PHY-MV-BE-T04 — Computational physics with focus on time-dependent quantum mechanics

Lecture held winter term 2022/23 by Prof. Dr. Daria Gorelova, Universität Hamburg

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Contents

1	Explicit forward Euler method	5
2	Runge-Kutta and related methods	7
3	Crank-Nicholson Method	9
4	Numerov method 4.1 General Numerov method	
5	Fast-Fourier-Transform and Split Operator	13
6	Time-dependent Perturbation Theory and Numerical Integration	15
7	Monte-Carlo Methods and Ising Model	17
8	Lanczos Algorithm	19
9	Machine Learning	21

1 Explicit forward Euler method

Consider a first-order differential equation with boundary condition,

Lecture 1–24.10.2022

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = f(y(t), t) \text{ with } y(t_0) = y_0.$$
(1.1)

As f is a known function, we know the derivative at t_0 :

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t}\bigg|_{t=t_0} = f(y(t_0), t_0) = f(y_0, t_0) \tag{1.2}$$

This is enough information to write down the tangent line of the solution at $t = t_0$:

$$y_{\text{tangent}}(t) = y_0 + f(y_0, t_0)(t - t_0)$$
 (1.3)

We can now take some $t_1 > t_0$. Given t_1 is close enough to t_0 ,

Put some graphic here

2 Runge-Kutta and related methods

3 Crank-Nicholson Method

Lecture 3 – 14.11.2022

4 Numerov method

4.1 General Numerov method

Lecture 4 – 21.11.2022

Within the Numerov method, differential equations of the following type are treated:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = U(x) + V(x)y(x) \tag{4.1}$$

Exercise 4 – 21.11.2022

4.2 Anharmonic oscillator and shooting

In the exercise, the Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(x) + V(x)\Psi(x) = E\Psi(x)$$
(4.2)

with the potential

5 Fast-Fourier-Transform and Split Operator

Lecture 5 - 28.11.2022

6 Time-dependent Perturbation Theory and Numerical Integration

Lecture 6 - 05.12.2022

7 Monte-Carlo Methods and Ising Model

 $\begin{array}{c} Lecture \ 7 - \\ 12.12.2022 \end{array}$

8 Lanczos Algorithm

Lecture 8 – 19.12.2022

9 Machine Learning

 $\begin{array}{c} \text{Lecture} \\ 9\text{--}16.01.2023 \end{array}$