Ideal F-v motor, Heavy Spring, Rounded Latch @ Constant Force
30 Jan 2020

Parameters:
Motor: $F_{\max}$, d, $v_{\max}$
Spring: m_{spr}
Latch: F_l, m_l, v_{l0}, R
Projectile: m
Timing: t_l (time of latch releasing), t_{to} (time of projectile taking off)
Non-dimensionalized variables: see pg. 7, supplemental.

Prior to latch release ($t \leq t_l$)
Latch moves in y direction, projectile in x direction (like supplemental).
Position of the rounded latch, w/r/t time (assume $y(0) = 0$) for a constant force:

$$y(t) = v_0 t + \frac{1}{2}\frac{F_{\max}}{m_l}\, t^2$$

Position of projectile while in contact with latch:

$$x(t) = R\left(1 - \sqrt{1 - \left(\frac{y(t)}{R}\right)^2}\right)$$

Velocity of projectile, in terms of $y(t)$:

$$v(t) = \frac{y(t)y'(t)}{R\sqrt{1 - \left(\frac{y(t)}{R}\right)^2}}$$

Acceleration:

$$a(t) = \frac{1}{R\sqrt{1 - \left(\frac{y(t)}{R}\right)^2}}\left(y'(t)^2 + y(t)y''(t) + \frac{y(t)^2 y'(t)^2}{R^2 - y(t)^2}\right)$$

Game-plan: By leaving these equations in terms of $y(t)$, we can declare $y(t)$ as a separate function in Matlab. This hopefully will also allow changing $y(t)$ easily, if needed (e.g. for testing/debugging).

Sanity checking: Plugging in $y(t) = v_0$ in each of these equations yielded identical results to equations 27-29 in supplemental text.

Non-dimensionalizing: Yields exact same equations (with tildas)

Solving for time of latch release t_l
Using the energy argument of section 6 of supplemental,

$$\frac{1}{2}(1-\tilde{x})^2 + \frac{1}{2}\left(\tilde{m} + \frac{\tilde{m}_{\text{spr}}}{3}\right)\tilde{v}^2 - \int_0^{\tilde{x}} f_{\text{latch}}(x')\,\mathrm{d}x' = \textit{constant}.$$

$$\rightarrow \left(\tilde{m} + \frac{\tilde{m}_{\text{spr}}}{3}\right)\tilde{a}(\tilde{t_l}) = 1 - \tilde{x}(\tilde{t_l})$$

We can substitude the equations for $x(t)$ and $a(t)$ above into this expression, and solve for t_l.

Solving for velocity at takeoff $v_{\mathbf{to}}$:
Based on conservation of energy: $\tilde{U}(\tilde{t_l}) + \tilde{KE}(\tilde{t_l}) = \tilde{KE}(\tilde{v_{\text{to}}}))$, we get equation 39 in supplemental:

$$\tilde{v_{\text{to}}} = \sqrt{\tilde{v_l}^2 + \frac{1}{\tilde{m} + \frac{\tilde{m}_{\text{spr}}}{3}}(1-\tilde{x}_l)^2}$$

where $\tilde{v}_l$ and $\tilde{x}_l$ are the equations for $\tilde{v}(\tilde{t})$ and $\tilde{x}(\tilde{t})$ above, evaluated at $\tilde{t}_l$.

After latch release $t_l < t < t_{\mathbf{to}}$:
Equations 61-67 in supplemental.

After takeoff $t > t_{\mathbf{to}}$:

$$x(t) = v_{\text{to}}t$$

$$v(t) = v_{\text{to}}$$

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Before, all derivations assumed an ideal k value for the spring; i.e. $k_{optimal} = F_{max}/d$. Now, we will mdoify these derivations for a non-ideal spring stiffness.

New Parameters:
E, young's modulus (stiffness of material)
A, cross-sectional area of spring
L, rest length of the spring
$k = \frac{EA}{L}$

Dimensionless values:
$\tilde{k} = \frac{k}{k_{optimal}} = \frac{kd}{F_{max}}$

Solving for time of latch release t_l
Modifying the potential energy to include $\tilde{k}$,

$$\frac{1}{2}\tilde{k}(\tilde{x}_{max} - \tilde{x})^2 + \frac{1}{2}\left(\tilde{m} + \frac{\tilde{m}_{\mathrm{spr}}}{3}\right)\tilde{v}^2 - \int_0^{\tilde{x}} f_{\mathrm{latch}}(x')\,\mathrm{d}x' = constant.$$

$$\rightarrow \left(\tilde{m} + \frac{\tilde{m}_{\mathrm{spr}}}{3}\right)\tilde{a}(\tilde{t}_l) = \tilde{k}\left(\tilde{x}_{max} - \tilde{x}(\tilde{t}_l)\right)$$

where

$$\tilde{x}_{max} = \frac{x_{max}}{d} = \frac{F_{max}/k}{d} = \frac{1}{\tilde{k}}.$$

and $\tilde{x}_{max}$ can never exceed 1, since the spring can never be contracted past the motor's max range d.

Solving for velocity at takeoff $v_{\mathbf{to}}$:

$$\tilde{v_{\mathrm{to}}} = \sqrt{\tilde{v_l}^2 + \frac{\tilde{k}}{\tilde{m} + \frac{\tilde{m}_{\mathrm{spr}}}{3}}(\tilde{x}_{max} - \tilde{x}_l)^2}$$

After latch release $t_l < t < t_{\mathbf{to}}$:
In equation 61, switch in maximum position $\tilde{x}_{max}$ and add spring constant $\tilde{k}$ to give

$$\tilde{x}(\tilde{t}) = \tilde{x}_{max} - \tilde{v}_{to}\sqrt{\frac{\tilde{m} + \frac{\tilde{m}_{spr}}{3}}{\tilde{k}}}\cos\left(\sqrt{\frac{\tilde{k}}{\tilde{m} + \frac{\tilde{m}_{spr}}{3}}}\;\tilde{t} + \tilde{\phi}\right)$$

In equations 62-67 in supplemental, replace $\tilde{m} + \frac{\tilde{m}_{spr}}{3}$ terms with $(\tilde{m} + \frac{\tilde{m}_{spr}}{3})/\tilde{k}$. In $\tilde{\phi}$, replace $1 - \tilde{x}_l$ with $\tilde{x}_{max} - \tilde{x}_l$.