Theorem 0.1

Every subgroup of a cyclic group G is cyclic

§0.1 Proof

Let $G =< x >$. Let $H \subset G$ be a subgroup.

Show that $H =< y >$ for some y.

If $H = \{e\}$ then $H =< e >$ ✓.

Wha t is H contains an element $g \neq e$

Let $S = \{n > 0 : x^n \in H\}$

$S \neq \varnothing$ because it at least must include (review this proof that $S \neq \varnothing$.

Next step:

By W.O.P, let m be the least element of S.

Claim: $H =< y >$ where $y = x^m$

Proof of claim: Let $h \in H$ Show that $h \in< y >$ which means $h = y^q$ for some q.

Since $h \in G =< x >$, $h = x^a$ for some $a \in \mathbb{Z}$

By the division algorithm: $a = mq + r$ where $0 \leq r < m$

If $r = 0$ then $h = x^a = x^{mq} = x^{m^q} = y^q$ ✓

If $r > 0$ the $hy^{-q} = x^{mq+r}x^{m^{-q}} = x^{mq+r}x^{-mq} = x^r$, but this would contradict that m is least element of S because... i'm not sure why review this

Cor 4.11. Subgroups of $\mathbb{Z}$ are $< n >= n\mathbb{Z}$ (notation)

Prop 4.12. Let G by cyclic of order n $\Rightarrow x^n = e$. Suppose $G =< x >$. This means that $(x^k = e) \Leftrightarrow n|k$.

Proof "$\Leftarrow$" If $k = n * l$, then x^k