Math 100 Homework 3

1- **Acknowledgment Statement:** Please write the following statement as the answer to Exercise 1 and place your signature right below the statement.

"I acknowledge that it is my responsibility to carefully read the class notes before attempting the homework problems. I understand that what is in the class notes is the minimum I should know, and I should not expect to pass this course if I do not fully understand the material covered in the class notes." Ruben Ortega Ruben Ortega

2- What does it mean to say that the matrix U is an echelon form of the matrix A? We say matrix U was derived From Matrix A and made into Echelon Form.

3- What do we mean by pivot positions of a matrix A? What do we mean by pivot columns of a matrix A?
Pivot positions - is a leading entry in a reduced Echelon Form Matrix
Pivot Column - A column with a pivot position

4- Determine the size of each matrix. Also, determine the number of equations (if you believe this question is nonsense, please explain why).

4-1) $\begin{bmatrix} 1 & 4 & 3 \\ -1 & 0 & 5 \end{bmatrix}$ 2x3

4-2) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \\ 0 & 0 \\ \sqrt{2} & 12 \end{bmatrix}$ 4x2

4-3) $\begin{bmatrix} 1 & 4 & 6 & 7 \\ -1 & 0 & 8 & -5 \\ 0 & 0 & 0 & 1 \\ \sqrt{2} & 12 & 10 & 3 \end{bmatrix}$ 4x4

Never said it was augmented matrix so No equations They are just matrices,

5- Follow the flowchart in Handout1 to find the general solution of the system. Your solution must exactly mimic our solutions in class; do not use any creativity or miss any step from the flowchart. If the system has no solution, you ~~may write, 'the system~~ is inconsistent'.

5-1) $\begin{cases} 2x_1 - 6x_3 = -8 \\ x_2 + 2x_3 = 3 \\ 3x_1 + 6x_2 - 2x_3 = -4 \end{cases}$ $\begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ Pivot columns 1,2,3 No Free variable

System is Consistent but only 1 solution

5-2) $\begin{cases} 2x_1 + 4x_2 + 4x_3 = 4 \\ x_2 - 2x_3 = -2 \\ 2x_1 + 3x_2 = 0 \end{cases}$ $\begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ Pivot columns 1,2,3 No Free variable

System is Consistent but only 1 solution

5-3) $\begin{cases} x_1 - 2x_2 - x_3 = 4 \\ -2x_1 + 4x_2 - 5x_3 = 6 \end{cases}$ $\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ Pivot columns 1,3 Free variable x_2

System is Consistent & has 1 Free variable so infinite solutions

5-4) $\begin{cases} x_1 - 2x_2 - x_3 + 3x_4 = 0 \\ -2x_1 + 4x_2 + 5x_3 - 5x_4 = 3 \\ 3x_1 - 6x_2 - 6x_3 + 8x_4 = 2 \end{cases}$ $\begin{bmatrix} 1 & -2 & -1 & 3 & 0 \\ -2 & 4 & 5 & -5 & 3 \\ 3 & -6 & -6 & 8 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 10/3 & 0 \\ 0 & 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ Pivot columns 1,3,RHS

System is inconsistent

5-5) $\begin{cases} 3x_2 - 6x_3 + 6x_4 + 4x_5 = -5 \\ 3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9 \\ 3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15 \end{cases}$ $\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$ Pivot columns 1,2,5 Free variables = x_3, x_4

System is Consistent & 3 Free Variable so infinite Solution

5-6) $\begin{cases} x_1 + 6x_2 + 2x_3 - 5x_4 - 2x_5 = -4 \\ 2x_3 - 8x_4 - x_5 = 3 \\ x_5 = 7 \end{cases}$ $\begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 6 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$ Pivot columns 1 3 5 Free variables x_2 x_4

System is Consistent & 2 Free Variables Infinite solution

Bold Problem

6- Choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) infinitely many solutions. Give separate answers for each part.

$$\begin{aligned} x_1 - 3x_2 &= 1 \\ 2x_1 + hx_2 &= k \end{aligned} \qquad \begin{bmatrix} 1 & -3 & 1 \\ 2 & h & k \end{bmatrix}$$

$$\begin{bmatrix} x_1 - 3x_2 \\ 2x_1 + hx_2 \end{bmatrix} = \begin{bmatrix} 1 \\ k \end{bmatrix}$$

No Solution $h=-6$ $k=5$

Unique Solution $h=1$ $k=5$

infinate many Solution $h=-6, k=2$

$h=1$ $k\neq 1$ $\quad h=6$ $\quad k\neq 2$

$1 \quad -3 \quad 1$

$1x_1 - 3x_2 = 1$

$2x_1 + -6x_2 = 5$

$$\begin{bmatrix} 1 & -3 & 1 \\ 2 & -6 & 5 \end{bmatrix}$$

$3 \quad -9 \quad 6$

$3x_1 - 9x^2 = 6$

$h+6=0$

$h=-6$

$k-2\neq 0$

$k=2$

$3x_1 + -9x_2 = 6$

$-3 \quad\quad -3$

$9x_2 = -9$

$\frac{9x_2}{9} = \frac{-9}{9}$

$x_2 = -1$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & h-2(-3) & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & h+6 & k-2 \end{bmatrix}$$