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The United Nations declared 2025 the 'International Year of Quantum Science and Technology' [1]. This is an effort is to raise awareness of the importance of quantum science and its applications, which focuses in 3 key areas: quantum computing, quantum communications and quantum sensors. One effect underlying many of these applications is the phenomenon of superconductivity. For example, superconducting qubits are a promising platform for scalable quantum computing [2, 3] and the Josephson effect [4] can be used to build extremely sensitive measurement devices for magnetic fields [5] or voltages [6].

Superconductivity was discovered in 1911, when Heike Onnes measured that the electrical resistance of Mercury suddenly vanished completely when cooling it below 4 K [7]. In the following decades, more effects in superconducting materials were discovered such as the Meissner effect, the perfect expulsion of external magnetic fields [8]. The theoretical description

Description: macroscopic quantum mechanics

BCS theory

Unconventional Superconductivity

In 1986 and 1987, superconductivity with very high T_C of 30 K (the highest T_C until then was 23.7 K in Nb_3Ge) was discovered in cuprates [9, 10]. Cuprate superconductors are made up of layers of cooper oxide and charge reservoirs in between. The specific charge reservoir layers determine the properties of the superconducting and varying them lead to the discovery of a rich zoo of materials with high T_C [11]. Cuprates are the prime example of the class of superconductors that cannot be explained by BCS theory. They are called unconventional. Mechanisms of pairing are something else that the electron-phonon interaction in BCS-theory, i.e. purely electronic mechanisms. In Cuprates, parent state is a Mott insular (insulator emerging from strong electronic correlations). This means it cannot be explained by BCS theory, because there the parent state is a Fermi liquid.

Some more words on characterization: unconventional, high TC, strongly correlated/coupling

HTSC not only have higher T_C, so that lower-cost cooling methods can be employed, they also support stronger currents and can withstand higher magnetic fields until SC breaks down. These three parameters span the critical surface

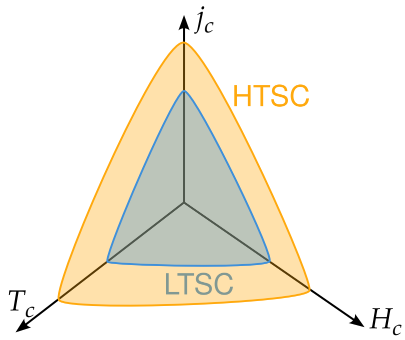

Figure 1.1 – Critical surface of a superconductor. For practical applications, this surface is desired to be as large as possible, making it possible to carry high currents and generate strong magnetic fields while not needing to cool the superconductor to very low temperatures. This generally is the case for high-temperature superconductors in comparison to low-temperature superconductors.

(see fig. 1.1), should be largest for technical applications. So for HTSC, largest commercial application to date is in magnetic resonance imaging, a medical technique using strong magnetic fields and field gradients [12]. There is cable development [13, 14] utilizing the high critical currents.

Critical currents and magnetic fields closely linked to two length scales in SCs: the coherence length ξ_0 and the London penetration depth $\lambda_{L,0}$. Coherence length is linked to size of Cooper pairs, London penetration depth describes how far magnetic fields penetrate into a SC. These length scales are a further characterization of SCs. For example, SC can be split up into type I and type II. The difference is in the response to magnetic fields. In type I SC, sc order is destroyed for fields higher than a critical magnetic field H_C. Type II superconductors have two critical fields H_{C_1,C_2}, $H_{C_1} > H_{C_2}$. For fields higher than H_{C_1}, the superconducting state is completely destroyed. For fields higher than H_{C_2} but lower than H_{C_1}, the superconducting order is not destroyed, but magnetic field lines penetrate the material. To characterize SC materials, especially SCs with strong correlations it is very desirable to calculate the length scales in ab-initio frameworks.

Experimental measurement of length scales

BCS-BEC Crossover

Topic in the context of high T_C SCs. Framework independent of pairing mechanism. Two regimes can be connected by

Regimes

Figure 1.2 shows the

describe figure

The intermediate region is characterized by the fact that the

difference between condensing and SC

[15]

BEC regime: TC is given by DS

Optimization of TC, need access to SC length scales

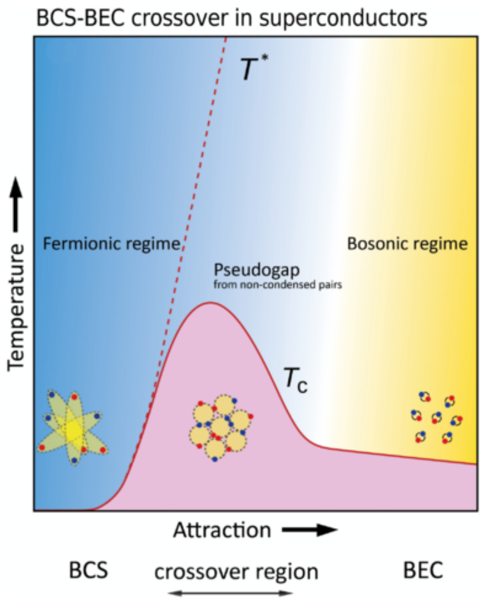

Figure 1.2 – BCS-BEC crossover. Reprinted figure with permission from [15]. Copyright 2024 by the American Physical Society.

Graphene Structures as a Platform for Correlated Physics

Introduction to Graphene historically

Following the 2018 discovery of superconductivity in twisted bilayer Graphene [16], graphene-based systems gained a renewed interest as a platform for strongly correlated physics. Two methods to engineer strong electron correlations emerged: twisted multilayer systems [16–20] and multilayer systems without twisting, such as Bernal bilayer, ABC or ABCA layered systems [21]. Through different means, electrons in these systems become localized so that interaction effects get more strongly pronounced. Connecting both kind of systems is the strong quantum geometry coming from the Graphene Dirac cones [22], which plays a role in stabilizing superconducting [17, 23] and magnetic order [24, 25].

One interesting development in is in twisted multilayer systems, first realized as twisted bilayer Graphene [16]. In comparison to the complex crystal structure of e.g. the Cuprates, twisted multilayer systems have a very simple structure and can be tuned very easily: the angle of twist between the layers can be easily accessed experimentally. The defining feature of these systems are flat electronic bands due to folding of the Brilluoin zone. Superconductivity in these systems is enhanced due to the fact that in the flat bands, interactions

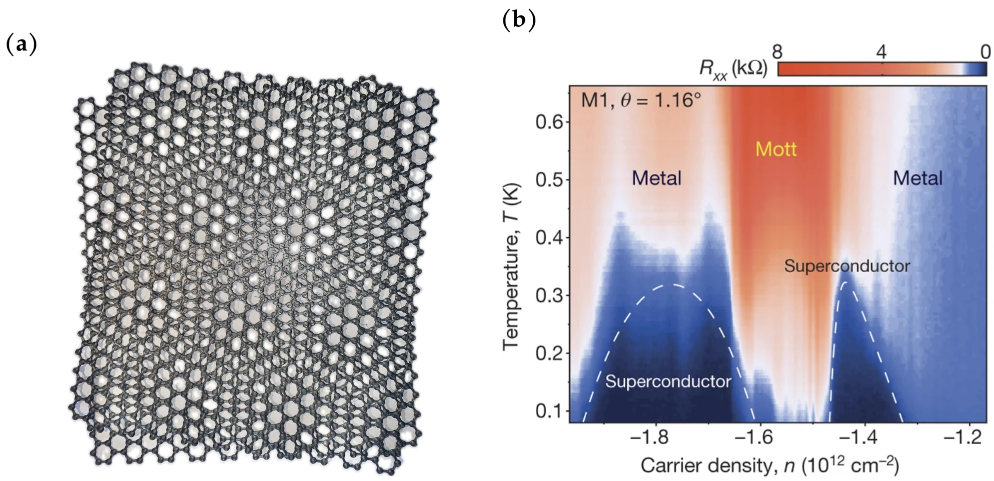

Figure 1.3 – (**a**) Twisted Bilayer Graphene (**b**) Reproduced from [16] with permission from Springer Nature.

between the electrons are very strongly enhanced. Thus these systems are a very interesting playground to study strongly correlation effects in general and superconductivity in particular.

Organization of this thesis

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