1 Conclusion

Summary

The goal of this thesis was to explore the phenomenon of superconductivity via the calculation of the coherence length and the London penetration depth, which connect to the pairing temperature and the superfluid phase stiffness respectively. This was done by placing a Finite Momentum Pairing (FMP) constraint on the order parameter and analyzing the suppression of the order parameter and the superconducting current.

The FMP method is based on the phenomenological Ginzburg-Landau theory, which is reviewed in the theoretical foundations. It is then explained how introducing a finite momentum to the order parameter gives access to the superconducting length scales. To calculate the length scales in microscopic theories, a way to introduce the finite momentum into these theories is needed. In the thesis, this is done for two theories: Bardeen-Cooper-Schrieffer (BCS) theory and Dynamical Mean Field Theory (DMFT). This section also introduces how the geometry of the space of quantum states (characterized by the quantum metric) connects to the superfluid weight, which is especially important in the context of superconductors with flat electronic bands.

The next chapter introduces a decorated graphene model which hosts a flat band and inherits robust quantum geometry from the underlying graphene band structure.

In the last chapter, the FMP method is used to explore the superconducting length scales. The BCS formulation is applied to the decorated graphene model, exploring how the superconductivity depends on the hybridization and how the quantum geometry of the model influence superconductivity. DMFT calculations are numerically significantly more complex and implementing it for the decorated graphene is beyond the scope of this thesis, so it is applied to a simpler model in the one band Hubbard model on a square lattice, exploring the phenomenon of the BCS-BEC crossover and seeing how the BCS and DMFT method differ.

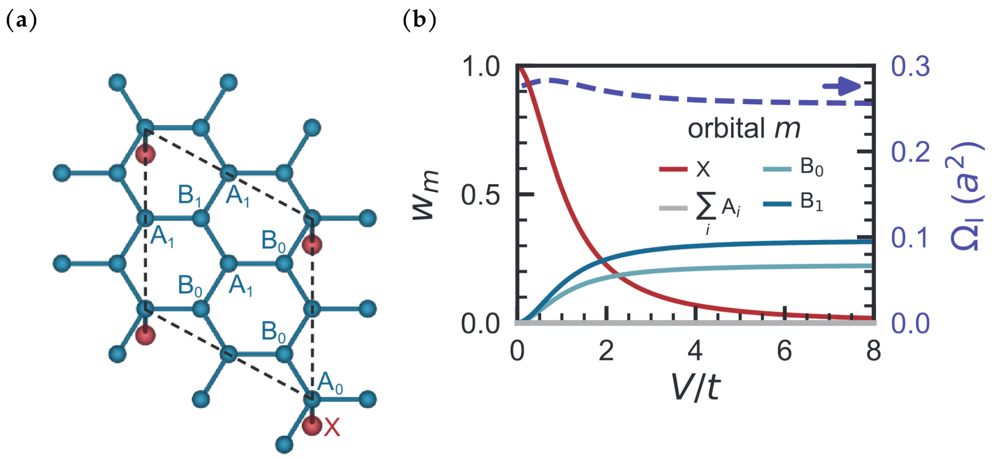

Figure 1.1 – Decorated graphene with smaller impurity density. (**a**) Unit cell with 1/8 impurity coverage. (**b**) Site-resolved orbital weight w_m and minimal Wannier spread Ω_I of the flat band. Importantly, Ω_I does not vanish for $V \to \infty$. Taken from [1].

Outlook

Starting from the mean-field results for the decorated graphene model discussed in **??**, further investigation into the interesting superconducting behavior with the hybridization V and why the superfluid weight does not follow the quantum metric. To this end, it is also interesting to extend mean-field treatment to the more realistic model with lower impurity density, see chapter 1. This model has a different $V \to \infty$ limit, retaining a finite quantum metric in this regime, which will in turn influence superconductivity.

DMFT and BCS: see whether it is a limitation of the BCS or the method in this model.

Better wording

As seen in **??**, including the full local fluctuations with DMFT reveals physics beyond the mean-field level, so investigating the decorated graphene model in both the structure treated in this thesis and with the smaller impurity density is an interesting next step.

In twisted Bilayer graphene, the fact that quantum geometry is important for superconductivity in the material is well established [2]. The pairing mechanism in twisted bilayer graphene is still an open question so using the FMP method to calculate ξ_0 and $\lambda_{\mathrm{L},0}$ to enable a more rigorous comparison to experiment can guide in this exploration, and using the FMP method on the mean-field level as developed in this thesis can be a tool for exploration with lower computational cost.