I Superconductivity

This chapter gives an introduction to the phenomenology and theory of superconductivity. Superconductivity describes the phenomenon of the electrical resistance of a metal suddenly dropping to zero below a critical temperature.

At the beginning of the 20th century,

More details on history

Discovered in mercury in 1911 by Heike Onnes [**onnes1911further**].

Meissner effect, leads to supercurrent

Some words: why is it interesting?

I.1 BCS Theory

Following [**colemanIntroductionManyBodyPhysics2015**].

Theoretical description of SC: 1956 by Bardeen, his postdoc Leon Cooper and the graduate in the group, J. Robert Schrieffer. Description is based on the fact, that the Fermi sea is unstable towards development of bound pairs under arbitrarily small attraction [**cooperBoundElectronPairs1956**]. These bound electrons show bosonic behaviour and

Why supercurrent?

This model Hamiltonian

BCS hamiltonian, pairing

It can be connected to

Mean-field

The final element in this description was the origin of the attractive interaction between electrons, which Bardeen, Cooper and Schrieffer identified as a retarded electron-phonon interaction [**bardeenTheorySuperconductivity1957**].

Phonon interaction

This so-called BCS-theory of superconductivity is very successful in explaining experimental results in many compounds, Surprisingly, it

Mean field level can already explain a lot

BCS-theory gave a microscopic explanation to a phenomenological description of superconductivity pioneered by Fritz London in 1937 [**londonNewConceptionSupraconductivity193**
This descriptions is based on a one-particle wavefunction $\phi(x)$

Later, this one-particle wa vefunction was identified as the order parameter in the developing GL-theory of phase transitions [**landauTheorySuperconductivity1965**]. GL-theory is discussed in more detail in section I.2. This explains the Meissner effect and in turn the supercurrent.

More recent developments: strongly correlated superconductors

I.2 Ginzburg-Landau theory of superconductivity,Phase transitions and broken symmetry

Following [**colemanIntroductionManyBodyPhysics2015**].

I.2.1 Order parameter concept

Landau theory: phase transitions (e.g. iron becomes magnetic, water freezes, superfluidity/superconductivity) are associated with the development of an order parameter when the temperature drops below the transition temperature T_C.

$$|\psi| = \begin{cases} 0 \; , \; T > T_C \\ |\psi_0| > 0 \; , \; T < T_C \end{cases} \tag{I.1}$$

Landau theory does not need microscopic expression for order parameter, it provides corse-grained description of the properties of matter. The order parameter description is good at length scales above ξ_0, the coherence length (e.g. size of Cooper pairs for SC).

I.2.2 Landau theory

Basic idea of Landau theory: write free energy as function $F[\psi]$ of the order parameter. Region of small ψ, expand free energy of many-body system as simple polynomial:

$$f_L = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4 \tag{I.2}$$

Provided r and u are greater that 0: minimum of $f_L[\psi])$ lies at $\psi = 0$. Landau theory assumes: at phase transition temperature r changes sign, so:

$$r = a(T - T_C) \tag{I.3}$$

Minimum of free energy occurs for:

$$\psi = \begin{cases} 0 \\ \pm\sqrt{\frac{a(T_C - T)}{u}} \end{cases} \tag{I.4}$$

Two minima for free energy function for $T < T_C$. With this, we can extract T_C from the knowledge of the dependence of $|\psi|^2$ on T via a linear fit. This is only valid for an area near T_C (where Landau theory holds), but can be used to get T_C from microscopic theories.

Could put a bit more into here about second order phase transition

Going from a one to a n-component order parameters, OP acquires directions and magnitude. Particularly important example: complex or two component order parameter in superfluids and superconductors:

$$\psi = \psi_1 + \mathrm{i}\psi_2 = |\psi| e^{\mathrm{i}\phi} \tag{I.5}$$

The Landau free energy takes the form:

$$f[\psi] = r(\psi^* \psi) + \frac{u}{2}(\psi^* \psi)^2 \tag{I.6}$$

As before:

$$r = a(T - T_C) \tag{I.7}$$

Figure I.1 shows the Landau free energy as function of ψ.

Rotational symmetry, because free energy is independent of the global phase of the OP:

$$f[\psi] = f[e^{\mathrm{i}a}\psi] \tag{I.8}$$

In this 'Mexican hat' potential: order parameter can be rotated continuously from one broken-symmetry state to another. If we want the phase to be rigid, we need to introduce an There is a topological argument for the fact that the phase is rigid. This leads to Ginzburg-Landau theory. Will see later: well-defined phase is associated with persistent currents or superflow.

I.2.3 Ginzburg-Landau theory I: Ising order

Landau theory: energy cost of a uniform order parameter, more general theory needs to account for inhomogenous order parameters, in which the

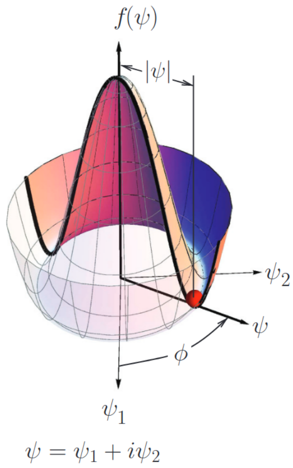

Figure I.1: *Mexican hat potential*

amplitude varies or direction of order parameter is twisted -> GL theory. First: one-component, 'Ising' order parameter. GL introduces additional energy $\delta f \propto |\Delta\psi|^2$, $f_{GL}[\psi, \Delta\psi] = \frac{s}{2}|\Delta\psi|^2 + f_L[\psi(s)]$, or in full:

$$f_{GL}[\psi, \Delta\psi, h] = \frac{s}{2}(\Delta\psi)^2 + \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4 - h\psi \tag{I.9}$$

What is the h here?

What is c?

length scale/correlation length

GL theory is only valid near critical point, where OP is small enough to permit leading-order expansion. Dimensional analysis shows: $\frac{c}{r} = L^2$ has dimension of length squared. Length scale introduced by

I.2.4 Ginzburg-Landau theory II: complex order and superflow

Now: GL theory of complex or two-component order parameters, so superfluids and superconductors. Heart of discussion: emergence of a 'macroscopic wavefunction', where the microscopic field operators $\hat{\psi}(x)$ acquire an expectation value:

$$\langle\hat{\psi}(x)\rangle = \psi(x) = |\psi(x)|e^{i\theta(x)} \tag{I.10}$$

Reminder: Field operators are the real space representations of creation/annihilation operators. They can be thought of the super position of all ways of creating a particle at position x via the basis coefficients.

Magnitude determines density of particles in the superfluid:

$$|\psi(x)|^2 = n_s(x) \tag{I.11}$$

Density operator is

$$\hat{\rho} = \hat{\psi}(x)\hat{\psi}^{\dagger}(x) \tag{I.12}$$

so expectation value of that is the formula above.

Twist/gradient of phase determines superfluid velocity:

$$\mathbf{v}_s(x) = \frac{\hbar}{m}\Delta\phi(x) \tag{I.13}$$

We will derive this later in the chapter. Counterintuitive from quantum mechanics: GL suggested that $\Phi(x)$ is a macroscopic manifestation of a macroscopic number of particles condensed into precisely the same quantum state. Emergent phenomenon, collective properties of mater not a-priori self-evident from microscopic physics.

GL free energy density for superfluid (with one added term in comparison to Landau energy):

$$f_{GL}[\psi, \Delta\psi] = \frac{\hbar^2}{2m}|\Delta\psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4 \tag{I.14}$$

Interpreted as energy density of a condensate of bosons in which the field operator behaves as a complex order parameter. Gives interpretation of gradient term as kinetic energy:

energy density of bosonic field? -> for comparison!

$$s|\Delta\psi|^2 = \frac{\hbar^2}{2m}\langle\Delta\hat{\psi}^{\dagger}\Delta\hat{\psi}\rangle \implies s = \frac{\hbar^2}{2m} \tag{I.15}$$

As in Ising order: correlation length/GL-coherence length governs characteristic range of amplitude fluctuations of the order parameter:

$$\xi = \sqrt{\frac{s}{|r|}} = \sqrt{\frac{\hbar^2}{2m|r|}} = \xi_0\left(1 - \frac{T}{T_C}\right)^{-\frac{1}{2}} \tag{I.16}$$

where $\xi_0 = \xi(T = 0) = \sqrt{\frac{\hbar^2}{2maT_C}}$ is the coherence length. Beyond this length: only phase fluctuations survive. Freeze out fluctuations in amplitude

Compare with Ising order, especially dependence on T

Compare with Ising order. Is that derived or postulated?

(no x-dependence in amplitude) $\psi(x) = \sqrt{n_s} e^{i\phi(x)}$, then $\Delta\psi = i\Delta\phi\psi$ and $|\Delta\psi|^2 = n_s(\Delta\phi)^2$, dependency of kinetic energy on the phase twist is (bringing it into the form $\frac{m}{2}v^2$):

$$\frac{\hbar^2 n_s}{2m}(\Delta\phi)^2 = \frac{m n_s}{2}(\frac{\hbar}{m}\Delta\phi)^2 \tag{I.17}$$

So twist of phase results in increase in kinetic energy, associated with a superfluid velocity:

$$\mathbf{v}_s = \frac{\hbar}{m}\Delta\phi \tag{I.18}$$

For interpretation of superfluid states: coherent states. These are eigenstates of the field operator

$$\hat{\psi}(x)\,|\psi\rangle = \psi(x)\,|\psi\rangle \tag{I.19}$$

and don't have a definite particle number. Importantly, this small uncertainty in particle number enables a high degree of precision in phase (which is the property of a condensate).

Phase rigidity and superflow: in GL theory, energy is sensitive to a twist of the phase. Substitute $\psi = |\psi|e^{i\phi}$ into GL free energy, gradient term is:

$$\Delta\psi = (\Delta|\psi| + i\Delta\phi|\psi|)e^{i\phi} \tag{I.20}$$

So:

$$f_{GL} = \frac{\hbar}{2m}|\psi|^2(\Delta\phi)^2 + \left[\frac{\hbar}{2m}(\Delta|\psi|)^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4\right] \tag{I.21}$$

The second term resembles GL functional for an Ising order parameter, describes energy cost of variations in the magnitude of the order parameter.

Here: particle-current operator, especially for coherent state, connection with phase twist

I.3 Coherence length and penetration depth in strongly correlated superconductors

From [**wittBypassingLatticeBCSBEC2024**].

In most materials: Cooper pairs do not carry finite center-of-mass momentum. In presence of e.g. external fields or magnetism: SC states with FMP might arise.

Theory/procedure in the paper: enforce FMP states via constraints on pair-center-of-mass momentum $\mathbf{q}$, access characteristic lenght scales ξ_0, λ_L through analysis of the momentum and temperature-dependent OP. Constrain for FF-type pairing:

$$\psi_{\mathbf{q}}(\mathbf{r}) = |\psi_{\mathbf{q}}| e^{\mathbf{i}\mathbf{q}\mathbf{r}} \tag{I.22}$$