Summary

The goal of this thesis was to explore the superconductivity in flat-band systems and characterize the coherence length and the London penetration depth, which connect to the pairing temperature and the superfluid phase stiffness respectively. This was done by placing a Finite Momentum Pairing (FMP) constraint on the order parameter and analyzing the suppression of the order parameter and the superconducting current.

The FMP method is based on the phenomenological Ginzburg-Landau theory, which is reviewed in the theoretical foundations. It is then explained how introducing a finite momentum to the order parameter gives access to the superconducting length scales. To calculate the length scales in microscopic theories, a way to introduce the finite momentum into these theories is needed. In the thesis, this is done for two theories: Bardeen-Cooper-Schrieffer (BCS) theory and Dynamical Mean Field Theory (DMFT). This chapter also introduces how the geometry of the space of quantum states (characterized by the quantum metric) connects to the superfluid weight, which is especially important in the context of superconductors with flat electronic bands.

The next chapter introduces a decorated graphene model which hosts a flat band and inherits robust quantum geometry from the underlying graphene band structure.

In the last chapter, the FMP method is applied. For the decorated graphene model, the hybridization V between the Gr_A and X sites is shown to be important for superconductivity. The magnitude of the superconducting gap is given by the orbital weight of the flat band for the corresponding orbital. When V is increased, the flat band switches from X to Gr_B character, so in turn the largest value of the superconducting gap between the orbitals switches over from the X to the Gr_B . The superconducting transition temperature T_C follows this maximum, so it has a minima at the point of the switchover V = 1.46t.

The coherence length ξ_0 similarly follows the orbital weight: the orbital with the largest gap shows the shortest coherence length. It is demonstrated that in line with previous works on flat-band systems that there is just the geometric contribution to the superfluid weight when the gap separating the flat and dispersive bands is smaller than the interaction U. The results for the superfluid weight from the FMP method only qualitatively agree with these results calculated from linear response theory, but especially in the low V regime in which the the superfluid weight is peaked, the FMP does not give reliable results.

In the DMFT implementation of the FMP method, the phenomenon of the BCS-BEC crossover (here implemented for the one-band Hubbard model) is investigated. In

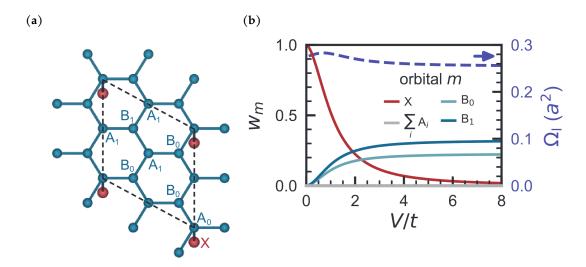


Figure 1.1 – Decorated graphene with smaller impurity density. (a) Unit cell with $^{1}/8$ impurity coverage. (b) Site-resolved orbital weight w_m and minimal Wannier spread Ω_l of the flat band. Importantly, Ω_l does not vanish for $V \to \infty$. Taken from [2].

particular it is demonstrated that the pairing temperature (from mean-field theory) diverges from the superconducting transition temperature $T_{\rm C}$ when pair condensation is suppressed by the attractive interaction. At the same time, the coherence length ξ_0 is reduced and reaches a constant value and the superfluid weight $D_{\rm S} \propto \lambda_{\rm L,0}^{-2}$ goes from a constant value to 0 for larger interaction.

Outlook

The decorated graphene model investigated in this thesis is a toy model for the graphene adatom heterostructures realized in ref. [1]. A more realistic model that is also explored in ref. [2] is a model with a smaller density of X atoms in the unit cell, as shown in fig. 1.1a. In this case, the quantum metric does not vanish for $V \to \infty$, compare fig. 1.1b. This is due to the fact that the orbital weight spreads over all B sublattice sites. As shown in ??, a robust quantum metric supports superconductivity, so investigating this model is an interesting further pathway.

As seen in ??, including the full local fluctuations with DMFT reveals physics beyond the mean-field level, so investigating the decorated graphene model in both the structure treated in this thesis and with the smaller impurity density is an interesting next step.

In twisted Bilayer graphene, the fact that quantum geometry is important for superconductivity in the material is well established [3]. The pairing mechanism in twisted bilayer graphene is still an open question so using the FMP method to calculate ξ_0 and $\lambda_{\rm L,0}$ to enable a more rigorous comparison to experiment can guide in this exploration, and using the FMP method on the mean-field level as developed in this thesis can be a tool for exploration with lower computational cost.