PHY-MV-FN-T28 – Condensed-matter theory - special topics

Lecture held summer term 2022 by Prof. Dr. Tim Wehling, Universität Hamburg

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Part I.

Entrée – Revisiting topics from PHY-MV-FN-T14 (Condensed Matter Theory)

1. Second Quantization

We want to describe a system of identical particles (electrons, photons, neutrons, ...). Each particle has a corresponding single-particle Hilbert space: Lecture 1 – 04.04.2022

$$\mathcal{H}_1^i = \operatorname{span}\{|\phi_\alpha^i\rangle\} \tag{1.1}$$

With the single-particle basis for the ith particle $|\phi_\alpha^i\rangle$. The index α is a complete system of single-particle quantum numbers, such as $\alpha = \{\mathbf{k}, n, \sigma\}$. A general N particle Hilbert space is then constructed as

$$\mathcal{H}_N = \mathcal{H}_1^1 \otimes \mathcal{H}_1^2 \otimes \ldots \otimes \mathcal{H}_1^N \,. \tag{1.2}$$

In quantum mechanics, identical particles are indistinguishable, so there should be no measurable consequences of particle permutations (swapping two particles). This means, that only states invariant under particle permutations reflect physical reality.

A more formal treatment:

2. Interaction effects in solids and electronic correlations

Part II.

Premier plat principal – Green Function Formalism for Many-Body-Physics

3. Many-Body Green Functions

3.1. Reminder: Time evolution pictures in Quantum Mechanics

Lecture 3 – 14.04.2022

Schrödinger picture: time evolution goes with the wave function, which is governed by the Schrödinger equation

$$i\hbar\partial_t |\Psi(t)\rangle = H |\Psi(t)\rangle \; . \tag{3.1}$$

The time evolution of expectation values is

Lecture 4 – 21.04.2022

3.2. Green Functions for Many-Body Systems

The fundamental idea of Green functions can be thought of as the process of putting a particle into a system, letting it propagate and then taking it out again. Because the particle is interacting with the full many-body system, this propagation encodes then information about this system.

As an introduction, we look at a single particle system with a Hamilton operator H. We want to understand the following process: an initial state $|\phi_\alpha\rangle$ at time $t_0 = 0$ is prepared, then a measurement of an observable A is taken at time t. The expectation value for the measurement is then:

$$\begin{aligned}
\langle A\rangle_t &= \left\langle \phi_\alpha \,\middle|\, e^{iHt} A e^{-iHt} \,\middle|\, \phi_\alpha \right\rangle & (3.2)\\
&= \langle\phi_\alpha| e^{iHt} (\sum_\beta |\phi_\beta\rangle \langle\phi_\beta|) A (\sum_{\beta'} |\phi_{\beta'}\rangle \langle\phi_{\beta'}|) e^{-iHt} |\phi_\alpha\rangle & (3.3)\\
&= & (3.4)
\end{aligned}$$

4. Matsubara Green Functions

5. Feynman Diagrams

Part III.

Cours intermédiaire – Superconductivity

6. Superconductivity

Part IV.

Deuxième plat principal –

7. Coherent States

8. Coherent State Path Integrals

9. Broken Symmetry and Collective Phenomena