

- 1- Acknowledgment Statement: Please write the following statement as the answer to Exercise 1 and place your signature right below the statement.

"I acknowledge that it is my responsibility to carefully read the class notes before attempting the homework problems. I understand that what is in the class notes is the minimum I should know, and I should not expect to pass this course if I do not fully understand the material covered in the class notes."

Andrew Ortega

See bottom of page for more neat writing

- 2- In class, we discussed two viewpoints regarding what linear algebra is about, namely, the *applied perspective* and the *abstract perspective*. Reproduce the descriptions discussed in class. *Applied - Linear Algebra is a collection of methods related to linear equations. Abstract - Linear Algebra is about vector spaces & linear transformations between vector spaces.*
- 3- In class, we mentioned that linear algebra can be viewed as a course about language. List the four languages that were discussed in class. *Linear Systems, Vectors in \mathbb{R}^n , Matrices, Vector Spaces*

- 4- For each problem, determine whether the equation is linear or nonlinear. If the equation is linear, list the unknowns and their coefficients.

- 4-a) $x_1 + x_2 = 5$ Yes coefficient = 1 unknown = x_1, x_2
- 4-b) $\sqrt{2}x_1 + 5x_2 - 6x_3 = 7$ Yes coefficient = $\sqrt{2}, 5, -6$ unknown = x_1, x_2, x_3
- 4-c) $x + y + z = 10$ Yes coefficient = 1 unknown = x, y, z
- 4-d) $xy + yz + xz = 20$ No
- 4-e) $x^2 + y^2 + z^2 = 1$ No

- 5- For each of the following linear systems, determine the number of equations and the number of unknowns.

- 5-a)

$$\begin{cases} 2x_1 - 6x_3 = -8 & 2 \text{ equations} \\ x_2 + 2x_3 = 3 & 3 \text{ unknowns } x_1, x_2, x_3 \end{cases}$$

- 5-b)

$$\begin{cases} x_1 - x_2 = 2 & 2 \text{ equations} \\ x_1 + x_2 = 5 & 2 \text{ unknowns } x_1, x_2 \end{cases}$$

- 5-c)

$$\begin{cases} x_1 + x_2 = 7 & 4 \text{ equations} \\ 10x_1 + 9x_2 = 8 & 2 \text{ unknowns } x_1, x_2 \\ -x_1 + 3x_2 = -2 \\ x_1 - x_2 = 6 \end{cases}$$

- 6- As discussed in class, the names we choose for the unknowns will not impact the solution set of the linear system. Rewrite the following linear system using the symbols x, y, z instead of x_1, x_2, x_3 , respectively.

$$\begin{cases} x_1 + \sqrt{3}x_2 + x_3 = 10 & x + \sqrt{3}y + z = 10 \\ 10x_1 + 2x_2 - x_3 = \sqrt{2} & 10x + 2y - z = \sqrt{2} \end{cases}$$

- 7- Consider the following linear system, which consists of one equation and three unknowns:

$$x_1 + x_2 + 2x_3 = 1$$

An infinite Amount of solutions

Clearly, $(x_1 = 0, x_2 = 0, x_3 = \frac{1}{2})$ is one solution of this system. List four other solutions for this system. How many solutions does this system have?

$$\begin{array}{l} x_1 = 1 \quad x_2 = 1 \quad x_3 = -\frac{1}{2} \\ x_1 = 2 \quad x_2 = 2 \quad x_3 = -\frac{3}{2} \\ x_1 = 3 \quad x_2 = 3 \quad x_3 = -\frac{5}{2} \\ x_1 = 4 \quad x_2 = 4 \quad x_3 = -\frac{7}{2} \end{array}$$

8- Consider the linear system

$$0x + 0y + 0z = 1$$

How many solutions does this system have?

None

9- Solve the linear system

$$\begin{cases} 3x_1 - x_2 = 5 & x_1 = 2 \\ -x_1 + x_2 = -1 & x_2 = 1 \end{cases}$$

Draw the corresponding lines in the x_1x_2 -plane and identify the solution as the point of intersection.

10- Is there a linear system with exactly five solutions?

No only 1 solution, no solution, or infinite solution

11- Solve the following nice linear system by backward substitution.

$$\begin{cases} 2x_1 + 5x_2 + 6x_3 = 7 \\ x_2 + x_3 = 4 \\ 3x_3 = 6 \end{cases}$$

$$x_3 = 2, x_2 = 2,$$

$$x_2 + 2 = 4$$

$$x_2 = 2$$

$$2x_1 + 5(2) + 6(2) = 7$$

$$2x_1 + 10 + 12 = 7$$

$$2x_1 + 22 = 7$$

$$2x_1 = -15 \quad x_1 = -7.5$$

$$\begin{aligned} x_3 &= 2 \\ x_2 &= 2 \\ x_1 &= -7.5 \end{aligned}$$

$$9, \quad 3x_1 - x_2 = 5$$

$$-x_1 + x_2 = -1$$

$$x_2 = -1 + x_1$$

$$3x_1 - (-1 + x_1) = 5$$

$$3x_1 + 1 - x_1 = 5$$

$$3x_1 - x_1 = 4$$

$$\frac{2x_1}{2} = \frac{4}{2}$$

$$x_1 = 2$$

$$x_1 = 2$$

$$-2 + x_2 = -1$$

$$+2 \quad +2$$

$$x_2 = 1$$

2. In class we discussed 2 view points regarding what Linear algebra is about, namely the applied perspective & the abstract perspective. Reproduce the descriptions discussed in class. Abstract - Linear Algebra is about vector spaces & Linear transformations between vector spaces. Applied - Linear Algebra is a collection of methods related to linear equations.

3. In class we mentioned that Linear Algebra can be viewed as a course about Languages. List the 4 languages that were discussed in class. Linear Systems, Vectors in \mathbb{R}^N , Matrices, vector spaces