OSU Mathematical Contest for Modeling
Solution due Sunday, November 5th at 10am

Choose one of the two problems proposed below. Submit your model report and anything else that the problem might ask for in a zip file to ciocanel.1@osu.edu by **Sunday, November 5th at 10 am**. Reports received after this time will not be considered.

Your report should start with a cover sheet page that includes the following information: names of your team members, title of your report, and which problem you chose to solve. The rest of the pages in your report **should not include your team members' names**. Remember to reference your sources at the end of the report.

Problem 1: Thrones

White Walkers are ice creatures in the Game of Thrones universe. They are known to be much stronger than humans and are on a quest from the northern regions (north of Castle Black/The Wall) to the southern region of Westeros and eventually King's Landing (see map in Figure 1). White Walkers can resurrect dead humans into wights (see below), are resistant to fire, but may be killed by swords made out of a material called dragonglass.

Wights are not as powerful as White Walkers, but they can only be killed by fire, being broken apart, or stabbed with dragonglass. Additionally, they cannot swim or walk through water. The white walkers and wights intend to destroy humanity on their quest south. However, it appears that these creatures may move relatively slowly. In particular, the red line in the linked map (see References) shows their movement to The Wall in the same amount of time as the movement of a human leader (Jon Snow, by boat and on horseback) depicted by the green line.

The fans of Game of Thrones are concerned that the fight between white walkers and humans cannot be won by humans under any circumstance. The creators of the show have given your team the task of developing a model for the spread of the white walkers and wights into the Kingdom, with the goal of determining how fast they could travel to key strongholds in Westeros and whether there are any measures humans can take to protect themselves. Assume that there are roughly 1000 White Walkers with about 100 wights each and that they are already at Castle Black (also known as The Wall). Further assume that the human armies are spread out as follows: 80000 at Dragonstone, 8000 at Casterly Rock, 26500 in the North, 80000 at King's Landing, and 30000 at Iron Islands.

You must propose at least one strategy through which humans can prevent the white walkers taking over. Let's assume that in order for humans to survive, King's Landing must be free of white walkers. For humans to have a chance at victory, your strategy should also protect Winterfell, which is likely to be the White Walkers' first stop. Factors you **may** consider include:

- The location of decisive battles
- The strength of white walkers/wights compared to humans (for instance, you may assume 5 humans have the strength of one white walker, or leave this as a parameter)
- Whether all dead people or only a fraction are revived into wights
- The time it takes for armies to move to other locations
- The distribution of dragonglass across the Kingdom (you may assume that initially all dragonglass is at Dragonstone)
- How many people must be armed with dragonglass to prevent the whole population from becoming wights.

Note that you are not required to take all the above into account, but it is recommended that you consider as many items as possible.

Figure 1: Map of the Game of Thrones universe with key locations marked.

You may assume that the white walkers will stay on the roads denoted on the map, and that human armies can travel by foot or horse-driven carriage on the roads, or by boat (at technological levels of medieval Europe). You may use that the distance between Castle Black and Winterfell is 600 miles in a straight path (by raven).

How do your model predictions change if you further account for the fact that the Night King (the ruler of White Walkers & Wights) can also transform living human infants into White Walkers?

Besides your model report, your supervisor requires that you write a half-page non-technical summary of your approach and predictions for the spread of white walkers to be shared with the team.

References:
General: White Walkers
Map: see Figure 1
Map of movement: Game of Thrones Map
Population numbers: How many white walkers are there in Game of thrones

Problem 2: Rise of a Modeling Empire

Congratulations! Your team was just chosen to run a math modeling competition, much like the one you are participating in now, at a large online university with exactly 300 students intending to participate. Since the classes are online, the students unfortunately don't know each other, so you must help organize the students into 100 teams of three.

The competition relies on a variety of attributes in each team member; the 4 that we have data from the students for are:

(a) Mathematics (M) (Range from 0 to 10 with 10 being the highest)

(b) Computer programming (C) (Range from 0 to 10 with 10 being the highest)

(c) Writing skills (W) (Range from 0 to 10 with 10 being the highest)

(d) Personality (P) (Integer valued from 0 to 29, indicating one of 30 types of personalities).

The attributes for each individual student are known and are included in the data file provided.

It appears that mathematics and computer programming skills are not correlated. On the other hand, combined mathematics and computer programming skills that are lower than the median in the 300 students seem to correspond to higher writing skills. Finally, personality skills are also independent from all other skills.

It is not currently clear what a good predictor of the score of a team in the competition may be. Given your experience with math modeling contests, you are tasked with proposing at least one such predictor. Your predictor **should** include the following three facts. Note that you may interpret these facts in various ways but must provide justification for your approach:

- Higher mathematics (M) and computer programming (C) skills correlate with a better result; a single person seems to be able to "carry" the team, but other members also contribute with no negative contributions.
- Higher writing (W) skills may act like a "multiplier" for the M and C skills in the final score (or at least must contribute a lot more to the score than M and C). In addition, since no one person can write the whole report, the weakest writer will lower the writing score of a team.
- If the Personalities (P) for the entire team are very close, then the results are higher. Otherwise, if the personalities are too distinct, there is no teamwork and the team's results suffer. To keep the personalities in the range of the data (0-29), you may consider a metric on P to be the modulo distance. I.e. dist(2, 4) = 2 but also dist(2, 30) = 2 because $30 + 2 \equiv 2 \pmod{30}$.

In addition, you **may** choose to consider the following:

- Having a large imbalance between the computer programming and the mathematics skilla results in a poorer score.
- An "ego" effect: if two persons on the team have extremely high and similar skill sets in M, C or W, then there is a net decrease in the score.
- The score is rarely deterministic, and is likely to have a stochastic element.
- Any other sensible factor you might be observing now, or have experienced in such competitions.

Your team must first come up with one (or several) good score predictors as mentioned above.[1] Once your group chooses a good predictor, use it to answer questions (a, c), (b, c) or all 3 below:

(a) Determine the best team you can put together. Can you find the top possible 10 teams if the same students can be in more than one team? What if each student can only be in one team?

(b) Given two people, how do we search for the third person in a team in an informed way (i.e. without evaluating your predictor function with all 298 combinations)?

(c) Can you come up with algorithm(s) to organize students in 100 teams such that the average (or median) score of all teams (with each of the 300 students in only one a team) is higher than that of randomly picking 100 teams from the 300 people?

Besides your model report, the online contest organizer also asked that you write a half-page summary of your findings for the students still looking to complete their team.

Notes on the data set: Since these are not real data and anomalies can occur (such as perfect 10/10/10 attributes), we have included 3 different data sets drawn from the same distributions. You may choose from the three, but please only use **one** to work with for the entire problem.

[1] You may consider simpler predictors if you want for analysis.