Assignment 5 - The proton depth dose curve

due on October 27, 2025

Bethe-Bloch equation

The Bethe-Bloch equation describes the energy loss dE/dz of a charged particle due to collisions with electrons while passing through matter. The Bethe-Bloch equation for a relativistic proton with charge e moving along the z-axis is given by

$$-\frac{dE}{dz} = \frac{4\pi e^4}{m_e c^2} \frac{N_e}{\beta^2} \left(\frac{1}{4\pi\epsilon_0} \right)^2 \left[\ln \left(\frac{2 m_e c^2 \beta^2}{I(1-\beta^2)} \right) - \beta^2 \right] \tag{1}$$

where $\beta = v/c$. v is the velocity of the proton, which depends on the residual energy $E(z)$.

In this assignment, we assume water as the medium. The mean ionization potential of water is given by $I = 75eV$. The electron density N_e is given by

$$N_e = \rho \frac{N_A}{M_u} \left(\frac{Z}{A} \right)_{eff} \tag{2}$$

where ρ is the mass density and $\left(\frac{Z}{A}\right)_{eff}$ is the ratio of the number electrons and nucleons in water. All other fundamental constants have their usual meaning.

In SI units, these are given by:

$m_p = 1.672631\,10^{-27}\,[kg]$ (proton rest mass)
$m_e = 9.1093897\,10^{-31}\,[kg]$ (electron rest mass)
$\epsilon_0 = 8.854187817\,10^{-12}\,[\frac{C^2}{Jm}]$ (vaccum permittivity)
$e = 1.60217733\,10^{-19}\,[C]$ (charge of the electron)
$c = 2.99792458\,10^{8}\,[\frac{m}{s}]$ (speed of light)
$N_A = 6.0221367\,10^{23}$ [particles per mole] (Avogadro's number)
$M_u = 1.0\,10^{-3}$ [kg per mole] (molar mass)

Tasks

The theoretical background for solving these exercises is covered in Chapters 2.1 and 2.2 of the book *Proton Therapy Physics* (available on OLAT).

1) Review the approximate classical derivation of the Bethe-Bloch equation. Understand the main steps and how the most important terms and dependencies in the Bethe-Bloch equation arise.

2) The Bethe-Bloch equation represents an ordinary differential equation for the residual proton energy as a function of depth z. Numerical integration of the Bethe-Bloch equation yields the depth dose curve in continuous slowing down approximation (CSDA), i.e. the depth-dose curve in a hypothetical world without the effect of range straggling and nuclear interactions. Write a program to calculate the depth dose curve of proton beam from the Bethe-Bloch equation. Since this is not a class on numerical methods for ordinary differential equations, it is fully sufficient to use the Euler method for discretization, i.e.

$$\frac{E(z+\Delta z) - E(z)}{\Delta z} = S(E(z)) \tag{3}$$

where $S(E(z))$ is the stopping power given by the right hand side of the Bethe-Bloch equation. Of course, you are free to use something more elaborate such as a higher order Runge-Kutta method. Verify that the result is correct, e.g. by comparing the range of your proton beam with the true range.

3) Range straggling leads to smoothing of the Bragg peak in depth-direction, which can be described by convolving the depth-dose curve in CSDA with a Gaussian distribution. Assuming that the range is measured in centimeters, the standard deviation of the Gaussian is given by

$$\sqrt{\sigma_R^2} = 0.012\, R^{0.935} \tag{4}$$

Modify your CSDA depth-dose curve to account for range straggling.

4) Review the derivation of equation 4.

Hints

a) For a relativistic proton, the relation of kinetic energy E and velocity v is given by

$$E = \frac{m_p c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_p c^2 \tag{5}$$

This replaces the classical relation $E = \frac{1}{2} m_p v^2$.