§1 05-12

§1.1 Basic Propositional Logic

Lot's of confusion about what is meant by a true statement vs valid statement vs theorem.

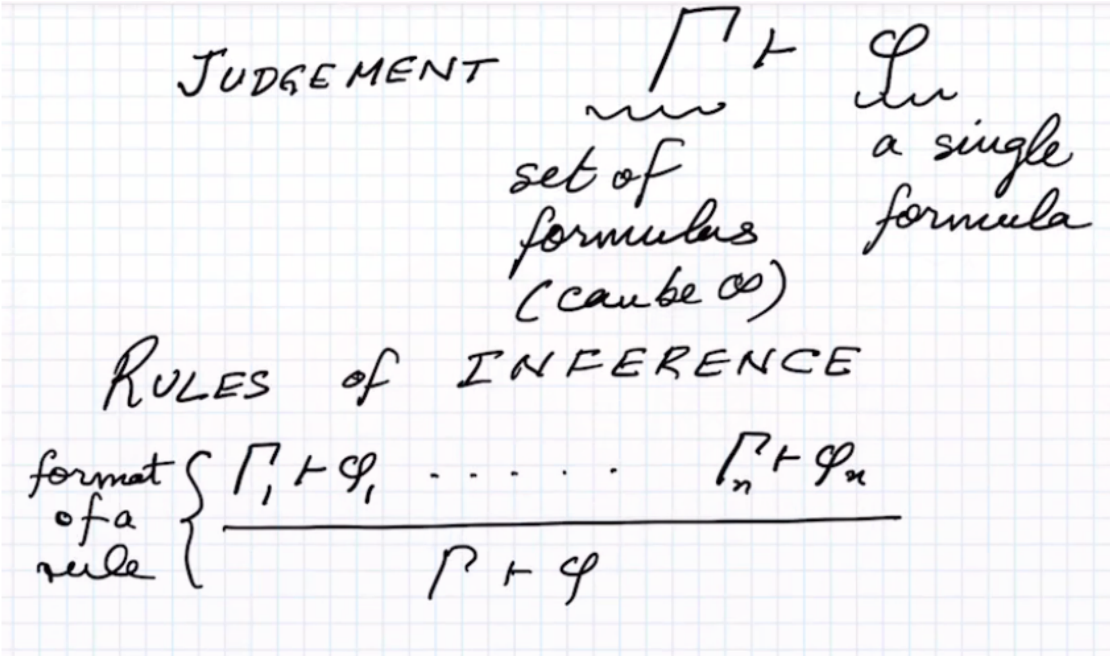
§1.1.1 Syntax

PROP $= \{p, q, r, \cdots\}$ are propositional variables.
φ, ψ are meta-variables for describing generic propositional formulas.
Formulas are defined inductively. Inductive because you have some formulas and you give rules for building new formulas from old ones.
This is syntax. What I'm allowed to write, but explaining anything about what they mean. Explaining what they mean is called semantics.

1. T for true is a formula, and $\perp$ for false is a forumla.
2. Any propositional variable is a formula
3. If φ is a formula, so is $\neg\phi$.
4. If φ and ψ are formulas, so are $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \Rightarrow \psi$

§1.1.2 Proof Theory

How to use and manipulate formulas in deduction. Proof theory is not about being true. It's about true.
Gamma is the set of assumptions you are making.
$\gamma \cup \{\psi\}$ but we write γ, ψ



AXIOM $\dfrac{}{\Gamma \vdash \varphi} \quad \varphi \in \Gamma$

RULES

Weakening $\dfrac{\Gamma \vdash \varphi}{\Gamma, \psi \vdash \varphi}$

$\Gamma \cup \{\psi\}$ but we write Γ, ψ

You can do introduction or elimination.
$\neg\varphi$ is a marco for $\varphi \Rightarrow bot$.
A proof is a tree whose leaves are axioms, the nodes are rules instances, and the rute is a single judgement $\gamma \vdash \varphi$.
If the root has the form $\vdash$ we say that φ is a theorem.

§1.1.3 Semantics

Interpret logical formulas as elements of a simple mathematical structure.

A valuation v : PROP $\to \{0, 1\}$. Extend v to a map on formulas by structural induction.

$$v(T) = tt \quad v(\bot) = ff$$
$$v(\varphi) \underbrace{\wedge}_{\text{Boolean algebra}} v(\varphi') = v(\varphi \underbrace{\wedge}_{\text{Syntax}} \varphi')$$

If $\models \varphi$, then $\forall v,\ v(\varphi) = tt$. Such a formula is called a tautology.

$\vdash$ denotes a syntactic implication while $\models$ denotes a semantic implication.

Theorem 1.1 (Soundness)
If $\gamma \vdash \varphi$ then $\gamma \models$ in particular if $\vdash \varphi$ then $\models \varphi$.

Theorem 1.2 (Completeness)
If $\gamma \models \varphi$ then $\gamma \vdash \varphi$.

Suppose that γ has no valuation such that $\forall \varphi \in \gamma,\ v(\varphi) = tt$. Then we say that γ is unsatisfiable.

Theorem 1.3 (Compactness)
If γ is unsatisfiable, then some finite subset of γ is unsatisfiable.

Remark 1.4. Tepmoral logic fails this property.

§1.2 Temporal Logic

as opposed to propositional logic. The basic atomic properties can change their truth values in time.
A mir Puuli showed this is very useful for reasoning about code execution especially concurrent programs.

§1.2.1 Syntax

PROP $= \{p, q, r, \dots\}$. Formulas $\neg, \wedge, \dots$. Two temporal operators O, u. If φ is a formula then 0φ is a formula. If φ, ψ are formulas, then $\varphi u \psi$ is a formula.
$\varphi = true|p|\varphi_1 \wedge \varphi_2|\neg\varphi|O\varphi|\varphi u \psi$
Intuition. Instead of a valuation we have a linear sequence of valuations. We will call these valuations states. The entire sequence is called a history.
Defined operators.

§1.3 Labelled Transition System

$(S, Act, \rightarrow, I, AP, L)$ S states perhaps infinite. Act actions. $\rightarrow \subseteq S \times A \times S$. represents function a $s \rightarrow s'$ $I \subseteq S$. initial states. AP are the set of atomic propositions.