I Ginzburg-Landau theory of superconductivity

I.1 Coherence length and penetration depth in strongly correlated superconductors

[**wittBypassingLatticeBCSBEC2024**]

Order parameter (OP) of a superconducting condensate with FMP has the form

$$\Psi_{\mathbf{q}}(\mathbf{r}) = |\Psi_{\mathbf{q}}| e^{i\mathbf{q}\mathbf{r}} \tag{I.1}$$

where $\mathbf{q}$ is the center-of-mass momentum of Cooper pairs.

FMP is well known from Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) theory, where the single-momentum phase used here corresponds to FF-type pairing.

What does that mean? More details on FFLO theory

I.2 Ginzburg-Landau description

First: Motivate how the FMP constraint relates to λ_L and ξ_0.

GL low-order expansion of the free energy density f_{GL} in terms of the FMP-constrained OP reads

$$1 \tag{I.2}$$

Fill in equation

More details on GL theory in general

The temperature dependent correlation length ξ appears as the natural length scale of the amplitude mode ($\propto \alpha$) and kinetic energy term

$$\xi(T) = \tag{I.3}$$

with the zero temperature value ξ_0 being the coherence length.

Fill in equation

I.3 Phase transitions and broken symmetry

Following [**colemanIntroductionManyBodyPhysics2015**].

I.3.1 Order parameter concept

This works also for phase transitions not dependent of temperatures, so e.g. in pressures?

Landau theory: phase transitions (e.g. iron becomes magnetic, water freezes, superfluidity/superconductivity) are associated with the development of an order parameter when the temperature drops below the transition temperature T_C

$$|\psi| = \begin{cases} 0 \ , \ T > T_C \\ |\psi_0| > 0 \ , \ T < T_C \end{cases} \tag{I.4}$$

Landau theory does not need microscopic expression for order parameter, it provides corse-grained description of the properties of matter. The order parameter description is good at length scales above ξ_0, the coherence length (e.g. size of Cooper pairs for SC).

I.3.2 Landau theory

Rest of Landau theory

Landau theory

Going from a one to a n-component order parameters, we can actually

Particularly important example: complex or two component order parameter in superfluids and superconductors:

$$\psi = \psi_1 + \mathrm{i}\psi_2 = |\psi|e^{\mathrm{i}\phi} \tag{I.5}$$

The Landau free energy takes the form:

$$f[\psi] = r(\psi^*\psi) + \frac{u}{2}(\psi^*\psi)^2 \tag{I.6}$$

Figure I.1 shows the Landau free energy as function of ψ.

In this ‘Mexican hat’ potential: order parameter can be rotated continuously from one broken-symmetry state to another. If we want the phase to be rigid, we need to introduce an There is a topological argument for the fact that the phase is rigid. This leads to Ginzburg-Landau theory. Will see later: well-defined phase is associated with persistent currents or superflow.

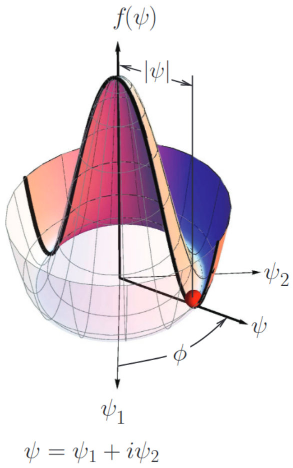

Figure I.1: *Mexican hat potential*

I.3.3 Ginzburg-Landau theory I: Ising order

Landau theory: energy cost of a uniform order parameter, more general theory needs to account for inhomogenous order parameters, in which the amplitude varies or direction of order parameter is twisted -> GL theory. First: one-component, 'Ising' order parameter. GL introduces additional energy $\delta f \propto |\Delta\psi|^2$, $f_{GL}[\psi, \Delta\psi] = \frac{s}{2}|\Delta\psi|^2 + f_L[\psi(s)]$, or in full:

$$f_{GL}[\psi, \Delta\psi, h] = \frac{s}{2}(\Delta\psi)^2 + \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4 - h\psi \tag{I.7}$$

GL theory is only valid near critical point, where OP is small enough to permit leading-order expansion.

What is the h here?

length scale/correlation length

I.3.4 Ginzburg-Landau theory II: complex order and superflow

Now: G-L theory of complex or two-component order parameters, so superfluids and superconductors. Heart of discussion: emergence of a 'macroscopic wavefunction', where the microscopic field operators $\psi(\hat{x})$ acquire an expectation value:

What exactly are field operators again?

$$\langle\psi(x)\rangle = \psi(x) = |\psi(x)|e^{i\theta(x)} \tag{I.8}$$

Magnitude determines density of particles in the superfluid:

More info on th
Does that come
later in chapter

$$|\psi(x)|^2 = n_s(x) \tag{I.9}$$

Twist/gradient of phase determines superfluid velocity:

$$\mathbf{v}_s(x) = \frac{\hbar}{m}\Delta\phi(x) \tag{I.10}$$

We will derive this later in the chapter. Counterintuitive from quantum mechanics: GL suggested that $\Phi(x)$ is a macroscopic manifestation of a macroscopic number of particles condensed into precisely the same quantum state. Emergent phenomenon, collective properties of mater not a-priori self-evident from microscopic physics.

GL free energy density for superfluid (with one added term in comparison to Landau energy):

$$f_{GL}[\psi, \Delta\psi] = \frac{\hbar^2}{2m}|\Delta\psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4 \tag{I.11}$$

energy density of bosonic field? -> for comparison!

Interpreted as energy density of a condensate of bosons in which the field operator behaves as a complex order parameter. Gives interpretation of gradient term as kinetic energy:

$$s|\Delta\psi|^2 = \frac{\hbar^2}{2m}\langle\Delta\hat{\psi}^\dagger\Delta\hat{\psi}\rangle \implies s = \frac{\hbar^2}{2m} \tag{I.12}$$

As in Ising order: correlation length/GL-coherence length governs characteristic range of amplitude fluctuations of the order parameter:

$$\xi = \sqrt{\frac{s}{|r|}} = \sqrt{\frac{\hbar^2}{2m|r|}} = \xi_0(1 - \frac{T}{T_C})^{-\frac{1}{2}} \tag{I.13}$$

Compare with Ising order, especially dependence on T

Compare with Ising order. Is that derived or postulated?

where $\xi_0 = \xi(T=0) = \sqrt{\frac{\hbar^2}{2maT_C}}$ is the coherence length. Beyond this length: only phase fluctuations survive. Freeze out fluctuations in amplitude (no x-dependence in amplitude) $\psi(x) = \sqrt{n_s}e^{\mathrm{i}\phi(x)}$, then $\Delta\psi = \mathrm{i}\Delta\phi\psi$ and $|\Delta\psi|^2 = n_s(\Delta\phi)^2$, dependency of kinetic energy on the phase twist is (bringing it into the form $\frac{m}{2}v^2$):

$$\frac{\hbar^2 n_s}{2m}(\Delta\phi)^2 = \frac{mn_s}{2}(\frac{\hbar}{m}\Delta\phi)^2 \tag{I.14}$$

So twist of phase results in increase in kinetic energy, associated with a superfluid velocity:

$$\mathbf{v}_s = \frac{\hbar}{m}\Delta\phi \tag{I.15}$$

Coherent states/Interpretation of states/Off-diagonal long-range order

Phase rigidity and superflow: in GL theory, energy is sensitive to a twist of the phase. Substitute $\psi = |\psi|e^{\mathrm{i}\phi}$ into GL free energy, gradient term is:

$$\Delta\psi = () \tag{I.16}$$

Here: particle-current operator, especially for coherent state, connection with phase twist

I.3.5 Ginzburg-Landau theory III: charged fields

Here: Meissner effect, etc