1 Many-Body Green Functions

1.1 Reminder: Time evolution pictures in Quantum Mechanics

Lecture 3, 14.04.2022

1.1.1 Schrödinger picture

In the Schrödinger picture, the time evolution goes with the wave function, which is governed by the Schrödinger equation

$$i\hbar\partial_t |\Psi(t)\rangle = H |\Psi(t)\rangle \ . \tag{1.1}$$

The time evolution of expectation values is then:

$$\langle A\rangle_t = \langle\Psi(t)| A |\Psi(t)\rangle \tag{1.2}$$

1.1.2 Heisenberg picture

In the Heisenberg picture, the time evolution goes with the operators:

Lecture 4, 21.04.2022

1.2 Green Functions for Many-Body Systems

The fundamental idea of Green functions can be thought of as the process of putting a particle into a system, letting it propagate and then taking it out again. Because the particle is interacting with the full many-body system, this propagation encodes then information about this system.

As an introduction, we look at a single particle system with a Hamilton operator H. We want to understand the following process: an initial state $|\phi_\alpha\rangle$ at time $t_0 = 0$ is prepared, then a measurement of an observable A is taken at time t. The expectation value for the measurement is then:

$$\begin{aligned}
\langle A\rangle_t &= \left\langle \phi_\alpha \,\middle|\, e^{iHt} A e^{-iHt} \,\middle|\, \phi_\alpha \right\rangle && (1.3)\\
&= \langle\phi_\alpha| e^{iHt} (\sum_\beta |\phi_\beta\rangle\langle\phi_\beta|) A (\sum_{\beta'} |\phi_{\beta'}\rangle\langle\phi_{\beta'}|) e^{-iHt} |\phi_\alpha\rangle && (1.4)\\
&= && (1.5)
\end{aligned}$$