§1 Lecture 10-07

Example 1.1

Definition 1.2. Let $S \subseteq \mathbb{R}$. The interior $\mathring{S}$ "S with dot on top" or int(S) is defined as:

$$\mathring{S} = \bigcup_{U \subset S, \quad U open} U$$

Note that $\mathring{S}$ is open as a union of open sets. It is the largest open subset of S.

Definition 1.3. Let $S \subset \mathbb{R}$. The closure $\tilde{S}$ of S is defined as:

$$\tilde{S} = \bigcap_{V \supset S, \quad V closed} V$$

Note that $\tilde{S}$ is closed as an intersection of closed sets.

Theorem 1.4

Let $S \subset \mathbb{R}$. Then $\mathring{S} = S \setminus \delta S$

Proof. 1. " $\subseteq$ ". Let $x \in \mathring{S} \Rightarrow \exists U$ open , $U \subset S$ with $x \in U$.
Thus $\exists \epsilon > 0 s.t. V_\epsilon(x) \subset U \subset S$.

□

Theorem 1.5

Let $S \subset \mathbb{R}$. Then $\mathbb{R} \setminus \tilde{S} = int(\mathbb{R} \setminus S)$.