§1 10-11

Lemma 1.1

Conjugacy is an equivalence relation.
$x \sim y$ if $x = gyg^{-1}$ for some $g \in G$

Theorem 1.2

Any two k-cycles in S_n are conugate. Moreover, any conjugate of a k-cycle is a k-cycle.

Proof. $\alpha = (a_1 a_2 \dots a_k) \quad \beta = (b_1 b_2 \dots b_k)$
Let σ be a bijection with $\sigma(b_i) = a_i$ for $1 \leq i \leq k$
Then $\sigma\beta\sigma^{-1} = \alpha$. Claiming that β and α are conjugate to one another.
If $x \notin \{a_1 \ \dots \ a_k\}$, then $\sigma\alpha\sigma^{-1}(x) = x$

Example 1.3

$\sigma\beta\sigma^{-1}(a_i) = \sigma\beta b_i = \sigma b_{i+1} = a_{i+1} = \alpha(a)$

□

§1.1 A_4 is the group of rigid motions of a tetrahedron

Note 1.4. A_4 is the subgroup of even elements in S_4.

$$\text{identity: } [e] = \{()\}$$
$$[(12)(34)] = \{(12)(34), (13)(24), (14)(23)\}$$
$$\text{Clockwise rotations about a face: } [(123)] = \{(123), (134), (142), (243)\}$$
$$\text{Counter clockwise rotations about a face: } [(132)] = \{(132), (143), (124), (234)\}$$

Note 1.5. A_4 has no 6 element subgroup even though 6 divides $|A_4|$

§2 Isomorphisms

Definition 2.1. $(G, \cdot)$ and $(H, \circ)$ are isomorphic if there exists a bijection $\phi : G \to H$ such that $\phi(a \cdot b) = \phi(a) \circ \phi(b)$ for all $a, b \in G$.
This would make ϕ an isomorphism.
$G"equalsignwithsimontop"H$

Note 2.2. Let $H \subset G$ be a subgroup. It is possible for $x \not\sim y$ in H while $x \sim y$ in G. This is a distinguishing between $\sim_H$ and $\sim_G$.

Example 2.3

$H \subset S_{17}$
$H = \langle(1\ 2 \dots 17)\rangle$
H is cyclic and abelian.
It has 17 conjugacy classes.
All nontrivial elements of H are conjugate in S_{17}.

Note 2.4. In an abelian group, two elements are conjugate iff they are equal.
If H is abelian, then $(x \sim y) \Leftrightarrow x = aya^{-1} = y$