№ 1

$$\begin{pmatrix} 3 & 2 \\ 5 & -2 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 & 5 \\ 4 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 5 & -2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 14 & -9 & 17 \\ 2 & -15 & 23 \\ -2 & 3 & -5 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 5 & -2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -20 & 46 \\ -92 & 34 \\ 14 & -10 \end{pmatrix}$$

№ 1.1

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 11 & 16 \\ 19 & 28 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} =$$
$$= \begin{pmatrix} -16 & 43 \\ -28 & 75 \end{pmatrix}$$

№ 3

Найдем характеристический полином матрицы: $P(\lambda) = det(A - \lambda E) =$

$$= \begin{vmatrix} 3-\lambda & 1 & 4 & 4 \\ -4 & -2-\lambda & -4 & -4 \\ -1 & 0 & 2-\lambda & -1 \\ 0 & -1 & -4 & -1-\lambda \end{vmatrix} \underset{\substack{R_1 : R_1 + (3-\lambda)R_3 \\ R_2 : R_2 - 4R_3}}{=} \begin{vmatrix} 0 & 1 & \lambda^2 - 5\lambda + 10 & \lambda + 1 \\ 0 & -2-\lambda & 4\lambda - 12 & 0 \\ -1 & 0 & 2-\lambda & -1 \\ 0 & -1 & -4 & -1-\lambda \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & \lambda^2 - 5\lambda + 10 & \lambda + 1 \\ -2-\lambda & 4\lambda - 12 & 0 \\ 1 & 4 & 1+\lambda \end{vmatrix} = (\lambda+1) \cdot \begin{vmatrix} 1 & \lambda^2 - 5\lambda + 10 & 1 \\ -2-\lambda & 4\lambda - 12 & 0 \\ 1 & 4 & 1 \end{vmatrix} \underset{R_1 : R_1 - R_3}{=}$$

$$= (\lambda+1) \cdot \begin{vmatrix} 0 & \lambda^2 - 5\lambda + 6 & 0 \\ -2-\lambda & 4\lambda - 12 & 0 \\ 1 & 4 & 1 \end{vmatrix} = (\lambda+1) \cdot \begin{vmatrix} 0 & \lambda^2 - 5\lambda + 6 \\ -2-\lambda & 4\lambda - 12 \end{vmatrix} =$$
$$= \underbrace{(\lambda+1)(\lambda+2)(\lambda-2)(\lambda-3)}_{P(\lambda)}$$

Найдем собственные числа матрицы:

$$P(\lambda) = 0 \leftrightarrow \left[\begin{array}{l} \lambda_1 = -2 \\ \lambda_2 = -1 \\ \lambda_3 = 2 \\ \lambda_4 = 3 \end{array} \right. \quad \begin{array}{c} \text{Собственные} \\ \text{числа матрицы} \end{array}$$

Найдем собственные вектора матрицы:

1. $\lambda = -2$

$$\left(\begin{array}{cccc|c} 5 & 1 & 4 & 4 & 0 \\ -4 & 0 & -4 & -4 & 0 \\ -1 & 0 & 4 & -1 & 0 \\ 0 & -1 & -4 & 1 & 0 \end{array} \right) \underset{\substack{R_1 : R_1 + R_2 + R_3 \\ R_2 : \frac{R_2 - 4R_3}{-20} \\ R_3 : -R_3}}{=} \left(\begin{array}{cccc|c} 0 & 1 & 4 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -4 & 1 & 0 \\ 0 & -1 & -4 & 1 & 0 \end{array} \right) \underset{\substack{R_4 : R_4 + R_1 \\ R_3 : R_3 + R_1 \\ R_1 : R_1 - 4R_2}}{=}$$

$$
= \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \leftrightarrow \begin{cases} x_1 + x_2 = 0 \\ x_2 - x_4 = 0 \\ x_3 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow \begin{cases} x_1 = -c_4 \\ x_2 = c_4 \\ x_3 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow X = c_4 \cdot \underbrace{\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}}_{\vec{v_1}}
$$

2. $\lambda = -1$

$$
\left(\begin{array}{cccc|c} 4 & 1 & 4 & 4 & 0 \\ -4 & -1 & -4 & -4 & 0 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & -1 & -4 & 0 & 0 \end{array}\right) \underset{\substack{R_1 : R_1 + R_2 \\ R_2 : R_2 - 4R_3 \\ R_3 : -R_3 \\ R_4 : -R_4}}{=} \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -16 & 0 & 0 \\ 1 & 0 & -3 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 \end{array}\right) \underset{\substack{R_2 : -\frac{R_2 + R_4}{12} \\ R_3 : R_3 + 3R_2 \\ R_4 : R_4 - 4R_2}}{=} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)
$$

$$
\leftrightarrow \begin{cases} x_1 + x_4 = 0 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow \begin{cases} x_1 = -c_4 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow X = c_4 \cdot \underbrace{\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{\vec{v_2}}
$$

3. $\lambda = 2$

$$
\left(\begin{array}{cccc|c} 1 & 1 & 4 & 4 & 0 \\ -4 & -4 & -4 & -4 & 0 \\ -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -4 & -3 & 0 \end{array}\right) \underset{\substack{R_2 : -\frac{R_2}{4} \\ R_3 : -R_3 \\ R_4 : -R_4 \\ R_1 : \frac{R_1 - R_2}{3}}}{=} \left(\begin{array}{cccc|c} 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 3 & 0 \end{array}\right) \underset{\substack{R_2 : R_2 - R_1 - R_3 \\ R_4 : -(R_4 - R_2 - 4R_1)}}{=} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)
$$

$$
\leftrightarrow \begin{cases} x_1 + x_4 = 0 \\ x_2 - x_4 = 0 \\ x_3 + x_4 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow \begin{cases} x_1 = -c_4 \\ x_2 = c_4 \\ x_3 = -c_4 \\ x_4 = c_4 \end{cases} \leftrightarrow X = c_4 \cdot \underbrace{\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}}_{\vec{v_3}}
$$

4. $\lambda = 3$

$$
\left(\begin{array}{cccc|c} 0 & 1 & 4 & 4 & 0 \\ -4 & -5 & -4 & -4 & 0 \\ -1 & 0 & -1 & -1 & 0 \\ 0 & -1 & -4 & -4 & 0 \end{array}\right) \underset{\substack{R_4 : R_4 + R_1 \\ R_2 : \frac{4R_3 - R_2}{5} \\ R_3 : -R_3}}{=} \left(\begin{array}{cccc|c} 0 & 1 & 4 & 4 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \underset{R_1 : \frac{R_1 - R_2}{4}}{=} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \leftrightarrow
$$

$$
\leftrightarrow \begin{cases} x_1 + x_3 + x_4 = 0 \\ x_2 = 0 \\ x_3 + x_4 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = -c_4 \\ x_4 = c_4 \end{cases} \leftrightarrow X = c_4 \cdot \underbrace{\begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}}_{\vec{v_4}}
$$

Ответ: $P(\lambda) = (\lambda + 2)(\lambda + 1)(\lambda - 2)(\lambda - 3)$

Собственные числа матрицы: $-2, -1, 2, 3$

Собственные вектора матрицы: $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}$

№ 3.1

Найдем характеристический полином матрицы: $P(\lambda) = det(A - \lambda E) =$

$$= \begin{vmatrix} 7-\lambda & -3 & 2 & -6 \\ 1 & -1-\lambda & -2 & -6 \\ -1 & 3 & 4-\lambda & 6 \\ -1 & -3 & 2 & 2-\lambda \end{vmatrix} \underset{\substack{R_1 : R_1 + R_2(\lambda-7) \\ R_3 : R_3 + R_2 \\ R_4 : R_4 + R_2}}{=} \begin{vmatrix} 0 & -\lambda^2+6\lambda+4 & 16-2\lambda & 36-6\lambda \\ 1 & -1-\lambda & -2 & -6 \\ 0 & 2-\lambda & 2-\lambda & 0 \\ 0 & -4-\lambda & 0 & -4-\lambda \end{vmatrix} =$$

$$= (\lambda-2)(\lambda+4) \cdot \begin{vmatrix} \lambda^2-6\lambda-4 & 2\lambda-16 & 6\lambda-36 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = (\lambda-2)(\lambda+4) \cdot [\begin{vmatrix} 2\lambda-16 & 6\lambda-36 \\ 1 & 0 \end{vmatrix} +$$

$$\begin{vmatrix} \lambda^2-6\lambda-4 & 2\lambda-16 \\ 1 & 1 \end{vmatrix}] = (\lambda-2)(\lambda+4) \cdot [36-6\lambda+\lambda^2-6\lambda-4+16-2\lambda] =$$

$$= (\lambda-2)(\lambda+4)(\lambda^2-14\lambda+48) = \underbrace{(\lambda+4)(\lambda-2)(\lambda-6)(\lambda-8)}_{P(\lambda)}$$

Найдем собственные числа матрицы:

$$P(\lambda) = 0 \leftrightarrow \left[\begin{array}{l} \lambda_1 = -4 \\ \lambda_2 = 2 \\ \lambda_3 = 6 \\ \lambda_4 = 8 \end{array} \right. \quad \text{Собственные числа матрицы}$$

Найдем собственные вектора матрицы:

1. $\lambda = -4$

$$\left(\begin{array}{cccc|c} 11 & -3 & 2 & -6 & 0 \\ 1 & 3 & -2 & -6 & 0 \\ -1 & 3 & 8 & 6 & 0 \\ -1 & -3 & 2 & 6 & 0 \end{array} \right) \underset{\substack{R_1 : \frac{R_1+6R_3+5R_4}{60} \\ R_4 : R_4 + R_2 \\ R_3 : \frac{R_3+R_2}{6}}}{=} \left(\begin{array}{cccc|c} 0 & 0 & 1 & 1 & 0 \\ 1 & 3 & -2 & -6 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \underset{R_2 : R_2 - 3R_3 + 6R_1}{=}$$

$$= \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \leftrightarrow \begin{cases} x_1 + x_3 = 0 \\ x_2 + x_3 = 0 \\ x_3 + x_4 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow \begin{cases} x_1 = c_4 \\ x_2 = c_4 \\ x_3 = -c_4 \\ x_4 = c_4 \end{cases} \leftrightarrow X = c_4 \cdot \underbrace{\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}}_{\vec{v_1}}$$

2. $\lambda = 2$

$$\left(\begin{array}{cccc|c} 5 & -3 & 2 & -6 & 0 \\ 1 & -3 & -2 & -6 & 0 \\ -1 & 3 & 2 & 6 & 0 \\ -1 & -3 & 2 & 0 & 0 \end{array} \right) \underset{\substack{R_1 : \frac{R_1+3R_3+2R_4}{12} \\ R_4 : \frac{R_3-R_4}{6} \\ R_3 : R_3 + R_2}}{=} \left(\begin{array}{cccc|c} 0 & 0 & 1 & 1 & 0 \\ 1 & -3 & -2 & -6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{array} \right) \underset{R_2 : R_2 + 3R_4 + 3R_1}{=}$$

$$= \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \leftrightarrow \begin{cases} x_1 + x_3 = 0 \\ x_2 + x_4 = 0 \\ x_3 + x_4 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow \begin{cases} x_1 = c_4 \\ x_2 = -c_4 \\ x_3 = -c_4 \\ x_4 = c_4 \end{cases} \leftrightarrow X = c_4 \cdot \underbrace{\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}}_{\vec{v_2}}$$

3. $\lambda = 6$

$$\left(\begin{array}{cccc|c} 1 & -3 & 2 & -6 & 0 \\ 1 & -7 & -2 & -6 & 0 \\ -1 & 3 & -2 & 6 & 0 \\ -1 & -3 & 2 & -4 & 0 \end{array}\right) \underset{\substack{R_3 : R_3 + R_1 \\ R_2 : \frac{R_1 - R_2}{4} \\ R_4 : -\frac{R_4 + R_1}{2}}}{=} \left(\begin{array}{cccc|c} 1 & -3 & 2 & -6 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -2 & 5 & 0 \end{array}\right) \underset{\substack{R_1 : R_1 + R_4 \\ R_4 : \frac{3R_2 - R_4}{5}}}{=}$$

$$= \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \leftrightarrow \begin{cases} x_1 - x_4 = 0 \\ x_2 + x_3 = 0 \\ x_3 - x_4 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow \begin{cases} x_1 = c_4 \\ x_2 = -c_4 \\ x_3 = c_4 \\ x_4 = c_4 \end{cases} \leftrightarrow X = c_4 \cdot \underbrace{\begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}}_{\vec{v_3}}$$

4. $\lambda = 8$

$$\left(\begin{array}{cccc|c} -1 & -3 & 2 & -6 & 0 \\ 1 & -9 & -2 & -6 & 0 \\ -1 & 3 & -4 & 6 & 0 \\ -1 & -3 & 2 & -6 & 0 \end{array}\right) \underset{\substack{R_2 : -\frac{R_2 + R_3}{6} \\ R_3 : \frac{R_3 - R_1}{6} \\ R_4 : R_4 - R_1 \\ R_1 : -R_1}}{=} \left(\begin{array}{cccc|c} 1 & 3 & -2 & 6 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \underset{\substack{R_1 : R_1 - 3R_3 \\ R_3 : \frac{R_2 - R_3}{2}}}{=}$$

$$= \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \leftrightarrow \begin{cases} x_1 + x_3 = 0 \\ x_2 + x_3 = 0 \\ x_3 - x_4 = 0 \\ x_4 = c_4 \end{cases} \leftrightarrow \begin{cases} x_1 = -c_4 \\ x_2 = -c_4 \\ x_3 = c_4 \\ x_4 = c_4 \end{cases} \leftrightarrow X = c_4 \cdot \underbrace{\begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}}_{\vec{v_4}}$$

Ответ: $P(\lambda) = (\lambda + 4)(\lambda - 2)(\lambda - 6)(\lambda - 8)$
Собственные числа матрицы: $-4, 2, 6, 8$

Собственные вектора матрицы: $\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$

№ 5

$$\begin{pmatrix} 7 & 2 & 2 & 4 \\ 2 & 4 & 1 & 4 \\ -2 & -1 & 1 & -3 \\ -2 & 1 & 2 & -2 \end{pmatrix}^{-1} = ?$$

$$\left(\begin{array}{cccc|cccc} 7 & 2 & 2 & 4 & 1 & 0 & 0 & 0 \\ 2 & 4 & 1 & 4 & 0 & 1 & 0 & 0 \\ -2 & -1 & 1 & -3 & 0 & 0 & 1 & 0 \\ -2 & 1 & 2 & -2 & 0 & 0 & 0 & 1 \end{array}\right) \underset{\substack{R_1 : R_1 + 3R_3 \\ R_2 : R_2 + R_3 \\ R_3 : R_4 - R_3 \\ R_4 : R_4 + 2R_1}}{=} \left(\begin{array}{cccc|cccc} 1 & -1 & 5 & -5 & 1 & 0 & 3 & 0 \\ 0 & 3 & 2 & 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 12 & -12 & 2 & 0 & 6 & 1 \end{array}\right) \underset{\substack{R_1 : R_1 - R_4 \\ R_2 : R_2 - R_3 + R_4 \\ R_3 : R_3 + 2R_4 \\ R_4 : -R_4}}{=}$$

$$= \left(\begin{array}{cccc|cccc} 1 & 0 & -7 & 7 & -1 & 0 & -3 & -1 \\ 0 & 0 & 13 & -12 & 2 & 1 & 8 & 0 \\ 0 & 0 & 25 & -23 & 4 & 0 & 11 & 3 \\ 0 & 1 & -12 & 12 & -2 & 0 & -6 & -1 \end{array}\right) \underset{\substack{R_2 \leftrightarrow R_4 \\ R_4 : 2R_4 - R_3 \\ R_3 : \frac{R_3 - 25R_4}{2} \\ R_1 : R_1 + 7R_4}}{=} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 14 & 32 & -22 \\ 0 & 1 & -12 & 12 & -2 & 0 & -6 & -1 \\ 0 & 0 & 0 & 1 & 2 & -25 & -57 & 39 \\ 0 & 0 & 1 & -1 & 0 & 2 & 5 & -3 \end{array}\right) =$$

$$\underset{\substack{R_2 : R_2 + 12R_4 \\ R_4 : R_4 + R_3 \\ R_3 \leftrightarrow R_4}}{=} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 14 & 32 & -22 \\ 0 & 1 & 0 & 0 & -2 & 24 & 54 & -37 \\ 0 & 0 & 1 & 0 & 2 & -23 & -52 & 36 \\ 0 & 0 & 0 & 1 & 2 & -25 & -57 & 39 \end{array}\right) \Longrightarrow$$

$$\Longrightarrow \begin{pmatrix} 7 & 2 & 2 & 4 \\ 2 & 4 & 1 & 4 \\ -2 & -1 & 1 & -3 \\ -2 & 1 & 2 & -2 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 14 & 32 & -22 \\ -2 & 24 & 54 & -37 \\ 2 & -23 & -52 & 36 \\ 2 & -25 & -57 & 39 \end{pmatrix}$$

№ 5.1

$$\begin{pmatrix} 7 & 4 & 0 & 0 \\ 12 & 7 & 0 & 0 \\ 16 & 2 & 17 & 24 \\ 9 & -1 & 12 & 17 \end{pmatrix}^{-1} =?$$

$$\left(\begin{array}{cccc|cccc} 7 & 4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 12 & 7 & 0 & 0 & 0 & 1 & 0 & 0 \\ 16 & 2 & 17 & 24 & 0 & 0 & 1 & 0 \\ 9 & -1 & 12 & 17 & 0 & 0 & 0 & 1 \end{array}\right) \underset{\substack{R_4 : R_4 - R_1 \\ R_3 : R_3 - 2R_1 \\ R_2 : 2R_1 - R_2 \\ R_1 : R_1 - 3R_2}}{=} \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & -5 & 3 & 0 & 0 \\ 2 & 1 & 0 & 0 & 2 & -1 & 0 & 0 \\ 2 & -6 & 17 & 24 & -2 & 0 & 1 & 0 \\ 2 & -5 & 12 & 17 & -1 & 0 & 0 & 1 \end{array}\right) \underset{\substack{R_4 : R_4 - R_3 \\ R_3 : R_3 - R_2 \\ R_2 : 2R_1 - R_2}}{=}$$

$$= \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & -5 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 & -12 & 7 & 0 & 0 \\ 0 & -7 & 17 & 24 & -4 & 1 & 1 & 0 \\ 0 & 1 & -5 & -7 & 1 & 0 & -1 & 1 \end{array}\right) \underset{\substack{R_1 : R_1 - R_2 \\ R_3 : R_3 + 3R_4 + 4R_2 \\ R_4 : R_4 - R_2}}{=} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 7 & -4 & 0 & 0 \\ 0 & 1 & 0 & 0 & -12 & 7 & 0 & 0 \\ 0 & 0 & 2 & 3 & -49 & 29 & -2 & 3 \\ 0 & 0 & -5 & -7 & 13 & -7 & -1 & 1 \end{array}\right) =$$

$$\underset{\substack{R_3 : 3R_3 + R_4 \\ R_4 : \frac{R_4 + 5R_3}{3}}}{=} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 7 & -4 & 0 & 0 \\ 0 & 1 & 0 & 0 & -12 & 7 & 0 & 0 \\ 0 & 0 & 1 & 2 & -134 & 80 & -7 & 10 \\ 0 & 0 & 0 & 1 & -219 & 131 & -12 & 17 \end{array}\right) \underset{R_3 : R_3 - 2R_4}{=} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 7 & -4 & 0 & 0 \\ 0 & 1 & 0 & 0 & -12 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 & 304 & -182 & 17 & -24 \\ 0 & 0 & 0 & 1 & -219 & 131 & -12 & 17 \end{array}\right)$$

$$\begin{pmatrix} 7 & 4 & 0 & 0 \\ 12 & 7 & 0 & 0 \\ 16 & 2 & 17 & 24 \\ 9 & -1 & 12 & 17 \end{pmatrix}^{-1} = \begin{pmatrix} 7 & -4 & 0 & 0 \\ -12 & 7 & 0 & 0 \\ 304 & -182 & 17 & -24 \\ -219 & 131 & -12 & 17 \end{pmatrix}$$

№ 6

$$\begin{cases} 12x_1 + 14x_2 - 15x_3 + 23x_4 + 27x_5 = 5 \\ 16x_1 + 18x_2 - 22x_3 + 29x_4 + 37x_5 = 8 \\ 18x_1 + 20x_2 - 21x_3 + 32x_4 + 41x_5 = 9 \\ 10x_1 + 12x_2 - 16x_3 + 20x_4 + 23x_5 = 4 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 12 & 14 & -15 & 23 & 27 & 5 \\ 16 & 18 & -22 & 29 & 37 & 8 \\ 18 & 20 & -21 & 32 & 41 & 9 \\ 10 & 12 & -16 & 20 & 23 & 4 \end{array}\right) \underset{\substack{R_4 : R_1 - R_4 \\ R_3 : R_3 - R_2 \\ R_2 : R_2 - R_1 \\ R_1 : R_1 - 2R_2 - 2R_3}}{\longrightarrow} \left(\begin{array}{ccccc|c} 0 & 2 & -3 & 5 & -1 & -3 \\ 4 & 4 & -7 & 6 & 10 & 3 \\ 2 & 2 & 1 & 3 & 4 & 1 \\ 2 & 2 & 1 & 3 & 4 & 1 \end{array}\right) \underset{\substack{R_4 : R_4 - R_3 \\ R_2 : R_2 - 2R_3 \\ R_3 : R_3 - R_1}}{\longrightarrow}$$

$$\longrightarrow \left(\begin{array}{ccccc|c} 0 & 2 & -3 & 5 & -1 & -3 \\ 0 & 0 & -9 & 0 & 2 & 1 \\ 2 & 0 & 4 & -2 & 5 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \leftrightarrow \begin{cases} 2x_1 + 4x_3 - 2x_4 + 5x_5 = 4 \\ 2x_2 - 3x_3 + 5x_4 - x_5 = -3 \\ -9x_3 + 2x_5 = 1 \\ x_4 = c_4 \\ x_5 = c_5 \end{cases} \leftrightarrow \begin{cases} x_1 = -\frac{53}{18}c_5 + c_4 + \frac{20}{9} \\ x_2 = \frac{5}{6}c_5 - \frac{5}{2}c_4 - \frac{5}{3} \\ x_3 = \frac{2}{9}c_5 - \frac{1}{9} \\ x_4 = c_4 \\ x_5 = c_5 \end{cases} \leftrightarrow$$

$$\leftrightarrow X = \begin{pmatrix} \frac{20}{9} \\ -\frac{5}{3} \\ -\frac{1}{9} \\ 0 \\ 0 \end{pmatrix} + c_4 \cdot \begin{pmatrix} 1 \\ -\frac{2}{5} \\ 0 \\ 1 \\ 0 \end{pmatrix} + c_5 \cdot \begin{pmatrix} -\frac{53}{18} \\ \frac{5}{6} \\ \frac{2}{9} \\ 0 \\ 1 \end{pmatrix}$$

№ 6.1

$$\begin{cases} 12x_2 - 16x_3 + 25x_4 = 29 \\ 27x_1 + 24x_2 - 32x_3 + 47x_4 = 55 \\ 50x_1 + 51x_2 - 68x_3 + 95x_4 = 115 \\ 31x_1 + 21x_2 - 28x_3 + 46x_4 = 50 \end{cases}$$

$$\left(\begin{array}{cccc|c} 0 & 12 & -16 & 25 & 29 \\ 27 & 24 & -32 & 47 & 55 \\ 50 & 51 & -68 & 95 & 115 \\ 31 & 21 & -28 & 46 & 50 \end{array}\right) \underset{\substack{R_3 : 2R_2 - R_3 \\ R_4 : R_4 - R_2 - R_3 \\ R_2 : \frac{R_2 - 2R_1}{3}}}{\longrightarrow} \left(\begin{array}{cccc|c} 0 & 12 & -16 & 25 & 29 \\ 9 & 0 & 0 & -1 & -1 \\ 4 & -3 & 4 & -1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \underset{\substack{R_2 : R_2 - 2R_3 \\ R_3 : R_3 - 4R_2 \\ R_1 \leftrightarrow R_2 \\ R_2 \leftrightarrow R_3}}{\longrightarrow}$$

$$\longrightarrow \left(\begin{array}{cccc|c} 1 & 6 & -8 & 1 & 9 \\ 0 & -27 & 36 & -5 & -41 \\ 0 & 12 & -16 & 25 & 29 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \underset{\substack{R_1 : 2R_1 - R_3 \\ R_2 : R_2 + 2R_3 \\ R_3 : R_3 + 4R_2}}{\longrightarrow} \left(\begin{array}{cccc|c} 2 & 0 & 0 & -23 & -11 \\ 0 & -3 & 4 & 45 & 17 \\ 0 & 0 & 0 & 205 & 97 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \leftrightarrow$$

$$\leftrightarrow \begin{cases} 2x_1 - 23x_4 = -11 \\ -3x_2 + 4x_3 + 45x_4 = 17 \\ 205x_4 = 97 \\ x_3 = c_3 \end{cases} \leftrightarrow \begin{cases} x_1 = -\frac{12}{205} \\ x_2 = \frac{176}{123} + \frac{4}{3}c_3 \\ x_3 = c_3 \\ x_4 = \frac{97}{205} \end{cases} \leftrightarrow X = \begin{pmatrix} -\frac{12}{205} \\ \frac{176}{123} \\ 0 \\ \frac{97}{205} \end{pmatrix} + c_3 \cdot \begin{pmatrix} 0 \\ \frac{4}{3} \\ 1 \\ 0 \end{pmatrix}$$

№ 7

$$\Delta_n = \begin{vmatrix} x+1 & x & x & \dots & x & x \\ x & x+2 & x & \dots & x & x \\ x & x & x+3 & \dots & x & x \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x & x & x & \dots & x+n-1 & x \\ x & x & x & \dots & x & x+n \end{vmatrix} \underset{\substack{C_n : C_n - C_{n-1} \\ C_{n-2} : C_{n-2} - C_{n-3} \\ \dots \\ C_2 : C_2 - C_1}}{=} \begin{vmatrix} x+1 & -1 & 0 & \dots & 0 & 0 \\ x & 2 & -2 & \dots & 0 & 0 \\ x & 0 & 3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x & 0 & 0 & \dots & n-1 & 1-n \\ x & 0 & 0 & \dots & 0 & n \end{vmatrix} =$$

$$= n\cdot \underbrace{\begin{vmatrix} x+1 & -1 & 0 & \dots & 0 & 0 \\ x & 2 & -2 & \dots & 0 & 0 \\ x & 0 & 3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x & 0 & 0 & \dots & n-2 & 2-n \\ x & 0 & 0 & \dots & 0 & n-1 \end{vmatrix}}_{\Delta_{n-1}} + (-1)^{n-1}x\cdot \underbrace{\begin{vmatrix} -1 & 0 & 0 & \dots & 0 \\ 2 & -2 & 0 & \dots & 0 \\ 0 & 3 & -3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1-n \end{vmatrix}}_{(-1)^{n-1}(n-1)!} = n\Delta_{n-1} + x(n-1)!$$

$\Delta_n = n\Delta_{n-1} + x(n-1)! = n(n-1)\Delta_{n-2} + x((n-1)! + n(n-2)!) = n(n-1)(n-2)\Delta_{n-3} +$
$+ x((n-1)! + n(n-2)! + n(n-1)(n-3)!) = \dots = n! \underbrace{\Delta_0}_{1} + x((n-1)! + n(n-2)! + \dots +$

$+ 1! \cdot n \cdot (n-1) \cdot \ldots \cdot 3 + 0! \cdot n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2)$

$\Delta_n = n! + x \cdot n! \cdot [\frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \dots + \frac{1}{1}]$

Ответ: $\Delta_n = n! + x \cdot n! \cdot [\frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \dots + \frac{1}{1}]$

№ 7.1

$$\Delta_n = \begin{vmatrix} 7 & 5 & 0 & \dots & 0 & 0 \\ 2 & 7 & 5 & \dots & 0 & 0 \\ 0 & 2 & 7 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 7 & 5 \\ 0 & 0 & 0 & \dots & 2 & 7 \end{vmatrix} = 7 \cdot \underbrace{\begin{vmatrix} 7 & 5 & 0 & \dots & 0 & 0 \\ 2 & 7 & 5 & \dots & 0 & 0 \\ 0 & 2 & 7 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 7 & 5 \\ 0 & 0 & 0 & \dots & 2 & 7 \end{vmatrix}}_{\Delta_{n-1}} - 5 \cdot \underbrace{\begin{vmatrix} 7 & 5 & 0 & 0 & \dots & 0 & 0 & 0 \\ 2 & 7 & 5 & 0 & \dots & 0 & 0 & 0 \\ 0 & 2 & 7 & 5 & \dots & 0 & 0 & 0 \\ 0 & 0 & 2 & 7 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 & 5 & 0 \\ 0 & 0 & 0 & 0 & \dots & 2 & 7 & 5 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 2 \end{vmatrix}}_{2\Delta_{n-2}} =$$

$= 7\Delta_{n-1} - 10\Delta_{n-2}$
$\Delta_0 = 1$
$G(z) = \Delta_0 + \Delta_1 \cdot z + \Delta_2 \cdot z^2 + \dots$ | Согласно теории: $G(z) = \frac{1+z(7-7\cdot 1)}{1-7z+10z^2}$

$G(z) = \frac{1+z(7-7\cdot 1)}{(1-5z)\cdot(1-2z)} = \frac{a}{1-5z} + \frac{b}{1-2z} \leftrightarrow 1 = a + b - z(2a+5b) \leftrightarrow \begin{cases} a+b=1 \\ 2a+5b=0 \end{cases} \leftrightarrow \begin{cases} a = \frac{5}{3} \\ b = -\frac{2}{3} \end{cases}$

$G(z) = \frac{5}{3} \cdot \frac{1}{1-5z} - \frac{2}{3} \cdot \frac{1}{1-2z} = \frac{5}{3} \cdot \sum\limits_{n=0}^{+\infty} [z^n \cdot 5^n] - \frac{2}{3} \cdot \sum\limits_{n=0}^{+\infty} [z^n \cdot 2^n] = \sum\limits_{n=0}^{+\infty} z^n \cdot \frac{5^{n+1} - 2^{n+1}}{3} \implies$
$\implies \Delta_n = \frac{5^{n+1} - 2^{n+1}}{3}$
Ответ: $\Delta_n = \frac{5^{n+1} - 2^{n+1}}{3}$

№ 7.2

$$\Delta_{n+1} = \begin{vmatrix} 1 & 0 & 0 & \dots & 0 & 1 \\ 1 & a_1 & 0 & \dots & 0 & 0 \\ 1 & 1 & a_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \dots & a_{n-1} & 0 \\ 1 & 0 & 0 & \dots & 1 & a_n \end{vmatrix} \underset{C_1 : C_1 - C_{n+1}}{=} \begin{vmatrix} 0 & 0 & 0 & \dots & 0 & 1 \\ 1 & a_1 & 0 & \dots & 0 & 0 \\ 1 & 1 & a_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \dots & a_{n-1} & 0 \\ 1-a_n & 0 & 0 & \dots & 1 & a_n \end{vmatrix} = a_n \cdot \underbrace{\begin{vmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & a_1 & 0 & \dots & 0 \\ 1 & 1 & a_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & a_{n-1} \end{vmatrix}}_{0} +$$

$$+ (-1)^n \cdot \begin{vmatrix} 1 & a_1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 1 & a_2 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & 1 & a_3 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & \dots & 0 & 1 & a_{n-1} \\ 1-a_n & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{vmatrix} \underset{\substack{R_{n-1} : R_{n-1} - a_{n-1}R_n \\ R_{n-2} : R_{n-2} - a_{n-2}R_{n-1} \\ \dots \\ R_1 : R_1 - a_1 R_2}}{=}$$

$$= (-1)^n \cdot \underbrace{\begin{vmatrix} 1 - a_1(1 - a_2(1 - \dots (1 - a_n))) & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 - a_2(1 - a_3(1 - \dots (1 - a_n))) & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 - a_{n-1}(1 - a_n) & 0 & 0 & 0 & \dots & 0 & 1 & 0 \\ 1 - a_n & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{vmatrix}}_{\text{Диагональная матрица}} = (-1)^n \cdot$$

$\cdot\, (1 - a_1(1 - a_2(1 - \dots - a_{n-1}(1 - a_n))))$

$\Delta_{n+1} = (-1)^n \cdot (-1)^n \cdot a_1 \cdot \ldots \cdot a_n + (-1)^n \cdot (-1)^{n-1} \cdot a_2 \cdot \ldots a_n + \ldots + (-1)^{n-1} a_n + (-1)^n =$
$= a_1 \cdot \ldots \cdot a_n - a_2 \cdot \ldots \cdot a_n + a_3 \cdot \ldots \cdot a_n + \ldots + (-1)^{n-1} \cdot a_n + (-1)^n$

Ответ: $\Delta_{n+1} = a_1 \cdot \ldots \cdot a_n - a_2 \cdot \ldots \cdot a_n + a_3 \cdot \ldots \cdot a_n + \ldots + (-1)^{n-1} \cdot a_n + (-1)^n$