begin{bmatrix}2\\0\\0\\1\\1\end{bmatrix}$ $\begin{bmatrix}-1\\0\\-1\\-2\\-1\end{bmatrix}+\begin{bmatrix}6\\0\\0\\3\\3\end{bmatrix}=\begin{bmatrix}5\\0\\-1\\1\\2\end{bmatrix}$

$(1\cdot -1)+(2\cdot 3)=5$
$(0\cdot -1)+(0\cdot 3)=0$
$(1\cdot -1)+(3\cdot 0)=-1$
$(2\cdot -1)+(3\cdot 1)=1$
$(1\cdot -1)+(1\cdot 3)=2$

(c) $\begin{bmatrix}1 & 2 & -1 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}$ Not Defined

8- Express each linear combination as the product of a matrix and a vector. (You do NOT need to compute the resulting vector.)

(a) $2\begin{bmatrix}1\\4\end{bmatrix}+5\begin{bmatrix}-1\\3\end{bmatrix}+17\begin{bmatrix}-3\\-2\end{bmatrix}$ $\begin{bmatrix}1 & -1 & -3\\ 4 & 3 & -2\end{bmatrix}\begin{bmatrix}2\\5\\17\end{bmatrix}$

(b) $-3\begin{bmatrix}2\\5\\0\end{bmatrix}+7\begin{bmatrix}-1\\0\\8\end{bmatrix}$ $\begin{bmatrix}2 & -1\\ 5 & 0\\ 0 & 8\end{bmatrix}\begin{bmatrix}-3\\7\end{bmatrix}$

(c) $\sqrt{5}\begin{bmatrix}4\\1\\0\\-9\end{bmatrix}-3\begin{bmatrix}0\\0\\0\\2\end{bmatrix}-8\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$ $\begin{bmatrix}4 & 0 & 1\\ 1 & 0 & 1\\ 0 & 0 & 1\\ 9 & 2 & 1\end{bmatrix}\begin{bmatrix}\sqrt{5}\\-3\\-8\end{bmatrix}$

9- Consider the following system of linear equations

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 1\\ 2x_1 + x_2 + 4x_3 &= 0\\ x_1 - 5x_2 \quad &= 2 \end{aligned}$$

9-1 $\underbrace{\begin{bmatrix}1 & 2 & -3\\ 2 & 1 & 4\\ 1 & -5 & 0\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}}_{\vec{x}}=\underbrace{\begin{bmatrix}1\\0\\2\end{bmatrix}}_{b}$

9-2 $x_1\begin{bmatrix}1\\2\\1\end{bmatrix}+x_2\begin{bmatrix}2\\1\\-5\end{bmatrix}+x_3\begin{bmatrix}-3\\4\\0\end{bmatrix}=\begin{bmatrix}1\\0\\2\end{bmatrix}$

9-3 $\begin{array}{cccc} x_1 & x_2 & x_3 & RHS \end{array}$ $\begin{bmatrix}1 & 2 & -3 & 1\\ 2 & 1 & 4 & 0\\ 1 & -5 & 0 & 2\end{bmatrix}$

reduced echelon $\begin{bmatrix}1 & 0 & 0 & 42/61\\ 0 & 1 & 0 & -16/61\\ 0 & 0 & 1 & -17/61\end{bmatrix}$

Pivot columns

9-4 System is consistent
No Free Variable
$x_3 = -17/61$ $x_2 = -16/61$ $x_1 = 42/61$

9-1) Write a matrix equation that is equivalent to the above linear system.
9-2) Write a vector equation that is equivalent to the above linear system.
9-3) What is the augmented matrix of the above linear system?
9-4) Follow the flowchart in Handout1 to find the general solution of the system. Your solution must exactly mimic our solutions in class; do not use any creativity or miss any step from the flowchart. If the system has no solution, you may write, 'the system is inconsistent'.

10- Mark each statement True or False. No explanations need be given.

(a) The matrix equation $\begin{bmatrix}2 & 0\\ 3 & 4\\ 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\mathbf{b}$ has a solution if and only if the vector $\mathbf{b}$ can be written as a linear combination of the vectors $\begin{bmatrix}2\\3\\1\end{bmatrix}$ and $\begin{bmatrix}0\\4\\-1\end{bmatrix}$. True

(b) The matrix equation $\begin{bmatrix}2 & 0\\ 3 & 4\\ 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\mathbf{b}$ has a solution if and only if the vector $\mathbf{b}$ is in the plane spanned by the vectors $\begin{bmatrix}2\\3\\1\end{bmatrix}$ and $\begin{bmatrix}0\\4\\-1\end{bmatrix}$. True

(c) If A is a 4×3 matrix, then each column of A is a vector in $\mathbb{R}^4$. True
d) If A is a 3×4 matrix, then each column of A is a vector in $\mathbb{R}^3$. True