# I Green's Function Formalism

Following [**Bruus_Flensberg_2004**]

Green's functions: method to encode influence of many-body effects on propagation of particles in a system.

Have different kinds of Green's functions, for example the retarded Green's function:

$$G^R(\mathbf{r}\sigma t, \mathbf{r}'\sigma' t') = -\mathrm{i}\Theta(t-t')\left\langle\{c_{\mathbf{r}\sigma}(t), c^\dagger_{\mathbf{r}\sigma}(t')\}\right\rangle \tag{I.1}$$

They give the amplitude of a particle inserted at point $\mathbf{r}'$ at time $t'$ to propagate to position $\mathbf{r}$ at time $t$. In a translation invariant system: can use $\mathbf{k}$ as a natural basis set:

$$G^R(\mathbf{k}, \sigma t, \sigma' t') = -\mathrm{i}\Theta(t-t')\left\langle\{c_{\mathbf{k}\sigma}(t), c^\dagger_{\mathbf{k}\sigma}(t')\}\right\rangle \tag{I.2}$$

Can define the spectral function from this:

$$A(\mathbf{k}\sigma, \omega) = -2\Im G^R(\mathbf{k}\sigma, \omega) \tag{I.3}$$

Mathematical technique to calculate retarded GFs involves defining GFs on imaginary times $\tau$:

$$t \to -\mathrm{i}\tau \tag{I.4}$$

where $\tau$ is real and has the dimension time. This enables the simultaneous expansion of exponential $e^{-\beta H}$ coming from the thermodynamic average and $e^{-\mathrm{i}Ht}$ coming from the time evolution of operators.

Define imaginary time/Matsubara GF $\mathcal{C}_{AB}(\tau, 0)$:

$$\mathcal{C}_{AB}(\tau, 0) = -\left\langle T_\tau(A(\tau)B(0))\right\rangle \tag{I.5}$$

with time-ordering operator in imaginary time:

Can prove from properties of Matsubara GF, that they are only defined for

$$-\beta < \tau < \beta \tag{I.6}$$

Due to this, the Fourier transform of the Matsubara GF is defined on discrete values:

$$\mathcal{C}_{AB}(\mathrm{i}\omega_n) = \int_0^\beta \mathrm{d}\tau \tag{I.7}$$

with fermionic/bosonic Matsubara frequencies

$$\omega_n = \begin{cases} \frac{2n\pi}{\beta} \text{ for bosons} \\ \frac{(2n+1)\pi}{\beta} \text{ for fermions} \end{cases} \tag{I.8}$$

Extrapolation of the Matsubara GF to zero is proportional to the density of states at the chemical potential. Gapped: density is zero (Matsubara GF goes to 0), metal: density is finite (Matsubara GF goes to finite value) [**Bruus_Flensberg_2004**].

## I.1 Nambu-Gorkov GF

Order parameter can be chosen as the anomalous GF:

$$\Psi = F^{\mathrm{loc}}(\tau = 0^-) \tag{I.9}$$

or the superconducting gap

$$\Delta = Z\Sigma^{\mathrm{AN}} \tag{I.10}$$

that can be calculated from the anomalous self-energy $\Sigma^{\mathrm{AN}}$ and quasiparticle weight $Z$