№ 1

a. $\lim\limits_{x\to-2}\left(\frac{\frac{2}{x-8}+\frac{x}{3x-4}}{\frac{x}{2x-16}+\frac{1}{3x-4}}\right)^{\left[\frac{0}{0}\right]} = \lim\limits_{x\to-2}\frac{\left(\frac{6x-8+x^2-8x}{(x-8)(3x-4)}\right)}{\left(\frac{3x^2-4x+2x-16}{(3x-4)(2x-16)}\right)} =$

$= \lim\limits_{x\to-2}\frac{2\cdot\cancel{(x-8)}\cdot\cancel{(3x-4)}\cdot(x^2-2x-8)}{\cancel{(x-8)}\cdot\cancel{(3x-4)}\cdot(3x^2-2x-16)} = 2\cdot\lim\limits_{x\to-2}\frac{\cancel{(x+2)}\cdot(x-4)}{\cancel{(x+2)}\cdot(3x-8)} =$

$= 2\cdot\frac{-6}{-14} = \frac{6}{7}$

b. $\lim\limits_{x\to3}\frac{3x-9}{\sqrt{-x^2-x+21}-x}^{\left[\frac{0}{0}\right]} = 3\cdot\lim\limits_{x\to3}\frac{(x-3)\cdot(x+\sqrt{-x^2-x+21})}{-2x^2-x+21} =$

$= -3\cdot\lim\limits_{x\to3}\frac{\cancel{(x-3)}\cdot(x+\sqrt{-x^2-x+21})}{\cancel{(x-3)}(2x+7)} = -3\cdot\lim\limits_{x\to3}\frac{x+\sqrt{-x^2-x+21}}{2x+7} =$

$= -3\cdot\frac{6}{13} = -\frac{18}{13}$

c. $\lim\limits_{x\to4}\left(e^x ln(\sqrt{x^2+1})\right) = e^4\cdot ln(\sqrt{17}) = \frac{e^4\cdot ln(17)}{2}$

№ 2

$$f(x) = \begin{cases} x^2-x-3 & \text{if } x<-2 \\ -1 & \text{if } x=-2 \\ x^2-1 & \text{if } (-2<x) \text{ and } (x<1) \\ 0 & \text{if } x=1 \\ 3x^2-2x-1 & \text{if } (1<x) \text{ and } (x<2) \\ 7 & \text{if } x=2 \\ x^2+3x & \text{if } 2<x \end{cases}$$

1. $a = -2$

$$\underbrace{\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-}(x^2 - x - 3)}_{f(x)=x^2-x-3 \text{ for } x<-2} = (4 - (-2) - 3) = 3$$

$$\underbrace{\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+}(x^2 - 1)}_{f(x)=x^2-1 \text{ for } x>-2} = 4 - 1 = 3$$

$$\left.\begin{array}{l} \lim\limits_{x\to -2^-} f(x) = 3 \\ \lim\limits_{x\to -2^+} f(x) = 3 \end{array}\right\} \implies \lim_{x\to -2} f(x) = 3$$

$f(-2) = -1 \neq \lim\limits_{x\to -2} f(x) \implies f(x)$ is not continuous at $x = -2$

2. $a = 1$

$$\underbrace{\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x^2 - 1)}_{f(x)=x^2-1 \text{ for } x<1} = 1 - 1 = 0$$

$$\underbrace{\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(3x^2 - 2x - 1)}_{f(x)=3x^2-2x-1 \text{ for } x>1} = 3 - 2 - 1 = 0$$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^-} f(x) = 0 \\ \lim\limits_{x\to 1^+} f(x) = 0 \end{array}\right\} \implies \lim_{x\to 1} f(x) = 0$$

$f(1) = \lim\limits_{x\to 1} f(x) \implies f(x)$ is continuous at $x = 1$

3. $a = 2$

$$\underbrace{\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(3x^2 - 2x - 1)}_{f(x)=3x^2-2x-1 \text{ for } x<2} = 3\cdot 2^2 - 2\cdot 2 - 1 = 7$$

$$\underbrace{\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x^2 + 3x)}_{f(x)=x^2+3x \text{ for } x>2} = 2^2 + 3\cdot 2 = 10$$

$$\left.\begin{array}{l}\lim_{x\to 2^-} f(x) = 7 \\ \lim_{x\to 2^+} f(x) = 10\end{array}\right\} \implies \lim_{x\to 2} f(x) \text{ DNE}$$

Since $\lim_{x\to 2} f(x)$ DNE $\implies$ $f(x)$ is not continuous at x = 2