Lorentz force on superconducting vortices near line defects

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Abstract

In type-II superconductors, magnetic flux can penetrate through as quantized vortices whose dissipative motion driven by the Lorentz force breaks superconductivity. Understanding vortex motion in homogeneous superconducting regions and near unavoidable structural defects is critical for superconducting applications. This writeup considers a scenario where a superconducting quantum interference device (SQUID) scans across a thin film superconductor with line defects. We first estimate the radial Lorentz force on a vortex in a homogeneous region with an analytical and numerical method utilizing the fast Fourier transform algorithm. We show that the vortex can be dragged by the SQUID tip that exerts around three femto-Newton for a film with a Pearl length of 400 micrometers and a SQUID height of 4 micrometers. Then, we estimate the Lorentz force on vortices near two parallel line defects. We show that the Lorentz force applied by the SQUID on vortices pinned on and between the line defects is enhanced for line defects with reduced Peral length. Vortices pinned on the line defects experience an enhancement perpendicular to the defects, whereas vortices in between line defects experience an enhancement along the lines. Our findings aid in a more precise estimation of Lorentz force on vortices adjacent to line defects in thin film superconductors and further aid in understanding vortex pinning and dynamics in superconductors with necessary structural defects. Our methods could be easily adapted for bulk superconductors and defects of other shapes.

I. MOTIVATION

Superconducting vortices in type-II superconductors are whirlpools of supercurrents surrounding a normal core that carry one flux quanta Φ_0. Vortices tend to pin their cores on naturally occurring defects such as atomic vacancies or twin boundaries. In the presence of a current, the current applies a Lorentz force on vortices that causes unwanted vortex motion and dissipation, which breaks the superconducting state. As such, tremendous effort has been made on pinning vortices to artificial pinning centers to reduce vortex motion and raise the critical temperature T_c. Understanding vortex motion is vital for superconducting applications.

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The Lorentz force on a vortex with flux lines in the z-direction is

$$F = \Phi_0 \vec{j} \times \hat{z} \tag{1}$$

The vortex pinning centers can be approximated as the local minima of a pinning potential. The shape of the pinning potential decides the magnitude of a depinning force, which is the minimum force needed to entirely or partially pull a vortex away from the original pinning center. While global transport measurements provide information on macroscopic vortex motion, scanning probes are ideal for studying local pinning potential, especially in superconductors with inhomogeneous or spatially modulated superfluid density. Various magnetic scanning probes can apply a Loretz force on vortices and imaging the resultant flux line distortion. Here, a Lorentz force is applied through the field coil of a scanning superconducting quantum interference device (SQUID). The SQUID has a pickup loop that precisely measures the magnetic flux penetrating through itself and a concentric field coil to apply a local magnetic field. The applied field induces a screening current in the superconductor $\vec{j}$ that exerts a Lorentz force on the vortex. The pickup loop then captures the change of flux line distribution due to vortex motion. In the rest of this write-up, we calculate the Lorentz force on a vortex in a thin film superconductor in the isotropic and inhomogeneous Meissner screening case with parallel line defects.

II. HOMOGENEOUS SUPERCONDUCTOR

Figure 1 demonstrates the problem setup. A thin film superconductor (green sheet) is located at z = 0 in the xy-plane, with the top surface at z = 0^+ and the bottom surface at z = 0^-. The pickup and field coil loops (pink-blue loop pair) are at z_0. The blue field lines illustrate the magnetic field created by the field coil. Outside the film, there is no bound current, and this magnetic field could be represented as the gradient of a source scalar potential ϕ^s. The superconductor generates a screening circular current $\vec{j}$ represented by the red dashed lines. Such a screening current creates a response field opposite the applied field. Above the thin film, this response field could also be represented as a scalar potential ϕ^r gradient.

Simplifying the field coil and pickup loop pair to two concentric loops, the Lorentz force

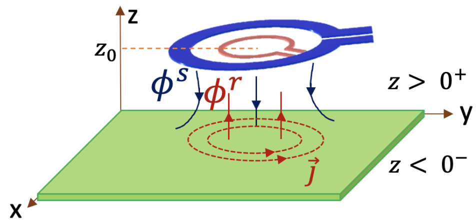

FIG. 1. Schematic representation of problem setup.

on a vortex in a thin film superconductor (in SI unit) was derived by Kogan [1]:

$$F(r) = -\Phi_0 I a \int_0^\infty dk \frac{k e^{-k z_0}}{1+\Lambda k} J_1(ka) J_1(kr) \tag{2}$$

where Φ_0, I, a are constants representing the flux quanta, field coil current, and field coil effective radius, respectively. r is the distance from the field coil center to the vortex center. The Lorentz force $F(r)$ is radial and integrated over the momentum k-space. Estimations for depinning force in bulk superconductors using similar expressions were given in [2, 3]. It should be mentioned that the force $F(r)$ has opposite signs with the radial distance r, so the force is an attractive force or dragging force when the SQUID scans across the sample.

A detailed derivation of (2) can be found in Ruby Shi's doctorate thesis. The general idea is to integrate (1) over the film thickness and relate the in-plane superfluid density $\vec{j}$ to the shear components of the magnetic field h using the London's equation

$$\vec{h} + \lambda^2 \nabla \times \vec{j} = 0 \tag{3}$$

The thin film case is easier than the bulk case as the magnetic field $\vec{h}$ has no z-dependence but rather discontinuities $h_{x,y}(x, y, 0^+) = -h_{x,y}(x, y, 0^-)$. Knowing that $\vec{j}$ has no z-dependence (See Pearl's original approximation of the thin film superconductor as a two-dimensional current sheet [4])

$$F_x(x, y, 0^+) = \Phi_0 \int_{0^-}^{0^+} dz j_y = \Phi_0 \int_{0^-}^{0^+} dz \frac{\partial}{\partial z} h_x^r = 2\Phi_0 h_x^r(x, y, 0^+) \tag{4}$$

The response field $h_x^r(x, y, 0^+)$ can be written as a Fourier sum of $h_x^r(\vec{k}, 0^+)$, which is also $ik_x \phi^r(\vec{k}, 0^+)$ in the two-dimensional Fourier space. In the thin film, we have $\phi^r(\vec{k}, 0^+) = \phi^s(\vec{k}, 0^+)/(1 + k\Lambda)$, where Λ is the Pearl length. We see that the Lorentz force calculation comes down to finding the correct ϕ_r whose gradient in the x-direction is the field h_x. To obtain the Lorentz force in the y-direction, replace the subscripts from x to y.

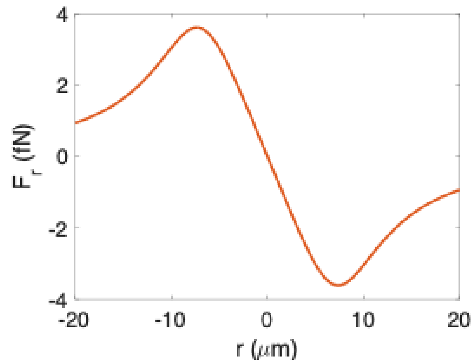

FIG. 2. Analytical calculation of the Lorentz force on a vortex with Pearl length $\Lambda = 420$ when the SQUID is positioned at 4.2 μm above.

It can be seen that the Lorentz force can be calculated in two methods: 1. Direcly plot the analytical solution according to equation (2). 2. Obtain the field component $h_x^r(\vec{k}, 0^+)$ in the Fourier space and numerically inverse Fourier transform to $h_x^r(x, y, 0^+)$.

Figure 2 plots the Lorentz force on a vortex with Pearl length $\Lambda = 420$ when the SQUID is positioned at 4.2 μm above using (2). The center of the SQUID is located at x = 0 and y = 0. It can be seen that the direction of the force is opposite to the sign of the SQUID-vortex displacement r, indicating the force is attractive.

Alternatively, MatLab's inverse fast Fourier transform algorism (FFT) could calculate the Lorentz force analytically. Figure 3 (a) shows the magnitude of the Lorentz force, with the direction indicated by the arrows. The center of the SQUID is not at (0, 0) but at the center of the black loops representing the pickup and field coil pair. The used parameters ($\Lambda = 420$ μm, $z_0 = 4.2$ μm) are the same as Fig .2, and one can see the agreement in force magnitude and direction between the two methods. Figure 3(b,c) plots the magnetic field components due to the screening currents. They can directly relate to the Lorentz force by (4).

The results from the two methods agree as expected. However, two precautions must be taken to yield correct results: 1. The k-space, especially the zero, must be carefully defined. Ideally, check the k-grid on a function with an analytical Fourier expression. 2. The range of the real space should be greater than the Pearl length for the k-space to resolve a singularity at $k = -1/\Lambda$ in the response potential $\phi^r = \phi^s/(1 + k\Lambda)$. More details can be found in Ruby Shi's thesis.

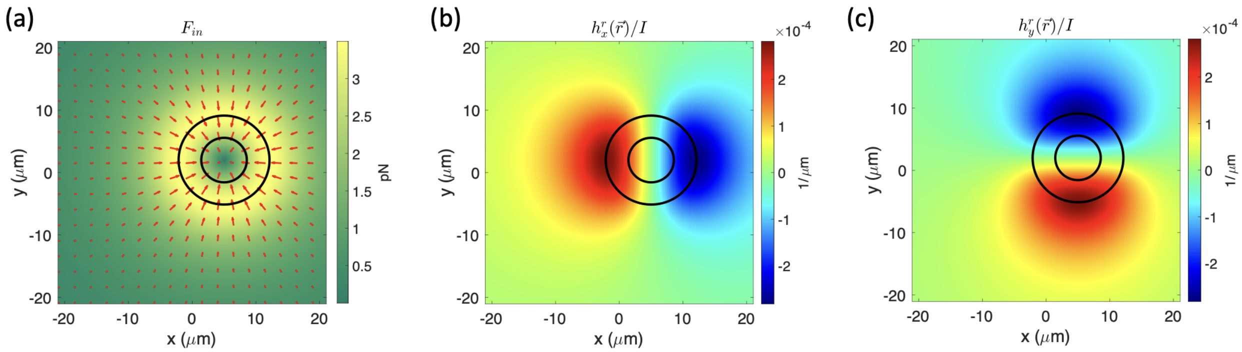

FIG. 3. Calculation of the Lorentz force using the inverse Fourier transform method. The black circles indicate the location of the pickup and field coil pair. The SQUID is positioned at 4.2 μm above over a superconductor with Pearl length $\Lambda = 420$ μm. (a) The Lorentz force on a vortex. The color depth indicates the force magnitude. The red arrows indicate the force direction. The xy-coordinates indicate the location of the vortex. As expected, the Lorentz force is radially symmetrical about the center of the field coil-pickup loop pair. (b, c) Magnetic field h_x, h_y normalized to current I. The field components are related to the Lorentz force in the corresponding direction by equation (4).

III. SUPERCONDUCTOR WITH LINE DEFECTS

Diamagnetic screening inhomogeneity is common in superconductors. Some inhomogeneities can be eliminated with improved sample synthesis; others are intrinsic to the superconductors due to structural defects. It is shown that the superfluid density can be reduced in cuprates and enhanced in pnictides along the twin boundaries, impacting vortex pinning and dynamics [2, 5]. In past literature, the twin boundaries were represented as lines of modified Pearl length [6]. This section considers the Lorentz force on a vortex sandwiched between two line Pearl length modifications. The results could be applied to study the anisotropic Lorentz force on vortices between twin boundaries.

In [6], Kogan and Kirtley approximated a line shape superfluid density defect centered at x $= x'$ as a delta function

$$\Lambda(x) = \Lambda_0 - \alpha^2\delta(x - x') \tag{5}$$

where the constant α with dimension of length indicates the strength of the defect. For a spatially inhomogeneous Λ, the London's equation in the z-direction is modified to

$$h_z^s + h_z^r + \Lambda\left(\frac{\partial h_x^r}{\partial x} + \frac{\partial h_y^r}{\partial y}\right) + \left(h_x^r \frac{\partial \Lambda}{\partial x} + h_y^r \frac{\partial \Lambda}{\partial y}\right) = 0 \tag{6}$$

It can be derived that the response potential takes the form

$$\phi^r = \frac{\phi^s}{1 + k\Lambda} + \delta\phi^r \tag{7}$$

The first term is the response potential for the isotropic case. The second term is an addition due to the presence of the line defect.

$$\delta\phi^r(\vec{k}, x_0, y_0) = \frac{\alpha^2}{k(1 + k\Lambda_0)} \int_\infty^\infty \frac{dq_x}{2\pi} \frac{(\vec{k}\cdot\vec{Q})\phi^s(\vec{Q}, x_0, y_0) e^{-i(k_x - q_x)x'}}{1 + \Lambda_0 Q} \tag{8}$$

where $\vec{Q} = (q_x, k_y)$. $\phi^s(\vec{Q}, x_0, y_0)$ is the source potential at (x_0, y_0). Kogan derived equation (8) for $y_0 = x' = 0$. Ruby's thesis shows the direct derivation of (8).

The source potential on the superconducting film for a field coil with current I and an effective radius a located at $\vec{r_0} = (x_0, y_0)$ and height z_0 is

$$\phi^s(\vec{k}, 0) = \frac{\pi I a}{k} e^{-kz_0 - i\vec{k}\cdot\vec{r_0}} J_1(ka) \tag{9}$$

Based on Kogan's previous derivation, we investigate the Lorentz force on a vortex sandwiched between two line defects. We clarify and rephrase our setup here: A superconducting thin film is in the xy-plane at $z = 0$. We place two line defects at $x = x'$ and $x = -x'$ on the film and investigate the Lorentz force on a vortex in the simulation area due to the screening current induced by a field coil located at (x_0, y_0, z_0). The line defects on the superconductor deem the screening current and, thereby, the Lorentz force anisotropic.

The line defects are represented as Pearl length modifications

$$\Lambda(x) = \Lambda_0 - \alpha^2\delta(x - x') - \alpha^2\delta(x + x') \tag{10}$$

Here, we assumed the line defects have the same strength α as is commonly the case for twin boundaries. The additional term to the response function is then modified to

$$\delta\phi^r(\vec{k}, x_0, y_0) = \frac{2\alpha^2}{k(1 + k\Lambda_0)} \int_\infty^\infty \frac{dq_x}{2\pi} \frac{(\vec{k}\cdot\vec{Q})\phi^s(\vec{Q}, x_0, y_0) cos((k_x - q_x)x')}{1 + \Lambda_0 Q} \tag{11}$$

substitute ϕ^s by (9)

$$\delta\phi^r(\vec{k}, x_0, y_0) = \frac{\alpha^2 I a}{k(1 + k\Lambda_0)} \int_\infty^\infty dq_x \frac{(\vec{k}\cdot\vec{Q}) e^{-Qz_0 - iq_x x_0 - ik_y y_0} J_1(Qa) cos((k_x - q_x)x')}{(1 + \Lambda_0 Q)Q} \tag{12}$$

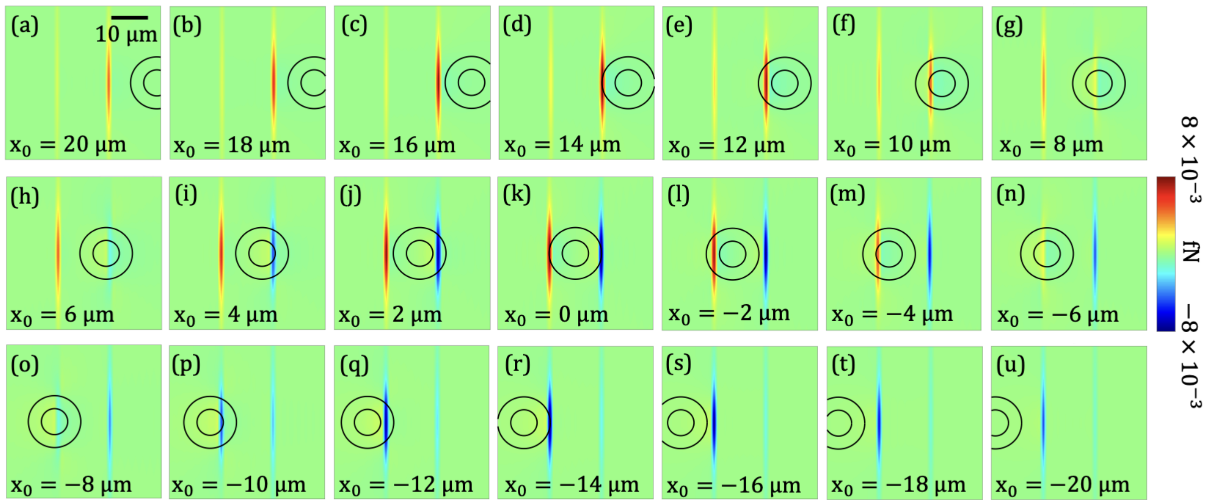

FIG. 4. Simulated Lorentz force in the x-direction on a vortex when two line-shape superfluid density defects are present. The color depth of each pixel indicates the Lorentz force on the vortex if it is pinned at the location. The parameters used in this simulation are: defect strength $\alpha = 1$ μm , field coil current $I = 3$ mA, defect location $x' = 7$ μm, Pearl length $\Lambda_0 = 400$ μm, and height $z_0 = 4$ μm.

First, we study F_x at $y_0 = 0$. We have

$$\delta h_x^r(\vec{k}, x_0, 0) = \frac{ik_x\alpha^2 Ia}{k(1+k\Lambda_0)} \int_{\infty}^{\infty} dq_x \frac{(\vec{k}\cdot\vec{Q})e^{-Qz_0 - iq_x x_0} J_1(Qa) cos((k_x - q_x)x')}{(1+\Lambda_0 Q)Q} \tag{13}$$

It can be seen that the additional field δh_x^r in k-space is a function of x_0, or the location of the field coil. Numerically inverse Fourier transform $\delta h_x^r(\vec{k})$ to obtain $\delta h_x^r(\vec{r})$, one can directly obtain $F_x(\vec{r})$ using (4) at a specific x_0. Figure 4 shows the simulation results at various x_0. The color depth of each pixel indicates the x-component of the Lorentz force if the vortex was pinned at the location. The parameters used in the simulation are listed in the figure caption. Two results can be inferred: 1. (For vortices pinned on line defects) Recall Fig. 3(b) that vortices left to the center of the field coil-pickup loop pair experience a positive force; it can be seen that vortices pinning on the line-shape superfluid density enhancements (signaled by a positive α) experience a greater attractive force to the center of the field coil. It has been shown that the superfluid density is reduced in bulk cuprates and vortices preferentially pin on twin boundaries [5]. Also, vortices are trapped in spatially modulated striaes where superfluid density is enhanced in thin film $La_{2-x}Ba_xCuO_4$ near the 1/8 anomaly [8]. Our results show that vortices pinned in these regions experience a

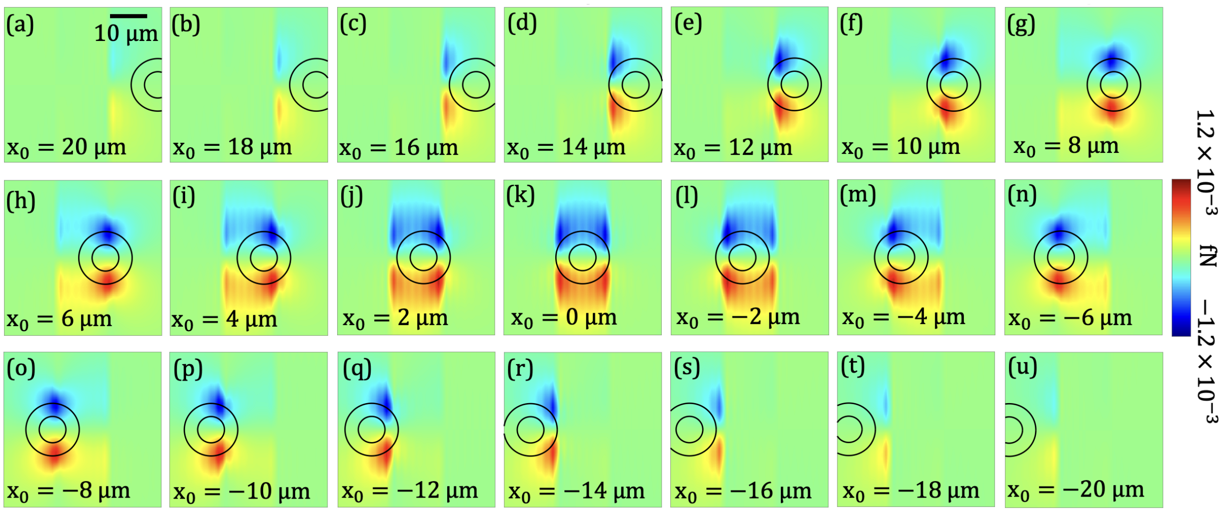

FIG. 5. Simulated Lorentz force in the y-direction on a vortex when two line-shape superfluid density defects are present. The color depth of each pixel indicates the Lorentz force on the vortex if it is pinned at the pixel location. The parameters used in this simulation are: defect strength $\alpha = 1$ μm , field coil current $I = 3$ mA, defect location $x' = 7$ μm, Pearl length $\Lambda_0 = 400$ μm, and height $z_0 = 4$ μm.

greater dragging force perpendicular to the defects, as the SQUID scans. 2. (For vortices not pinned on line defects) The modification to the Lorentz force at regions other than the superfluid density is not significantly altered, at least in our case where the separation of the line defects is 14 μm. Hence, the estimation of the depinning force using the isotropic model in the last section is justified if the vortex is not pinned on the defects.

Second, we study F_y at $y_0 = 0$ and various x_0,

$$\delta h_y^r(\vec{k}, x_0, 0) = \frac{ik_y \alpha^2 I a}{k(1 + k\Lambda_0)} \int_{\infty}^{\infty} dq_x \frac{(\vec{k} \cdot \vec{Q}) e^{-Qz_0 - iq_x x_0} J_1(Qa) cos((k_x - q_x)x')}{(1 + \Lambda_0 Q)Q} \tag{14}$$

Figure 5 shows the Lorezen force on a vortex as the SQUID scans horizontally. The color depth indicates the force magnitude experienced by a vortex pinned at the corresponding pixel, while the black circles indicate the SQUID location. The stripes feature most obvious in Fig. 5(i)–(m) are simulation artifacts due to a low number of pixels in consideration of the simulation time. Two results can be drawn: 1. Vortices pinned on the defects experience an enhanced dragging force, which is the greatest when the SQUID is directly above or between the defects, similar to the force in the x-direction. However, the y-direction force enhancement magnitude is around a factor of 10 less than the x-direction, indicating the primary force enhancement is perpendicular to the line defects for vortices pinned on the defects. 2. Vortices pinned between the line defects experience a slightly less y-direction dragging force than those pinned on the defects. This differs from the force enhancement in the x-direction, which primarily applies to vortices pinned on the defects. Adjusting the simulations in Fig. 4 to the same colormap scale with Fig. 5, one could see that the y-directional force enhancement on vortices pinned in between defects is more significant than the x-directional force enhancement, indicating that vortices pinned in between defects primarily experience a y-direction dragging force enhancement.

Last, the line defects can be modeled as a Gaussian function instead, allowing more precise estimation when the local Pearl length can be extracted from susceptibility measurements.

IV. CONCLUSIONS

Overall, the force applied by a scanning SQUID to vortices pinned on homogeneous regions of a thin film superconductor and vortices adjacent to line defects is studied. The Lorentz force applied by a SQUID on a vortex in the homogeneous region is an isotropic dragging force. The force magnetite applied to a vortex with a Pearl length of $\Lambda = 420$ μm is estimated to be a few picoNewtons for a SQUID with an effective field coil of 7.13 μm and positioned at 4 μm above the film. An analytical expression is given to calculate the Lorentz force on a vortex with any Pearl length and a SQUID of any size. The analytical expression converges with an alternative numerical method using the inverse Fourier transform. The numerical method calculates the dragging force applied on vortices pinned on and between two parallel line defects. In both the x- and y-direction, the presence of line-shape superfluid density enhanced (Pearl length reduced) defects enhances the dragging force applied by the SQUID. In particular, vortices pinned on the line defects experience a force enhancement perpendicular to the defects, whereas vortices between line defects experience a force enhancement along the lines. These findings aid in a more precise estimation of Lorentz force on vortices adjacent to line defects in thin film superconductors and further aid in understanding vortex pinning and dynamics in superconductors with necessary structural defects.

Our methods could be easily adapted for bulk superconductors and defects of other shapes.

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