Основы дискретной математики

№ 13г

Доказать $A \subseteq (B \cup C) \quad \leftrightarrow \quad (A \cap \overline{B}) \subseteq C$

В одну сторону: $\forall x \in A => x \in (B \cup C)$

1. $x \in A \ \& \ x \in B$
   $x \in A \ \& \ x \notin B \implies A \cap \overline{B} = \emptyset; \quad \emptyset \in C$
2. $x \in A \ \& \ x \notin B \implies x \in A \cap \overline{B} => x \in C$

В другую сторону:

1. $x \in A, x \in B \implies x \in (B \cup C)$
2. $x \in A, x \notin B, x \in C \implies x \in (B \cup C)$

№ 14б

Доказать: $A \triangle (B \triangle C) = (A \triangle B) \triangle C$

$A \triangle [(B \setminus C) \cap (C \setminus B)] = A \triangle [(B \cap \overline{C}) \cup (C \cap \overline{B})] = A \triangle X$
Пусть $X = (B \cap \overline{C}) \cup (C \cap \overline{B})$
$\overline{X} = \overline{(B \cap \overline{C}) \cup (C \cap \overline{B})} = (\overline{B} \cup C) \cap (\overline{C} \cup B)$
$A \triangle X = (A \setminus X) \cup (X \setminus A) = (A \cup \overline{X}) \cup (X \cup \overline{A}) = A \cap (\overline{B} \cup C) \cap (\overline{C} \cup B) \cup$
$\cup [\overline{A} \cap [(B \cap \overline{C}) \cup (C \cap \overline{B})]]$
$X = (\overline{B} \cap \overline{C}) \cup (C \cap \overline{C}) \cup (B \cap \overline{B}) \cup (B \cap C) = (\overline{B} \cap \overline{C}) \cup (B \cap C)$
$A \triangle X = (A \cap \overline{B} \cap \overline{C}) \cup (A \cap C \cap B) \cup (\overline{A} \cap B \cap \overline{C}) \cup (\overline{A} \cap C \cap B) =$
$[(A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap B \cap \overline{C})] \cup (A \cap C \cap B) \cup (\overline{A} \cap C \cap B) = [\overline{C} \cap ((A \cap \overline{B}) \cup (B \cap$
$\overline{A}))] \cup [C \cap ((A \cap \overline{B}) \cup (B \cap \overline{A}))] = [\overline{C} \cap (A \triangle B)] \cup [C \cap (\overline{A \triangle B})] = [A \triangle B] \triangle C$

Задача с листочка № 4

$U = \{\forall$ студентов 3 курса$\}, \quad |U| = 63$
$A = \{$студенты, слушающие прорицание$\}, \quad |A| = 16$
$B = \{...,$ магловедение$\}, \quad |B| = 37$
$|A \cap B| = 5$

$|A \cup B| = |A \setminus B| + |B \setminus A| - |A \cap B|$

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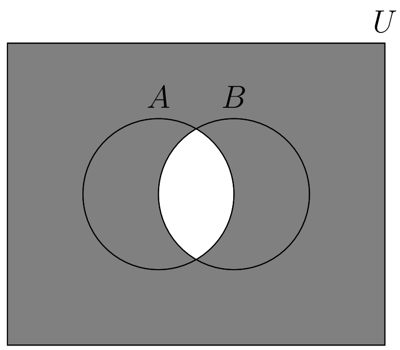
Def: $A * B = \overline{A \cap B} = \overline{A} \cup \overline{B}$



Доказать: $(A * B) * (A * B) = A \cap B$
$A * A = \overline{A} \cup \overline{A} = \overline{A} \implies = \overline{(A * B)} = \overline{\overline{A} \cup \overline{B}} = A \cap B$
Доказать: $(A * A) * (B * B) = A \cup B$
Аналогично: $(A * A) * (B * B) = \overline{A} * \overline{B} = \overline{\overline{A}} \cup \overline{\overline{B}} = A \cup B$

1 Решение уравнений и систем уравнений

№ 1

Доказать: $A \subseteq B \leftrightarrow A \cup B = B \leftrightarrow A \setminus B = \emptyset \leftrightarrow A \cap \overline{B} = \emptyset$

- Докажем, что $\overline{A} \cup B = U \implies A \subseteq B$:
  $x \in (\overline{A} \cup B) \leftrightarrow x \in \overline{A}$ или $x \in B$

$A \cup \overline{A} = U$
Пусть $x \in A$. Если x $\notin B \implies x \notin \overline{A} \implies x \notin \overline{A} \cup B \leftrightarrow$
$x \notin U$ - противоречие $\implies x \in B$

№ 2

Доказать: $A = B \leftrightarrow A \triangle B = \emptyset$

1. $A \triangle B = (A \setminus B) \cup (B \setminus A) = (A \cap \overline{B}) \cup (B \cap \overline{A})$

$$\begin{cases} \forall x \in A \implies x \in B \\ \forall x \in B \implies x \in A \end{cases} \leftrightarrow \begin{cases} A \subseteq B \implies A \cap \overline{B} = \emptyset \\ B \subseteq A \implies B \cap \overline{A} = \emptyset \end{cases} \implies (A \cap \overline{B}) \cup (B \cap \overline{A})$$

2. $A \triangle B = \emptyset = (A \setminus B) \cup (B \setminus B) \implies A \cap \overline{B} = \emptyset \implies$
$\implies A \subseteq B$. Аналогично: $B \subseteq A$

№ 3

Решить уравнение: $A \setminus X = B$
Пусть $C = A \setminus X \implies C = B \leftrightarrow C \triangle B = \emptyset$
$(A \setminus X) \triangle B = \emptyset = [(A \setminus X) \setminus B] \cup [B \setminus (A \setminus X)] = [(A \cap \overline{X}) \cap \overline{B}] \cup [B \cap \overline{(A \cap \overline{X})}] = [A \cap \overline{B} \cap \overline{X}] \cup [B \cap \overline{A}] \cup [B \cap X] = \emptyset \leftrightarrow$

$$\leftrightarrow \begin{cases} B \cap \overline{A} = \emptyset \\ B \cap X = \emptyset \\ [A \cap \overline{B}] \cap \overline{X} = \emptyset \end{cases} \leftrightarrow \begin{cases} B \subseteq A \\ X \subseteq \overline{B} \\ A \cap \overline{B} \subseteq X \end{cases}$$

$A \cap \overline{B} \subseteq X \subseteq \overline{B} \leftrightarrow X = (A \cap \overline{B}) \cup Z, \quad Z \subseteq \overline{A} \cap \overline{B}$

№ 4

Решить систему:

$$\begin{cases} A \cap X = \emptyset \\ B \cap \overline{X} = \emptyset \end{cases} \leftrightarrow \begin{cases} X \subseteq \overline{A} \\ B \subseteq X \end{cases} \leftrightarrow B \subseteq X \subseteq \overline{A} \implies$$

$$\implies X = B \cup Y, \quad Y \subseteq \overline{A} \cap \overline{B}$$

№ 27 Л&В

Решить систему:

$$\begin{cases} A \cap X = B \\ A \cup X = C \end{cases} \leftrightarrow \begin{cases} (A \cap X) \triangle B = \emptyset \\ (A \cup X) \triangle C = \emptyset \end{cases} \leftrightarrow \begin{cases} [A \cap X \cap \overline{B}] \cup [B \cap \overline{A}] \cup [B \cap \overline{X}] = \emptyset \\ [A \cap \overline{C}] \cup [X \cap \overline{C}] \cup [C \cap \overline{AX}] = \emptyset \end{cases} \leftrightarrow$$

$$\leftrightarrow \begin{cases} B \cap \overline{A} = \emptyset \\ A \cap \overline{C} = \emptyset \\ (A \cap \overline{B} \cap X) \cup (\overline{C} \cap X) = [(A \cap \overline{B}) \cup \overline{C}]^{=\mathrm{S}} \cap X = \emptyset \\ [B \cup (C \cap \overline{A})]^{=\mathrm{T}} \cap \overline{X} = \emptyset \end{cases} \leftrightarrow$$

$$\leftrightarrow \begin{cases} B \subseteq A \subseteq C \\ T \subseteq X \subseteq \overline{S} \\ \overline{S} = \overline{(A \cap \overline{B}) \cup \overline{C}} = \overline{A \cap \overline{B}} \cap C = (\overline{A} \cup B) \cap C = \\ = (C \cap \overline{A}) \cup (B \cap C) = (C \cap \overline{A}) \cup B = T \end{cases} \leftrightarrow \begin{cases} X = B \cup (C \cap \overline{A}) \\ B \subseteq A \subseteq C \end{cases}$$

№ 28 Л&В

$$\begin{cases} A \setminus X = B \\ X \setminus A = C \end{cases} \leftrightarrow \begin{cases} (A \setminus X) \triangle B = \emptyset \\ (X \setminus A) \triangle C = \emptyset \end{cases} \leftrightarrow \begin{cases} (A \cap \overline{X} \cap \overline{B}) \cup (B \cap \overline{A}) \cup (B \cap X) = \emptyset \\ (X \cap \overline{A} \cap \overline{C}) \cup (C \cap A) \cup (C \cap \overline{X}) = \emptyset \end{cases} \leftrightarrow$$

$$\leftrightarrow \begin{cases} B \subseteq A \subseteq \overline{C} \\ X \cap [B \cup (\overline{A} \cap \overline{C})] = \emptyset \\ \overline{X} \cap [C \cup (A \cap \overline{B})] = \emptyset \end{cases} \leftrightarrow \begin{cases} B \subseteq A \subseteq \overline{C} \\ [C \cup (A \cap \overline{B})] \subseteq X \subseteq \overline{[B \cup (\overline{A} \cap \overline{C})]} \end{cases}$$

$$\overline{[B \cup (\overline{A} \cap \overline{C})]} = \overline{B} \cap \overline{\overline{A} \cap \overline{C}} = \overline{B} \cap (A \cup C) = C \cup (A \cap \overline{B})$$

№ 30 Л&В

$$\begin{cases} A \setminus X = B \\ A \cup X = C \end{cases} \leftrightarrow \begin{cases} (A \setminus X) \triangle B = \emptyset \\ (A \cup X) \triangle C = \emptyset \end{cases} \leftrightarrow \begin{cases} (A \cap \overline{X} \cap \overline{B}) \cup (B \cap \overline{A}) \cup (B \cap X) = \emptyset \\ (A \cap \overline{C}) \cup (X \cap \overline{C}) \cup (C \cap \overline{A} \cap \overline{X}) = \emptyset \end{cases} \leftrightarrow$$

$$\leftrightarrow \begin{cases} B \subseteq A \subseteq C \\ X \cap (B \cup \overline{C}) = \emptyset \\ \overline{X} \cap ((C \cap \overline{A}) \cup (A \cap \overline{B})) = \emptyset \end{cases} \leftrightarrow \begin{cases} B \subseteq A \subseteq C \\ ((C \cap \overline{A}) \cup (A \cap \overline{B})) \subseteq X \subseteq \overline{B} \cap C \end{cases} \leftrightarrow$$

$$\leftrightarrow \begin{cases} X = \overline{B} \cap C \\ B \subseteq A \subseteq C \end{cases}$$

2 Декартово произведение множеств

$A = \{a_i | ...i \in (1, n)\}$
$B = \{b_j | j \in (1, m)\}$
$A \times B = \{(a_i; b_j) | i \in (1, n) \& j \in (1, m)\}$
$\{x, y\} \times \{1, 2, 3\} = \{(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)\}$
$|A \times B| = |A| \cdot |B|$
$A \times A = A^2$

2.1 Свойства декартого произведения

1. $A \times B = B \times A \leftrightarrow A = B$
2. $A \times B = A \times C \leftrightarrow B = C$