

# Lab 11: Reinforcement Learning

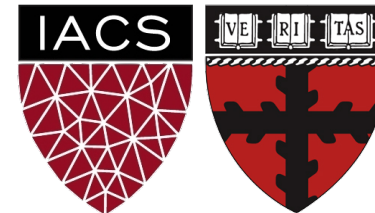
With a focus on Homework 8

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**Harvard IACS**

CS109B

Chris Tanner, Pavlos Protopapas, Mark Glickman

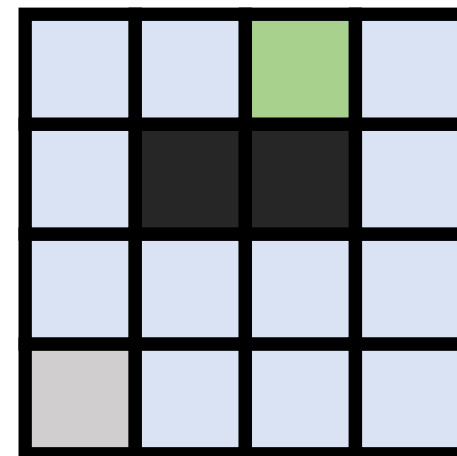


- Agent



The thing that operates  
within the environment

## RL Environment

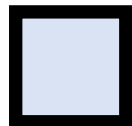


- Agent



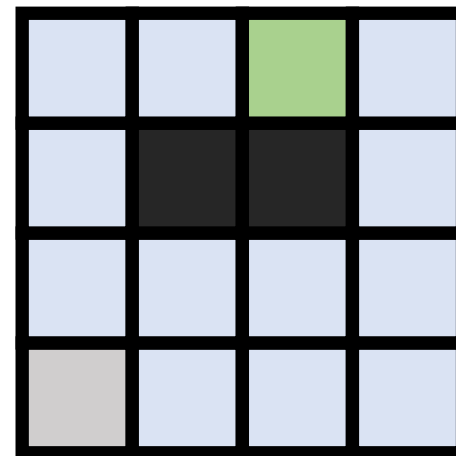
The thing that operates within the environment


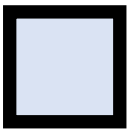

- States



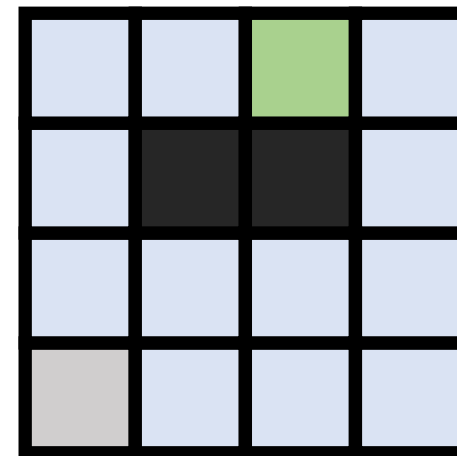
Static representations that define the current environment





## RL Environment



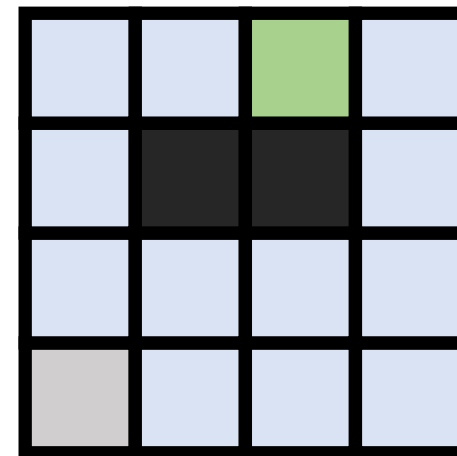
- Agent  The thing that operates within the environment
- States  Static representations that define the current environment
- Actions  An agent's operation that takes him/her from state  $s$  to state  $s'$

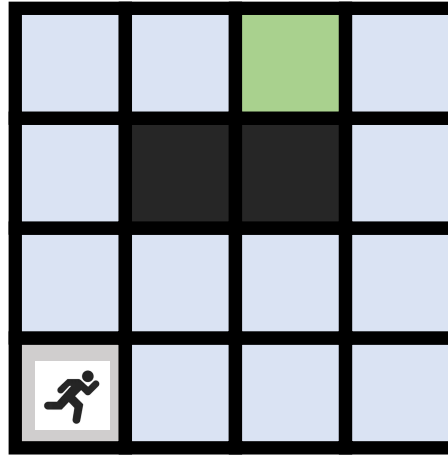
## RL Environment



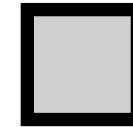
- Agent  The thing that operates within the environment
- States  Static representations that define the current environment
- Actions  An agent's operation that takes him/her from state  $s$  to state  $s'$
- Reward  A real-valued # that represents the goodness for the agent's being in a given state  $s$

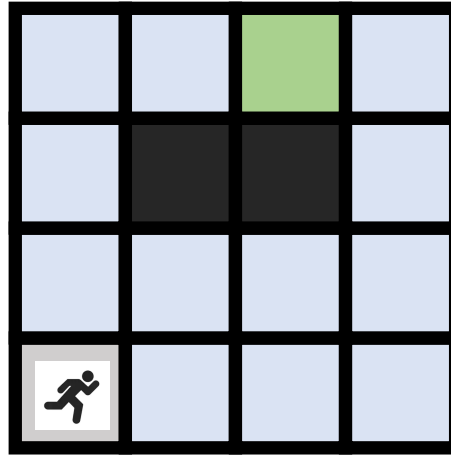
## RL Environment



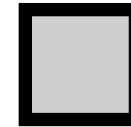


The agent  starts in an initial state  $s$

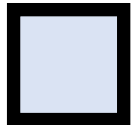


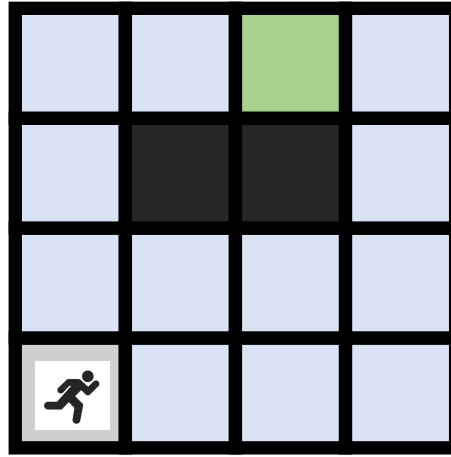


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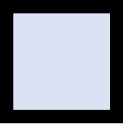


She performs an action  $a \rightarrow$  and becomes in state  $s'$



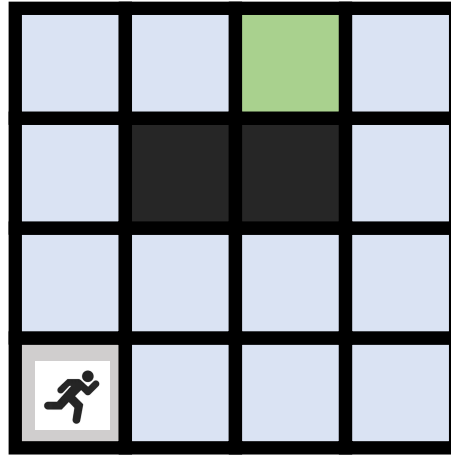


The agent  starts in an initial state  $s$  


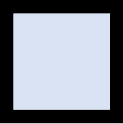
She performs an action  $a$   and becomes in state  $s'$  

Being in each state  $s'$  yields a reward  $r$  (e.g, 3.6)



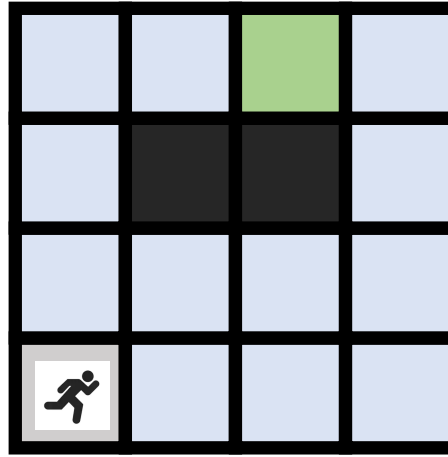


The agent  starts in an initial state  $s$  

She performs an action  $a$   and becomes in state  $s'$  

Being in each state  $s'$  yields a reward  $r$  (e.g, 3.6)

How do we determine how to move from state-to-state so as to receive maximum reward  $r$ ?



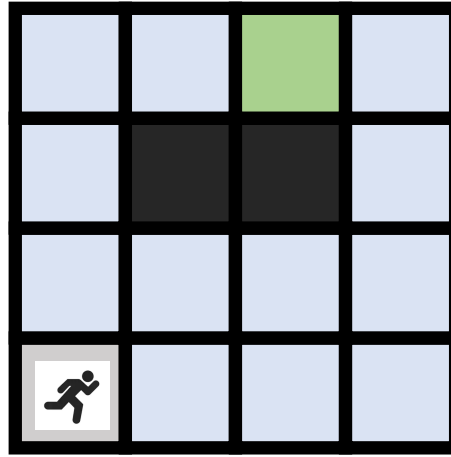
The agent  starts in an initial state  $s$

Sh

That's the entire crux of reinforcement learning!

Be

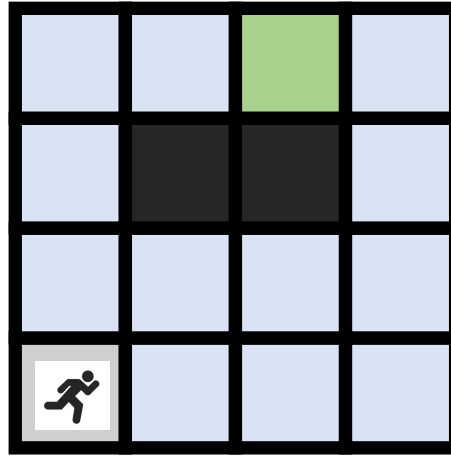
How do we determine how to move from state-to-state so as to receive maximum reward  $r$ ?



Given a state  $\mathbf{s}$ , and an action  $\mathbf{a}$ , estimate the reward  $\mathbf{r}$ .

A policy  $\pi(\mathbf{s})$  takes a state  $\mathbf{s}$  and executes an action  $\mathbf{a}$

$$\pi(\mathbf{s}) \rightarrow \mathbf{a}$$



So, there can be many policies  $\pi_1, \pi_2, \dots, \pi_n$   
(some better than others).

which policy to execute?

How do we determine ~~how to move from state-to-state~~  
~~so as to receive maximum reward~~  $r$ ?

The one that gives us the highest reward!

Let's estimate each policy  $\pi_i$  via a "value-function"  $v_\pi(s)$ , where  $s$  is our starting state

In class, we saw that:

$$\begin{aligned} v_{\pi}(\mathbf{s}) &= \mathbb{E}_{\pi} [\mathbf{G}_t | \mathbf{S}_t = \mathbf{s}] = \mathbb{E}_{\pi} \left[ \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid \mathbf{s} \right] \\ &= \sum_{\mathbf{a}} \pi(\mathbf{a} | \mathbf{s}) \sum_{\mathbf{s}'} \sum_{r} p(\mathbf{s}', r | \mathbf{s}, \mathbf{a}) [r + \gamma v_{\pi}(\mathbf{s}')] \end{aligned}$$

In class, we saw that:

$$v_{\pi}(s) = \mathbb{E}_{\pi} [G_t | S_t = s] = \mathbb{E}_{\pi} \left[ \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | s \right]$$




The policy we're interested in


$$= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r | s, a) [r + \gamma v_{\pi}(s')]$$

In class, we saw that:

Our starting state  $s$


$$v_{\pi}(s) = \mathbb{E}_{\pi} [G_t | S_t = s] = \mathbb{E}_{\pi} \left[ \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | s \right]$$

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In class, we saw that:

Our starting state  $s$

Cumulative reward

$$v_{\pi}(s) = \mathbb{E}_{\pi} [G_t | S_t = s] = \mathbb{E}_{\pi} \left[ \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | s \right]$$

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In class, we saw that:

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## Cumulative reward

Unadjusted reward

$$v_{\pi}(s) = \mathbb{E}_{\pi} [G_t | S_t = s] = \mathbb{E}_{\pi} \left[ \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | s \right]$$

The policy we're interested in

$$= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a) [r + \gamma v_\pi(s')]$$

In class, we saw that:

Our starting state  $s$

Cumulative reward

Weakened based on how far into the future it is

Unadjusted reward

$$v_{\pi}(s) = \mathbb{E}_{\pi} [G_t | S_t = s] = \mathbb{E}_{\pi} \left[ \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | s \right]$$

The policy we're interested in

$$= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r | s, a) [r + \gamma v_{\pi}(s')]$$

$$v_{\pi}(s) = \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a) [r + \gamma v_{\pi}(s')]$$

## HW #1.1

state-to-state transitions **T**

reward **R**

**T**=

**R**=

$$v_{\pi}(s) = \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a) [r + \gamma v_{\pi}(s')]$$

## HW #1.2

- Estimate  $v_{\pi}$
- The above equation is general and works for stochastic situations.
- We have fixed state-transitions and rewards though (we can make our life easier).

$$\begin{aligned} v_{\pi}(s) &= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a) [r + \gamma v_{\pi}(s')] \\ &= \sum_a \pi(a|s) \sum_{s'} p(s', r|s, a) (R(s, a) + \gamma v_{\pi}(s')) \end{aligned}$$

## HW #1.2

- Estimate  $v_{\pi}$
- The above equation is general and works for stochastic situations.
- We have fixed state-transitions and rewards though (we can make our life easier).
- What can we define as a constant?

$$\begin{aligned}
v_{\pi}(s) &= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a) [r + \gamma v_{\pi}(s')] \\
&= \sum_a \pi(a|s) \sum_{s'} p(s', r|s, a) (R(s, a) + \gamma v_{\pi}(s'))
\end{aligned}$$

## HW #1.2

- As a sanity check, a geometric series is defined to have a sum:

$$a + ar + ar^2 + ar^3 + ar^4 + \dots = \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}, \text{ for } |r| < 1.$$