§1 Lecture 11-27

Example 1.1

$$\mathbb{Z}_3[x]/\langle x^3 + 2x + 1 \rangle$$

$x^3 + 2x + 1$ is irreducible because deg ≤ 3 and no zeros. Hence we obtain a field.

Elements s: $\{ax^2 + bx + c + I : a, b, c \in \mathbb{Z}_3\}$ where $I = \langle x^3 + 2x + 1 \rangle$. There are 3^3 elements because $\mathbb{Z}_3$ has 3 elements and there are three coefficients in each polynomial.

Then:

$$0 + I = (x^3 + 2x + 1 + I) = (x^3 + I) + (2x + 1 + I)$$

So $x^3 + I = (x + 2 + I)$.

Example 1.2 (Example of addition)

$$\begin{gathered}(x^2 + 2x + I) + (2x + 1 + I) \\ = x^2 + 4x + 1 + I \\ = x^2 + x + 1 + I\end{gathered}$$

Example 1.3 (Example of multiplication)

$$\begin{gathered}(2x + 1 + I)(x^2 + 2x + I) \\ = 2x^3 + x^2 + x^2 + 2x + I \\ = 2x^3 + 2x^2 + 2x + I \\ = 2(x + 2) + 2x^2 + 2x + I \\ = 2x^2 + x + 1\end{gathered}$$

Fact 1.4. The group of units of a finite field is cyclic!

Exercise 1.5. Find the inverse of $(x^2 + 1 + I)$.

It's inverse is $(x^2 + 1 + I)^2 5$.

Another way to do it would be to take $(ax^2 + bx + c + I)(x^2 + 1 + I) = 1 + I$ and solve the system of linear equations over $\mathbb{Z}_3$ just like you would in linear algebra.

§1.1 Classification of symmetries over E^2 plane

1. e identity
2. θ_p θ notation. Counterclockwise rotation about point $p \in E^2$.

3. translation $\vec{u} \to \vec{u} + v$
4. reflection over some line l
5. glide reflection a reflection and translation over the same line

Definition 1.6. A Freeze Group G is an infinite subgroup G of isometries(E^2) that is actually a subgroup of Isom(Strip) which is discrete in the sense that finitely many elements $g \in G$ have distance $(p, g(p)) < 1$.

Classification: 7 types of freeze groups.