Magnetic dipole simulation

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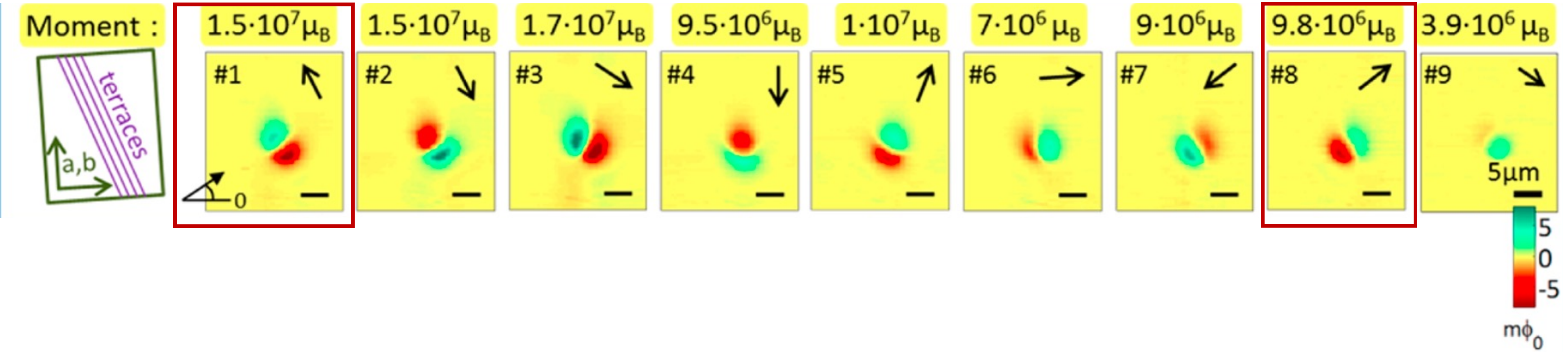
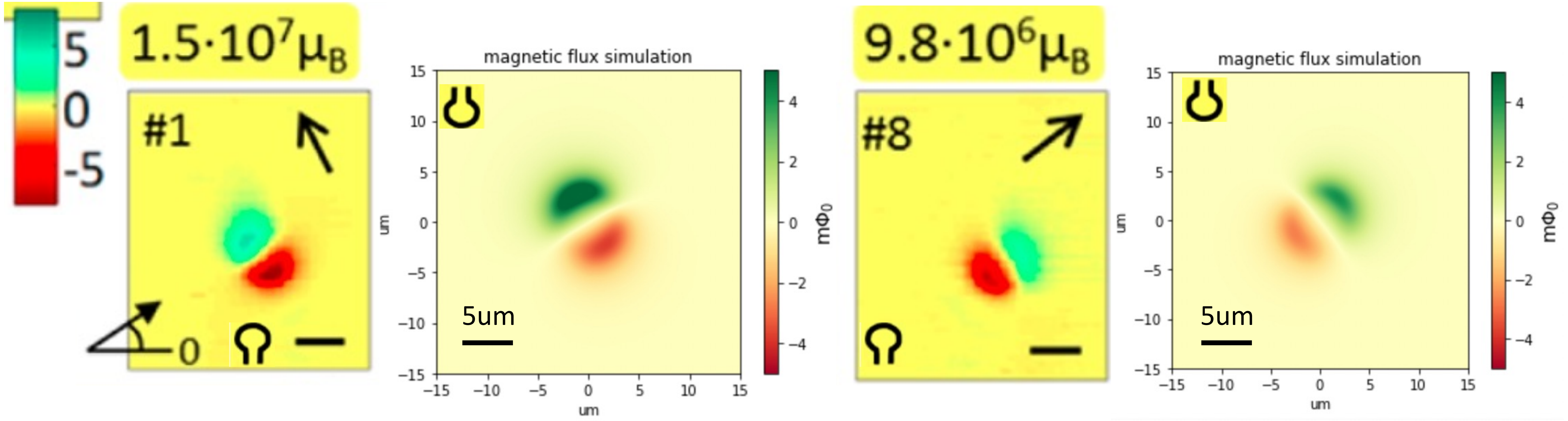
05/18/21

results





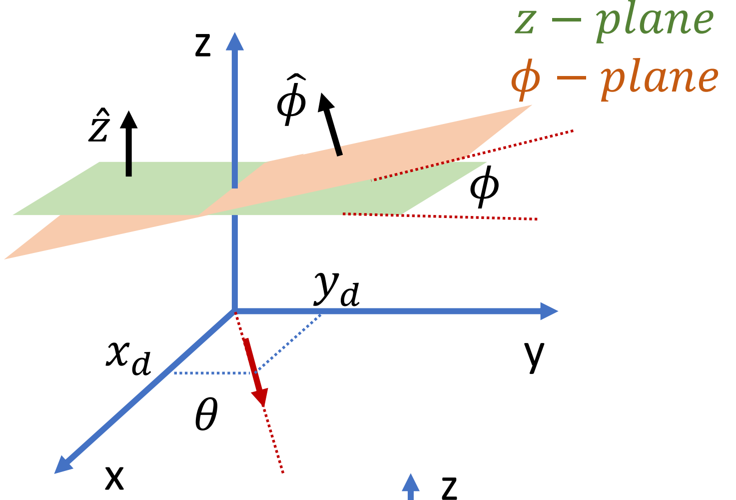
Beena's [dipole manipulation paper]

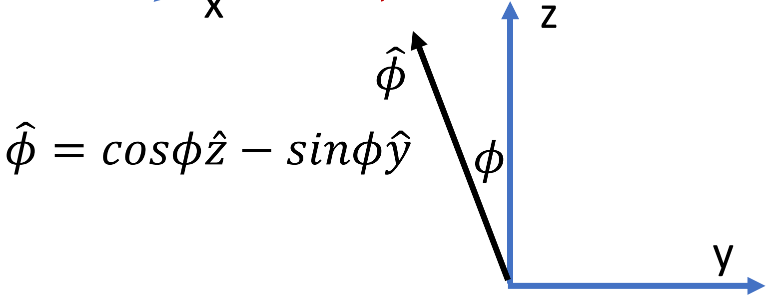
Math: B-expression

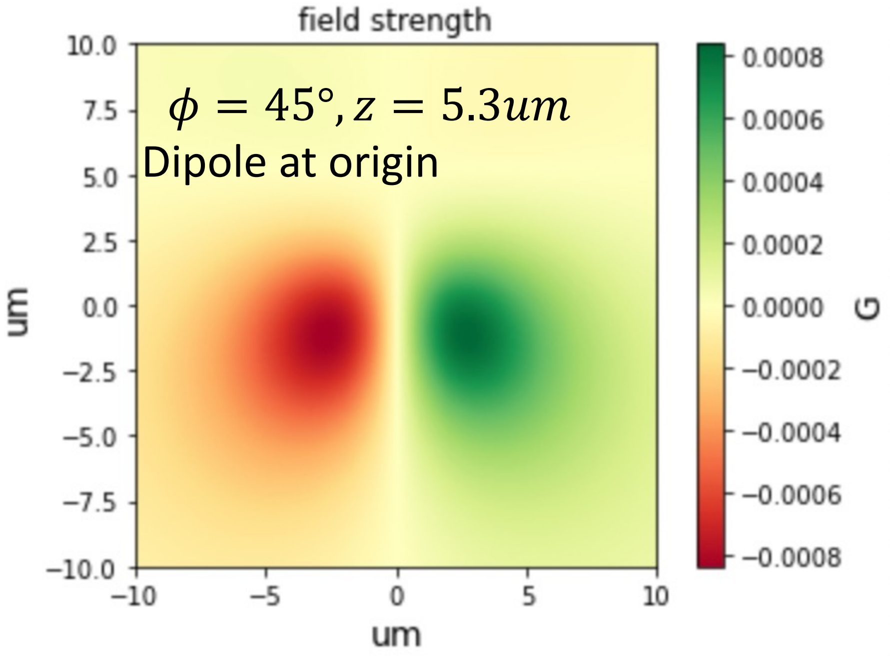
- In 3d space, magnetic field of a point dipole located at origin is $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\left[\frac{3\vec{r}(\vec{m}\cdot\vec{r})}{r^5} - \frac{\vec{m}}{r^3}\right]$
- That is, magnetic field of N Bohr magneton is $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} N\mu_B \left[\frac{3\vec{r}(\hat{m}\cdot\vec{r})}{r^5} - \frac{\hat{m}}{r^3}\right]$
- Now ignore constant coefficient $\frac{\mu_0}{4\pi} N\mu_B$ and consider a dipole located at $r_d = (x_d, y_d, z_d)$. At $\vec{r}$, it creates a magnetic field $\vec{B}(\vec{r}) = \frac{3(\vec{r}-\vec{r_d})(\hat{m}\cdot(\vec{r}-\vec{r_d}))}{|\vec{r}-\vec{r_d}|^5} - \frac{\hat{m}}{|\vec{r}-\vec{r_d}|^3}$
- Consider a dipole in xy-plane, rotated at θ to x-axis: $\hat{m} = (cos\theta, sin\theta, 0), \vec{r_d} = (x_d, y_d, 0)$
- Then $\hat{m} \cdot (\vec{r} - \vec{r_d}) = cos\theta(x - x_d) + sin\theta(y - y_d)$
- And $\vec{B}(x,y,z) = \frac{3[(x-x_d)\hat{x}+(y-y_d)\hat{y}+z\hat{z}]\,[cos\theta(x-x_d)+sin\theta(y-y_d)]}{[(x-x_d)^2+(y-y_d)^2+z^2]^{5/2}} - \frac{cos\theta\,\hat{x}+sin\theta\,\hat{y}}{[(x-x_d)^2+(y-y_d)^2+z^2]^{3/2}}$

Math: planes

- Now project vector B-field to a plane by multiplying $\vec{B}$ by the plane's unit normal vector
- For example: $B_z(x,y,z) = \vec{B}\cdot\hat{z} = \dfrac{3z\,[\cos\theta(x-x_d)+\sin\theta(y-y_d)]}{[(x-x_d)^2+(y-y_d)^2+z^2]^{5/2}}$
- Therefore: $B_\phi(x,y,z) = \vec{B}\cdot\hat{\phi} = \dfrac{3[-(y-y_d)\sin\phi+z\cos\phi]\,[\cos\theta(x-x_d)+\sin\theta(y-y_d)]}{[(x-x_d)^2+(y-y_d)^2+z^2]^{\frac{5}{2}}} + \dfrac{\sin\phi\sin\theta}{[(x-x_d)^2+(y-y_d)^2+z^2]^{3/2}}$
- Physically, the z-plane is the scanning plane – we conduct touchdowns to define a plane that is parallel to sample and scan at finite height. The ϕ-plane is SQUID pickup loop plane. We align the SQUID at a small angle tilt to the sample. The aligning angle is ϕ

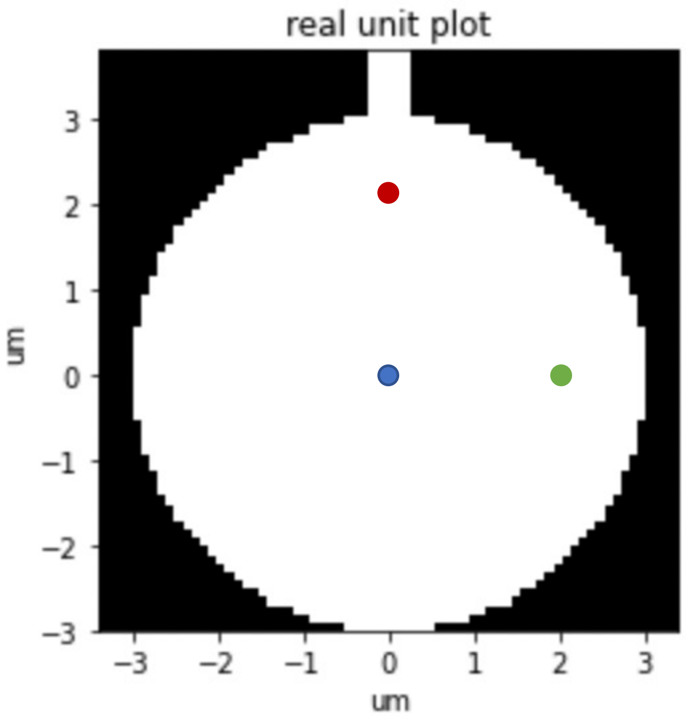






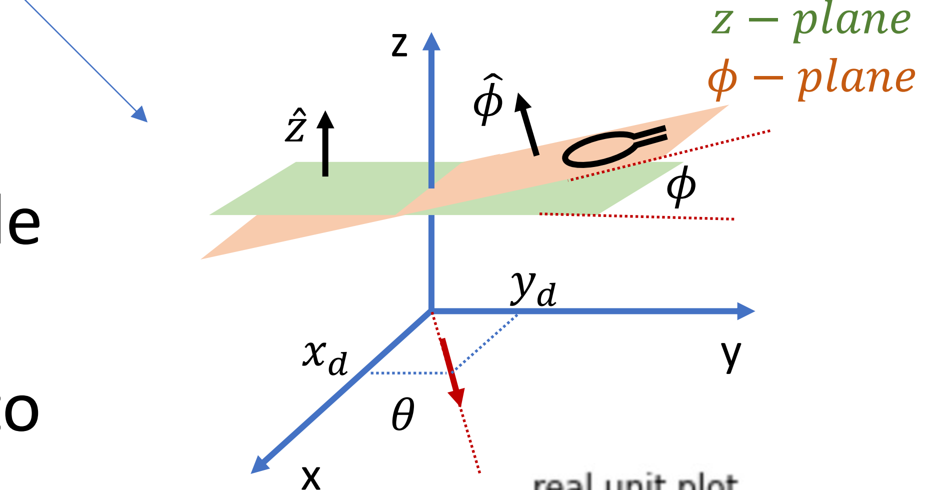
Math: SQUID layout

- As a result, for a small area segment of the pickup loop, $B_\phi(x, y, z)$ is what is sees, as the SQUID scans around sample surface.
- Of course, every small segment on SQUID pickup loop has a different height. I fix center of SQUID(blue dot in figure) at nominal scanning height.
- From the way we align, scanning height is higher where y > 0 in SQUID layout, and lower where y < 0.
- In addition, every small segment sees the dipole differently. If I fix dipole at center of scanning area, the center of SQUID sees a dipole at (0,0,0), whereas other segments see a displaced dipole. For example, green dot sees a dipole at (-2, 0), and red dot sees a dipole at (0, -2).



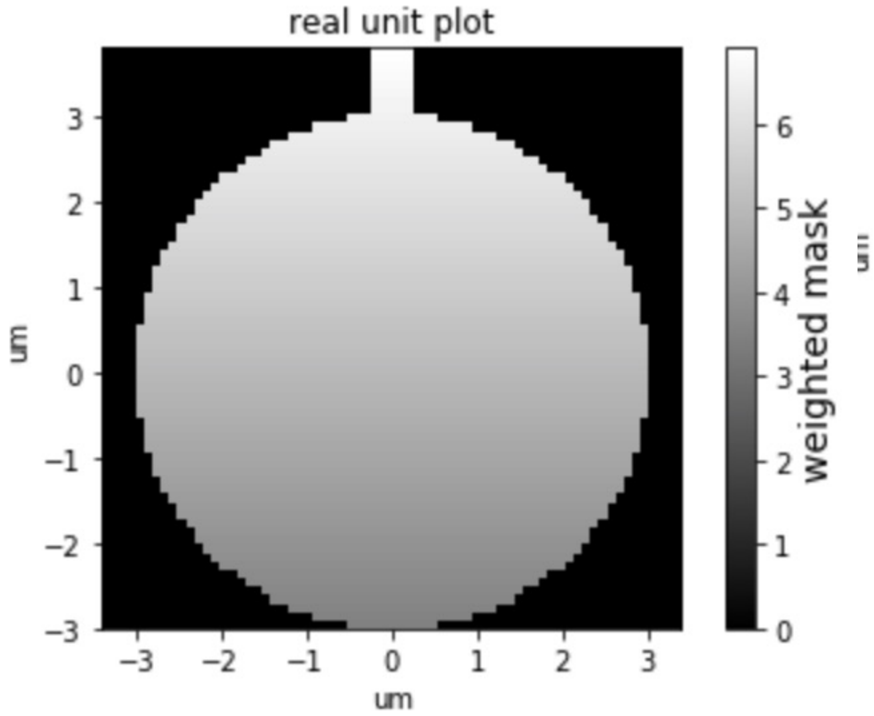
Real SQUID pickup loop layout. Center of SQUID(blue dot), or center of 3um radius circle is fixed at origin of SQUID's own coordinates.

Math: adjusted height

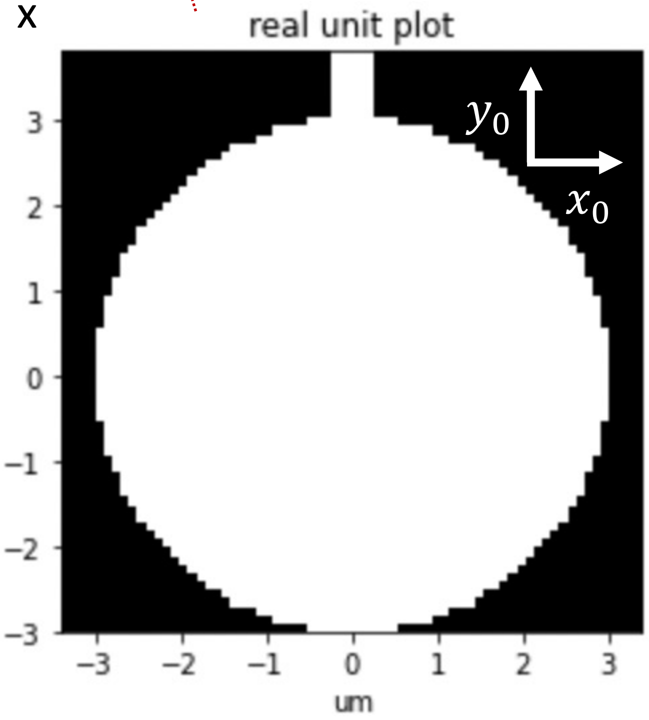


- As there is only one rotation angle here, it is a simple calculation. $\Delta z = \sin\phi y_0$
- Alternatively, one can project any point in ϕ-plane to z-axis. A point in ϕ-plane is $\mathrm{r}' = x_0\widehat{x'} + \mathrm{y}_0\widehat{y'}$. $\hat{z} \cdot r' = sin\phi y_0$. Same result.

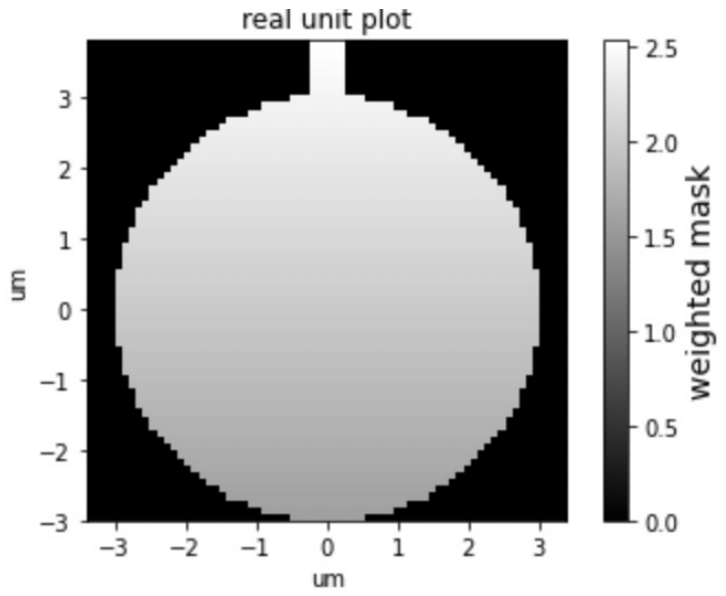
	Normal vector	In plane orthogonal vector
$z - plane$	$\hat{z}$	$\hat{x}, \hat{y}$
$\phi - plane$	$\hat{\phi} = cos\phi\hat{z} - sin\phi\hat{y}$	$\widehat{x'} = \hat{x}$, $\widehat{y'} = \cos\phi\hat{y} + \sin\phi\hat{z}$

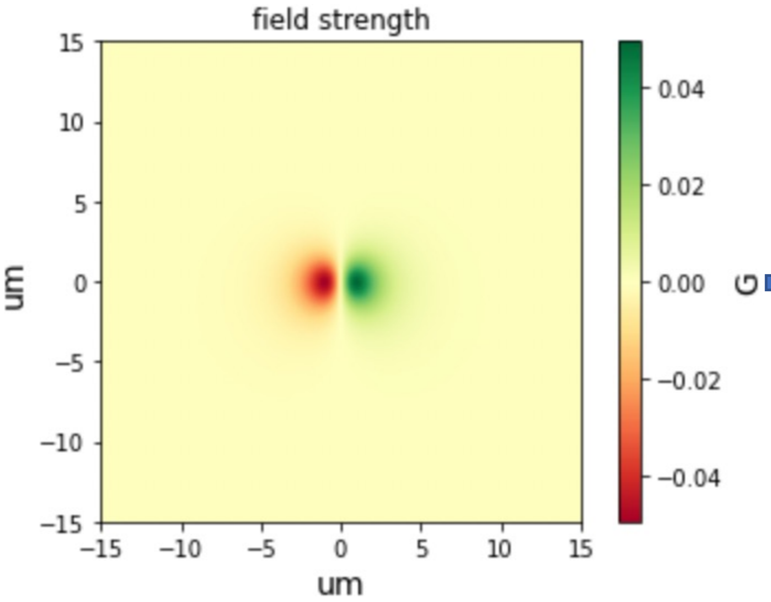


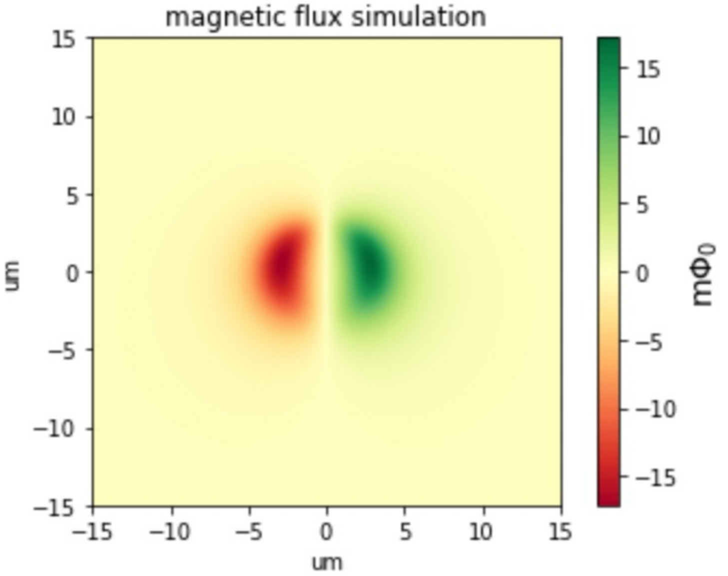
Nominal scanning height $z_0 =$ 5um, $\phi = 30°$



Logic diagram







$\phi = 8°, z = 2um$

Calculate adjusted height given scan height z and tilt angle ϕ

69 * 69 matrix

$\theta = 0°$, for center of pickup loop

Generate the magnetic field each layout segment sees to get a flux contribution

~1.8ms

Loop around every segment in dipole layout and take sum to obtain final simulation

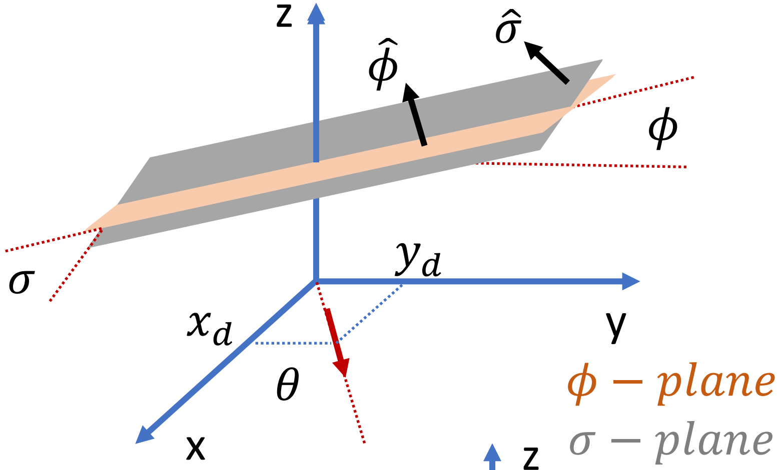
$$processing\ time = 69 * 69 * 1.8ms = 8.5s$$

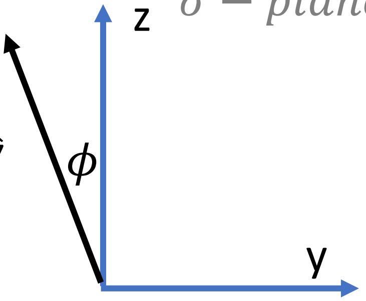
Math: two tilt angles

- Now rotate ϕ- plane along y-axis by σ degrees
- $B_\sigma(x,y,z) = \vec{B}\cdot\hat{\sigma} =$

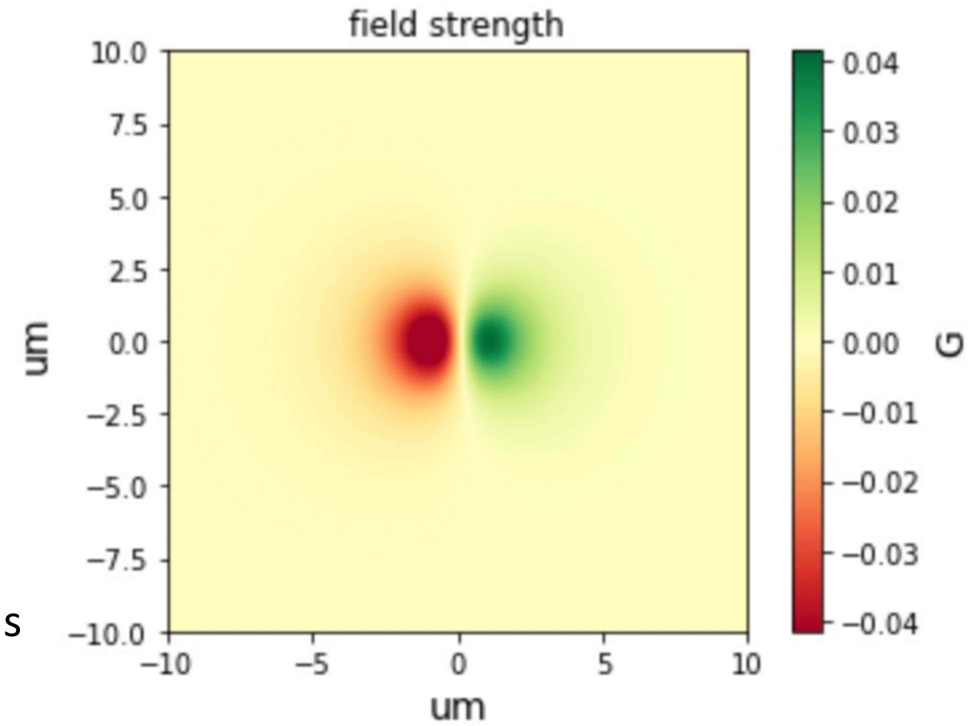
$$\frac{3[-(y-y_d)sin\phi cos\sigma+(x-x_d)sin\sigma+zcos\phi cos\sigma]\,[cos\theta(x-x_d)+sin\theta(y-y_d)]}{[(x-x_d)^2+(y-y_d)^2+z^2]^{\frac{5}{2}}}+$$

$$\frac{sin\phi sin\theta cos\sigma - cos\theta sin\sigma}{[(x-x_d)^2+(y-y_d)^2+z^2]^{3/2}}$$





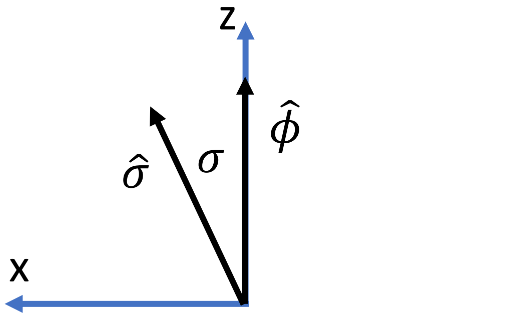
$\hat{\phi} = cos\phi\hat{z} - sin\phi\hat{y}$



$\phi = 8°, z = 2um$

Dipole at origin, $\sigma = 10°$

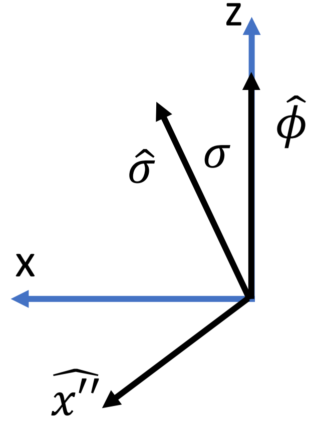
Asymmetrical about x-axis as expected



$\hat{\sigma} = cos\sigma\hat{\phi} + sin\sigma\hat{x}$

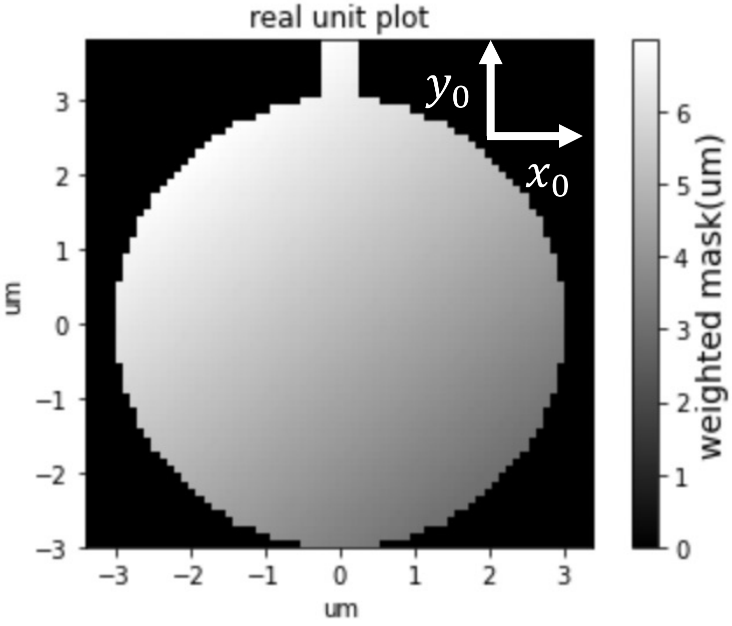
$= cos\sigma cos\phi\hat{z} - cos\sigma sin\phi\hat{y} + sin\sigma\hat{x}$

Math: adjusted height for two angles



- This is done by identifying correct in-plane unit vectors
- A point in σ-plane is $\mathrm{r}'' = x_0\widehat{x''} + \mathrm{y}_0\widehat{y''}$. Then $\hat{z} \cdot r'' = sin\phi y_0 - sin\sigma cos\phi x_0$

	Normal vector	In plane orthogonal vector
$z - plane$	$\hat{z}$	$\hat{x}, \hat{y}$
$\phi - plane$	$\hat{\phi} = cos\phi\hat{z} - sin\phi\hat{y}$	$\widehat{x'} = \hat{x}$, $\widehat{y'} = \mathrm{cos}\phi\hat{y} + \mathrm{sin}\phi\hat{z}$
$\sigma - plane$	$\hat{\sigma}$ $= cos\sigma cos\phi\hat{z}$ $- cos\sigma sin\phi\hat{y} + sin\sigma\hat{x}$	$\widehat{x''} = cos\sigma\widehat{x'} - sin\sigma\hat{\phi}$ $= cos\sigma\widehat{x'} + sin\sigma sin\phi\hat{y} - sin\sigma cos\phi\hat{z}$ $\widehat{y''} = \widehat{y'} = \mathrm{cos}\phi\hat{y} + \mathrm{sin}\phi\hat{z}$

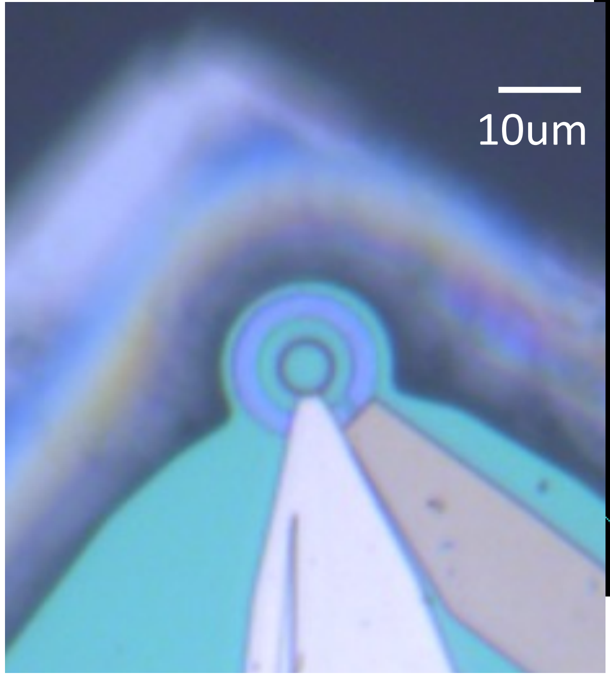


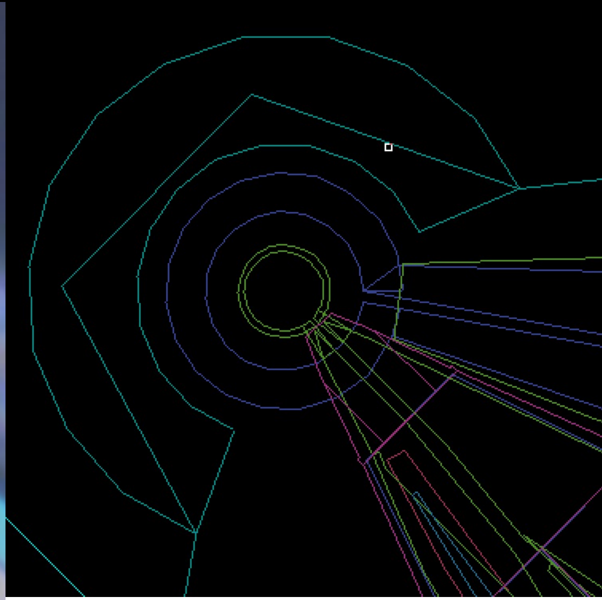
Nominal scanning height $z_0 = 5\mathrm{um}, \phi = 30°, \sigma = 30°$

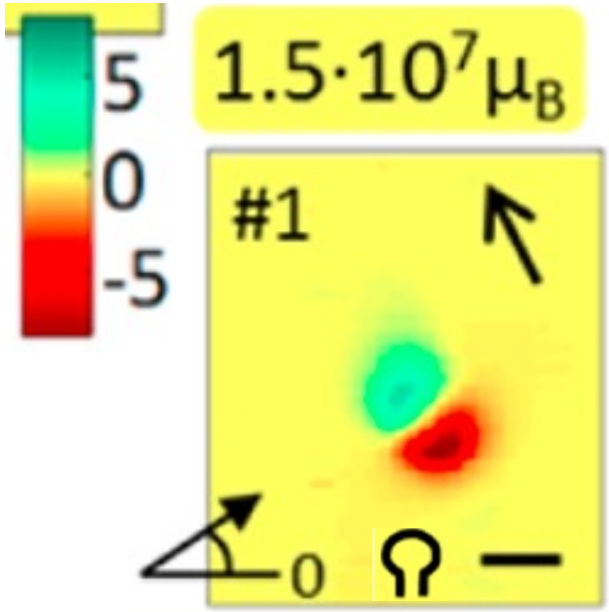
results

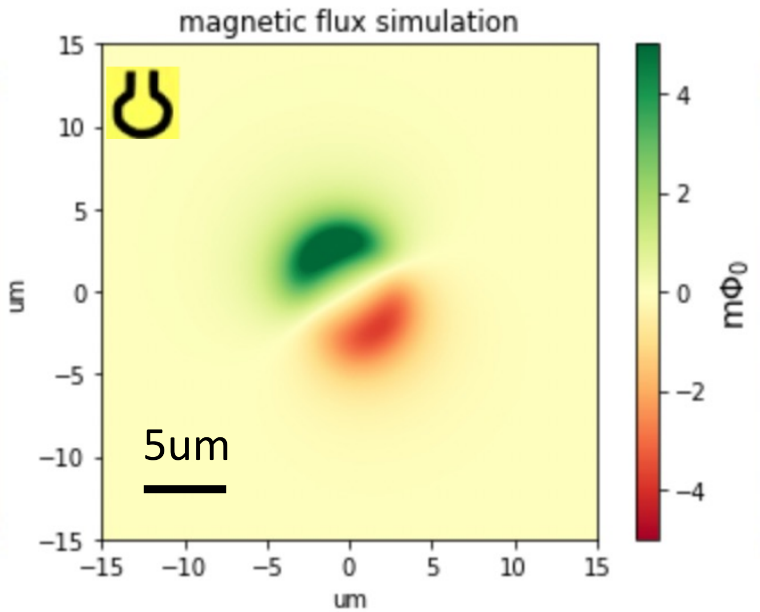
issue	Origin of issue
Simulations lack round feature	Dipole in Beena’s paper has a z component
Simulation peak to peak magnitude is smaller	I am not using the correct height (1.5um is more like it)

“The sensing area is ~7 µm away and 1 µm above the contact point ” – Beena’s paper

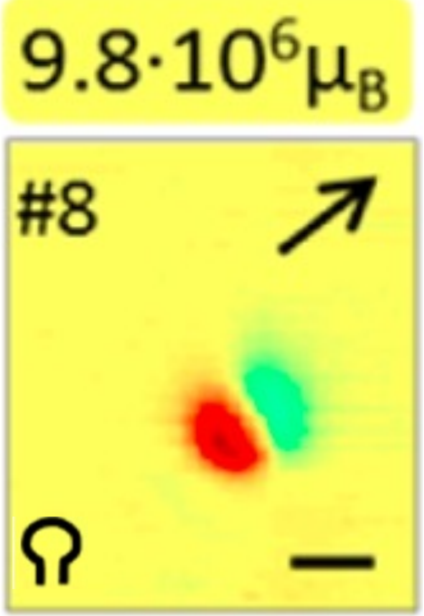


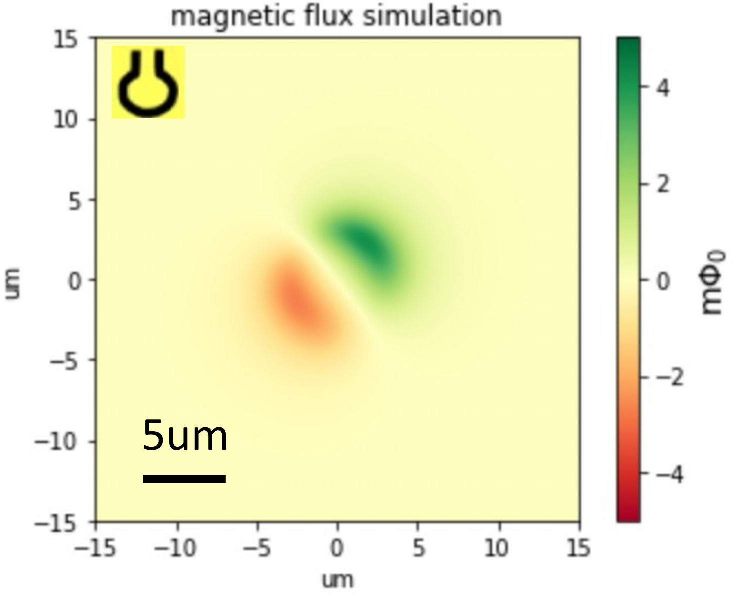






$\phi = 8°, z = 2um, \sigma = 2°$





$\phi = 8°, z = 2um, \sigma = 2°$

Beena’s dipole manipulation paper