

- 1- Acknowledgment Statement: Please write the following statement as the answer to Exercise 1 and place your signature right below the statement.

"I acknowledge that it is my responsibility to carefully read the class notes before attempting the homework problems. I understand that what is in the class notes is the minimum I should know, and I should not expect to pass this course if I do not fully understand the material covered in the class notes."

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- 2- For each of the following linear systems, write the corresponding augmented matrix.

2-1) $\begin{cases} 2x_1 - 6x_3 = -8 \\ x_2 + 2x_3 = 3 \end{cases} \rightarrow \left[\begin{array}{ccc|c} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \end{array} \right]$

2-2) $\begin{cases} x_1 - x_2 = 2 \\ x_1 + x_2 = 5 \end{cases} \rightarrow \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 1 & 1 & 5 \end{array} \right]$

2-3) $\begin{cases} x_1 + x_2 = 7 \\ 10x_1 + 9x_2 = 8 \\ -x_1 + 3x_2 = -2 \\ x_1 - x_2 = 6 \end{cases} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 7 \\ 10 & 9 & 8 \\ -1 & 3 & -2 \\ 1 & -1 & 6 \end{array} \right]$

- 3- For each of the following augmented matrices, write the corresponding linear system. Use the symbols x_1, x_2, \dots for the unknowns.

3-1) $\left[\begin{array}{ccc|c} 1 & 5 & 2 & -1 \\ 0 & 2 & -4 & 8 \\ 0 & 0 & 2 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \begin{cases} x_1 + 5x_2 + 2x_3 = -1 \\ 0x_1 + 2x_2 - 4x_3 = 8 \\ 0x_1 + 0x_2 + 2x_3 = 0 \end{cases}$

3-2) $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_1, R_3 \leftrightarrow R_1} \begin{cases} x_1 + 2x_2 + 4x_3 = 5 \\ 2x_1 + 4x_2 + 5x_3 = 4 \\ 4x_1 + 5x_2 + 4x_3 = 2 \end{cases}$

3-3) $\left[\begin{array}{cccc|c} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 3 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \begin{cases} x_1 + 0x_2 + 0x_3 + 0x_4 = 4 \\ 0x_1 + 1x_2 + 3x_3 + 0x_4 = -1 \\ 0x_1 + 0x_2 + 0x_3 + 2x_4 = 4 \\ 0x_1 + 0x_2 + 0x_3 + 0x_4 = 3 \end{cases}$

- 4- Mark each statement True or False. Justify each answer.

4-1) The matrix $A = \begin{bmatrix} 0 & 0 & 2 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 \end{bmatrix}$ is in echelon form. *No Not all Non zero rows are above zero rows*

4-2) The matrix $A = \begin{bmatrix} 1 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ is in reduced echelon form. *Yes it satisfies rule 1 & 2 to be echelon & rule 3 & 4 to be reduced echelon*

- 5- Consider the linear system $\begin{cases} 3x_1 - x_2 = 1 \\ -x_1 + x_2 = 1 \end{cases}$. Suppose we apply the following legitimate operation to the above system:

What is the resulting linear system? $\begin{cases} 3x_1 - x_2 = 1 \\ \frac{2}{3}x_2 = \frac{1}{3} \end{cases}$ *or* $\begin{cases} 3x_1 - x_2 = 1 \\ \frac{2}{3}x_2 = \frac{1}{3} \end{cases}$ *rewrite* $\begin{cases} 3x_1 - x_2 = 1 \\ \frac{2}{3}x_2 = \frac{1}{3} \end{cases}$

- 6- Please carefully review the examples solved in Part 2 of HW2 (the reading assignment) before attempting the remaining problems here.

Did that

6-1) Find the reduced echelon form of $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 4 \end{bmatrix}$. Make sure you follow the row reduction algorithm exactly as described in class.

6-2) Find the reduced echelon form of $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$. Make sure you follow the row reduction algorithm exactly as described in class.

$$= \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 4 \end{bmatrix}$$

7- Use the `rref` command in MATLAB to find the reduced echelon form of each of the following matrices. (Make sure to use "format rat" too.) After writing the reduced echelon form, circle the pivot positions in the original matrix and list the pivot columns.

7-1) $\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}$ ✓ $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ Pivot columns are columns 1, 2, 3

7-2) $\begin{bmatrix} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 3 & 3 \end{bmatrix}$ ✓ $\begin{bmatrix} 1 & 0 & -9 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ Pivot columns are columns 1, 2, 4, 5

7-3) $\begin{bmatrix} 1 & 5 & -2 & 0 & -7 \\ -3 & 1 & 9 & -5 & 9 \\ 4 & -8 & -1 & 7 & 0 \end{bmatrix}$ ✓ $\begin{bmatrix} 1 & 0 & 0 & 8/7 & -11/7 \\ 0 & 1 & 0 & -2/7 & -6/7 \\ 0 & 0 & 1 & -1/7 & 4/7 \end{bmatrix}$ Pivot columns are columns 1, 2, 3

Work for question 5

$$\begin{aligned} 5.) \text{ eq 1} &= 3x_1 - x_2 = 1 & \frac{1}{3}(3x_1 - x_2 = 1) &= 1x - \frac{1}{3}x_2 = \frac{1}{3} \\ \text{eq 2} &= -x_1 + x_2 = 1 \end{aligned}$$

$$\begin{aligned} \cancel{x_1} - \frac{1}{3}x_2 &= \frac{1}{3} \\ \cancel{x_1} + x_2 &= 1 \end{aligned} \quad \Rightarrow \quad \frac{2}{3}x_2 = 1\frac{1}{3}$$

$$-\frac{1}{3}x_2 + 1x_2 = \frac{2}{3}x_2$$

$$\frac{1}{3} + \frac{1}{3} \quad \frac{1}{3} + \frac{3}{9} = \frac{4}{9}$$

$$-\frac{1}{3} + \frac{1}{1} \quad -\frac{1}{3} + \frac{3}{3} = \frac{2}{3}$$

Work for question 6.1

$$6-1) \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 4 \end{bmatrix} \quad \begin{array}{l} \frac{-b}{a} \quad \frac{-1}{1} = -1(eq1) + eq2 \rightarrow eq2 \\ eq1 = -1 \quad -2 \quad 0 \quad -3 \\ eq2 = 1 \quad 2 \quad 0 \quad 4 \\ eq2 = 0 \quad 0 \quad 0 \quad -1 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 4 \end{bmatrix} \quad \begin{array}{l} eq1 \leftrightarrow eq2 \\ -\frac{1}{4} \end{array} \quad \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 3 & 4 \end{bmatrix} \quad \begin{array}{l} \frac{-3}{1} = -3(eq2) + (eq4) \rightarrow eq1 \\ 1(eq2) \quad 0 \quad 0 \quad -3 \quad -3 \\ +eq4 \quad 0 \quad 0 \quad 3 \quad 4 \\ eq4 \quad 0 \quad 0 \quad 0 \quad 1 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} -\frac{1}{4}(eq3) + eq4 \rightarrow 4 \\ 000- \\ eq4 \quad 0 \quad 0 \quad 0 \quad 1 \end{array} \quad \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} \frac{1}{4} eq3 \rightarrow eq3 \end{array}$$

$$\frac{-1}{4} \quad \frac{4}{1} \quad \frac{-4}{4} \quad \frac{1}{4} \quad \frac{4}{1} \quad \frac{4}{4}$$

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} -1(eq3) + (eq2) \rightarrow eq2 \\ 000-1 \\ 0011 \\ eq2 \quad 0010 \end{array} \quad \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} \frac{-3}{1} \\ -3(row3) + row1 \\ 000-3 \\ 1203 \\ row1 \quad 1200 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \checkmark$$

Work for question 6.2

$$\begin{array}{l}
 (-2) \left[\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ \textcircled{-1} & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{array} \right] \begin{array}{l} 1(eq1) + eq2 = eq2 \\ eq1 \ 1 \ 3 \ 0 \ 3 \\ eq2 \ 1 \ -1 \ -1 \ 1 \\ \hline 0 \ 2 \ 1 \ 4 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -4 & 2 & -8 \\ \textcircled{2} & 0 & 3 & -1 \end{array} \right] \begin{array}{l} -2(eq1) + eq4 \rightarrow \\ -2 \ -6 \ 0 \ -1 \\ 2 \ 0 \ 3 \ - \\ \hline 0 \ -6 \ 3 \ - \end{array} \\
 \\
 \left[\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & \textcircled{4} & 2 & -8 \\ 0 & -6 & 3 & -1 \end{array} \right] \begin{array}{l} 2(eq2) + eq3 \rightarrow eq3 \\ 0 \ 4 \ -2 \ 8 \\ 0 \ -4 \ 2 \ -8 \\ \hline 0 \ 0 \ 0 \ 0 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & \textcircled{-6} & 3 & -1 \end{array} \right] \begin{array}{l} 3(eq2) + eq4 \rightarrow \\ 0 \ 6 \ -3 \ 12 \\ 0 \ -6 \ 3 \ -7 \\ \hline 0 \ 0 \ 0 \ 5 \end{array} \\
 \\
 \frac{1}{2} \cdot \frac{2}{1} \cdot \frac{2}{2} \left[\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{array} \right] \begin{array}{l} eq3 \leftrightarrow eq4 \\ \textcircled{1} \ 3 \ 0 \ 3 \\ 0 \ \textcircled{2} \ -1 \ 4 \\ 0 \ 0 \ 0 \ \textcircled{5} \\ 0 \ 0 \ 0 \ 0 \end{array} \frac{1}{2}(eq2) \rightarrow eq2 \left[\begin{array}{ccc|c} \textcircled{1} & 3 & 0 & 3 \\ 0 & \textcircled{1} & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & \textcircled{5} \\ 0 & 0 & 0 & 0 \end{array} \right] \\
 \\
 \frac{1}{2} \cdot \frac{-1}{1} \cdot \frac{-1}{1} \left[\begin{array}{ccc|c} \textcircled{1} & 3 & 0 & 3 \\ 0 & \textcircled{1} & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & \textcircled{5} \\ 0 & 0 & 0 & 0 \end{array} \right] \frac{1}{5} eq1 \rightarrow eq1 \left[\begin{array}{ccc|c} \textcircled{1} & 3 & 0 & 3 \\ 0 & \textcircled{1} & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} -3(eq2) + eq1 \rightarrow eq1 \\ 0 \ -3 \ \frac{3}{2} \ 6 \\ 1 \ 3 \ 0 \ 3 \\ \hline 1 \ 0 \ \frac{3}{2} \ 9 \end{array} \\
 \\
 \frac{1}{5} \cdot \frac{5}{1} \cdot \frac{5}{1} \left[\begin{array}{ccc|c} \textcircled{1} & 0 & \frac{3}{2} & 9 \\ 0 & \textcircled{1} & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} -3(eq2) + eq1 \rightarrow eq1 \\ 0 \ 0 \ -2 \\ 0 \ 1 \ -\frac{1}{2} \ 2 \\ 0 \ 1 \ -\frac{1}{2} \ 0 \end{array} \left[\begin{array}{ccc|c} \textcircled{1} & 0 & \frac{3}{2} & 9 \\ 0 & \textcircled{1} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 \end{array} \right] \\
 \\
 \begin{array}{l} -3 \cdot \frac{-1}{2} \cdot \frac{3}{2} \\ 1 \quad 2 \quad 2 \end{array} \left[\begin{array}{ccc|c} \textcircled{1} & 0 & \frac{3}{2} & 9 \\ 0 & \textcircled{1} & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} -2(eq3) + eq2 \rightarrow eq2 \\ 0 \ 0 \ -2 \\ 0 \ 1 \ -\frac{1}{2} \ 2 \\ 0 \ 1 \ -\frac{1}{2} \ 0 \end{array} \left[\begin{array}{ccc|c} \textcircled{1} & 0 & \frac{3}{2} & 9 \\ 0 & \textcircled{1} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 \end{array} \right] \\
 \\
 \begin{array}{l} -9(eq3) + eq1 \rightarrow eq1 \\ 0 \ 0 \ 0 \ -9 \\ 1 \ 0 \ \frac{3}{2} \ 9 \\ \hline 1 \ 0 \ \frac{3}{2} \ 0 \end{array} \left[\begin{array}{ccc|c} \textcircled{1} & 0 & \frac{3}{2} & 0 \\ 0 & \textcircled{1} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 \end{array} \right]
 \end{array}$$