題目4

Q: Find the solution of the following equation with respect to θ:

$$A\cos\theta + B\sin\theta + C = 0$$

A:

Let $x_1 = \cos\theta$ and $x_2 = \sin\theta$, then the solution is given by the intersection of the circle and the line:

$$x_1^2 + x_2^2 = 1$$
$$Ax_1 + Bx_2 + C = 0$$

We reformulate the equations in a parametric form:

$$|x|^2 = 1$$
$$x(t) = \mathbf{a} + t\mathbf{b}$$

where $x = (x_1, x_2)$, $\mathbf{a} = (0, -\frac{C}{B})$, $\mathbf{b} = (-\frac{C}{A}, \frac{C}{B})$, and t is a parameter. The intersection points satisfy the following equation:

$$|\mathbf{a} + t\mathbf{b}|^2 = 1$$

which can be solved for t to find the intersection points:

$$t_{1,2} = \frac{-\mathbf{a}\cdot\mathbf{b} \pm \sqrt{(\mathbf{a}\cdot\mathbf{b})^2 - |\mathbf{b}|^2(|\mathbf{a}|^2 - 1)}}{|\mathbf{b}|^2}$$