§1 01-06

http://www.math.mcgill.ca/ daimon /courses /algebra /algebra.html

Assignments: due on wednesday at burnside floor 10. Handed back on Monday

office hourse mw 10:35 - 11:45

Midterm feb 19

Friday: no lecture this week

Textbook: "Linear algebra and geometry" Kostakin makte

Definition 1.1 (Linear Algebra)**.** The study of vector spaces over a field and of the maps between them.

Homomorphism aka linear transformation. Studying linear transformations between vector spaces.

Groups are an abstraction of the notion of symmetry.

Rings are an abstraction of the notion of numbers.

Vector spaces arose as a model of physical space.

Example 1.2

Prototypical

1. $\mathbb{R}$
2. $\mathbb{R}^2$
3. $\mathbb{R}^3$

§1.1 Abstractions

1. $\mathbb{R}^n$. n-dimensional euclideon space
2. Replace $\mathbb{R}$ by a general field $F \to F^n$
   Allow you to study some interesting and practical ideas.

Definition 1.3. Fix a field F (e.g. $\mathbb{R}, \mathbb{C}, \mathbb{Q}, \mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}, \mathbb{F}_{p^n} = \mathbb{F}_p[x]/(q(x)) \deg(q) = n, q$ irreducible

A vector space over $\mathbb{F}$ is a set V equipped with the following structures:

1. A binary operation.
$$+ : V \times V \to V$$
$$(v_1, v_2) \to v_1 + v_2$$
2. A scalar multiplication
$$\cdot : F \times V \to V$$
$$(\lambda, v) \to \lambda \cdot v$$

Subject to the following axioms.

1. $(V, +)$ is an abelian group, i.e. $\exists$ a mutual (identity) element for $+$.
   a) Identity:
$$0_V \text{ such that } 0_V + w = w + 0_V = w \forall w \in V$$

b) Commutative:

$$v_1 + v_2 = v_2 + v_1 \forall v_1, v_2 \in V$$

c) Inverses:

$$\forall v \in V, \exists v' \text{ such that } v + v' = v' + v = 0_V$$

Note 1.4. $v' = -v$

d) Associativity

$$(v_1 + v_2) + v_3 = v_1 + (v_2 + v_3)$$

2. Multiplication rules

Identity

$$1 \cdot v = v$$

Associativity

$$\lambda_1, \lambda_2 \in F, v \in V$$
$$\lambda_1(\lambda_2 v) = (\lambda_1 \lambda_2)v$$

3) Distributive Laws

a)

$$\lambda(v_1 + v_2) = \lambda v_1 + \lambda v_2$$

b)

$$(\lambda_1 + \lambda_2)v = \lambda_1 v + \lambda_2 v$$

§1.2 Consequences of these axioms

1.

$$0 \cdot w = 0_V$$
$$(0 + 0) \cdot w = 0 \cdot w + 0 \cdot w = 0 \cdot w \Rightarrow 0_V = 0 \cdot w$$

2.

$$(-1) \cdot w = -w$$
$$(1 + (-1) \cdot w = 1 \cdot w + (-1) \cdot w$$
$$\Rightarrow 0_V = 0 \cdot w = w + (-1)w$$

Example 1.5 1. Euclidean space $\mathbb{R}^n$ is a vector space over $\mathbb{R}$.

2. Solutions of linear equations
Let $x_1, \dots, x_n, a_1, dots, a_n \in F$

$$a_1 x_1 + \cdots + a_n x_n = 0$$

If $(x_1, \dots x_n)$ and $(x_1', \dots, x_n')$ are solutions to $(*)$, then so is $\lambda(x_1, \dots, x_n)$ and $(x_1, \dots, x_n) + (x_1' + \dots, x_n')$
More generally you can set up a series of these equations. Let S be the set of solutions of this set of equations. It is a vector subspace of F^n. Homogeneous vs non homogeneous
Let $\sim S$ be solutions to (**) where replace 0 with constants.
$\sim S$ is either empty, or it is a coset for S in F^n. If $x_1^0, \dots, x_n^0) \in \sim S$, then $\sim S = (x_1^0, \dots, x_n^0) + S$

§1.3 Linear Differential Equations

$a_0(x), a_1(x), \dots, a_n(x)$ functions from $\mathbb{R} \to \mathbb{R}$

$$f : \mathbb{R} \to \mathbb{R} \text{ such that}$$
$$a_0(x)f(x) + a_1(x)f'(x) + \cdots + a_n(x)f^{(n)}(x) = 0$$
$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f + \frac{d}{dx}g$$

Something about no closure under multiplication but yes under addition