收益率
Yield Rate

孟生旺
中国人民大学统计学院
http://blog.sina.com.cn/mengshw

主要内容

- 收益率与净现值
- 币值加权收益率
- 时间加权收益率
- 再投资与修正收益率
- 收益分配
  - 投资组合法
  - 投资年度法

收益率的定义

- **问题：** 已知一个项目的现金流，如何评价其收益的高低？
- **方法一：净现值法**（net present value，*NPV*）

$$NPV(i)=\sum_{t=0}^{n}v^{t}R_{t}$$

$$=\text{资金流入的现值}-\text{资金流出的现值}$$

- 净现值大于零，表示项目可行。
- 净现值越大，表示收益越高。

- **方法二：收益率法**
  - **收益率**（**yield rate**）：使得资金流入的现值与资金流出的现值相等时的利率 。
    - 也称为**内部报酬率**（internal rate of return，IRR）
  - 资金流入的现值与资金流出的现值之差就是净现值，所以收益率也是使得净现值等于零的利率：

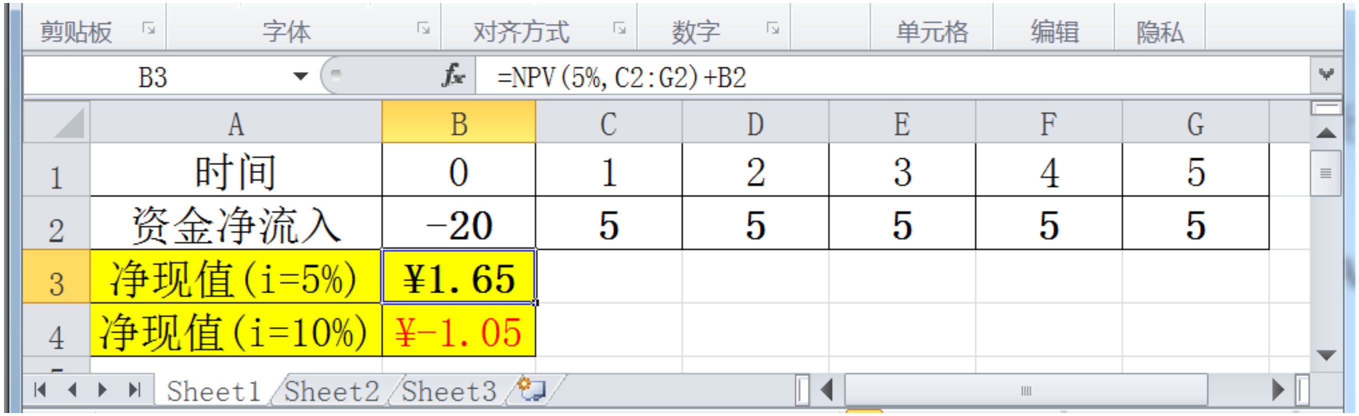
$$NPV(i) = \sum_{t=0}^{n} v^t R_t = 0$$

  - 收益率越高，表示项目的投资价值越高。

例： 如果期初投资20万元，可以在今后的5年内每年末获得5万元的收入。 假设投资者A的资金成本为5%，投资者B的资金成本为10%。 请通过净现值法和收益率法分别分析投资者A和投资者B的投资决策。

解： （1）该项目的净现值为 $-20+5\cdot a_{\overline{5}|}$

注：NPV函数中不含0点的现金流，需要单独加上。



剪贴板 字体 对齐方式 数字 单元格 编辑 隐私

B3 =NPV(5%,C2:G2)+B2

	A	B	C	D	E	F	G
1	时间	0	1	2	3	4	5
2	资金净流入	-20	5	5	5	5	5
3	净现值(i=5%)	¥1.65					
4	净现值(i=10%)	¥-1.05					

Sheet1 Sheet2 Sheet3

（2）令净现值等于零，即 $-20+5\cdot a_{\overline{5}|}=0$

- 可以计算出该项目的收益率为7.93%，
- 大于A的资金成本（5%），可行
- 小于B的资金成本（10%），不可行

剪贴板 字体 对齐方式 数字

B3 =IRR(B2:G2)

	A	B	C	D	E	F	G
1	时间	0	1	2	3	4	5
2	资金净流入	-20	5	5	5	5	5
3	收益率	7.93%					

Example

- Project P requires an investment of 4000 at time 0. The investment pays 1000 at time 1 and 4000 at time 2.
- Project Q requires an investment of x at time 2. The investment pays 2000 at time 0 and 4000 at time 1.
- The net present values of the two projects are equal at an interest rate of 10%.
- Calculate the yield rate of project Q.

t	0	1	2
P	-4000	1000	4000
Q	2000	4000	$-x$

Solution:

$$-4000 + 1000v + 4000v^2 = 2000 + 4000v - xv^2$$

$$\Rightarrow x = 6560$$

$$2000 + 4000v - 6560v^2 = 0 \quad \Rightarrow \quad i = 6.88\%$$

```
f = function(x) {
    v = 1.1^(-1)
    -4000 + 1000 * v + 4000 * v^2 - 2000 - 4000 * v + x * v^2
}
x0 = uniroot(f, c(0, 10000))$root
x0
## [1] 6560

f2 = function(i) {
    v = 1/(1 + i)
    2000 + 4000 * v - x0 * v^2
}
uniroot(f2, c(0, 1))$root
## [1] 0.0688
```

收益率的唯一性

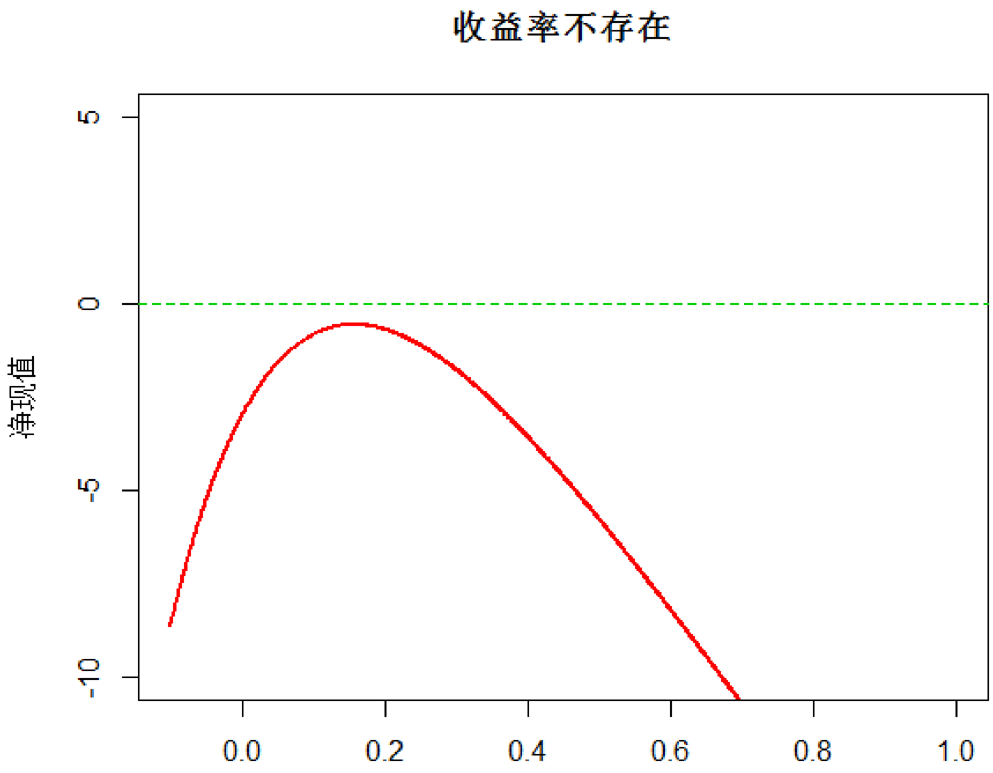
- **例（不存在）：** $R_0 = -100,\ R_1 = 230,\ R_2 = -133$，求收益率。
- **解:** **净现值为**

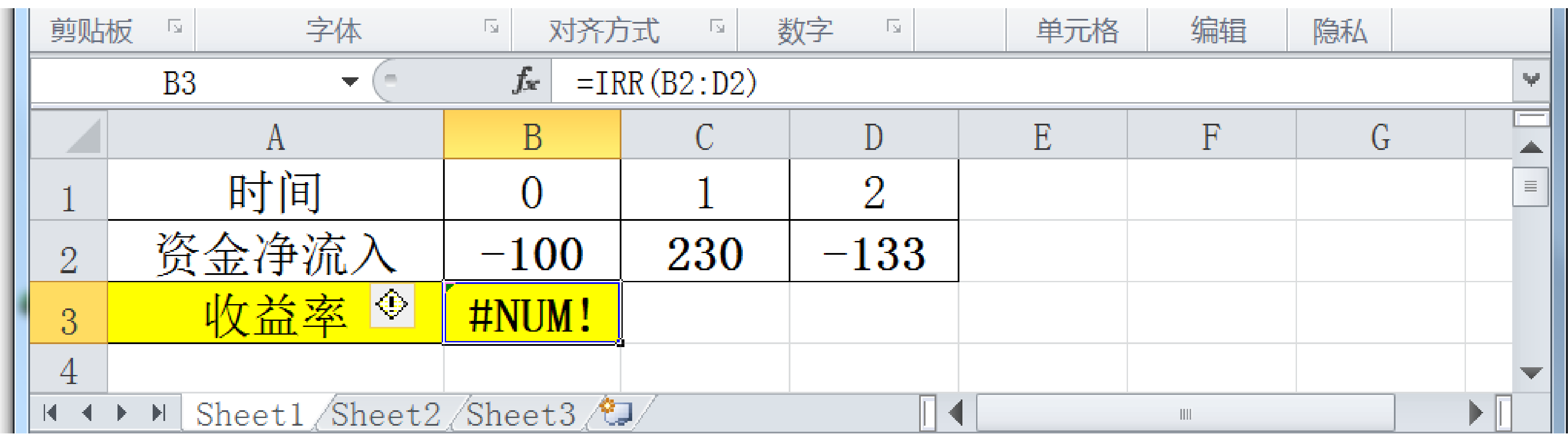
$$NPV(i) = -100 + 230v - 133v^2$$

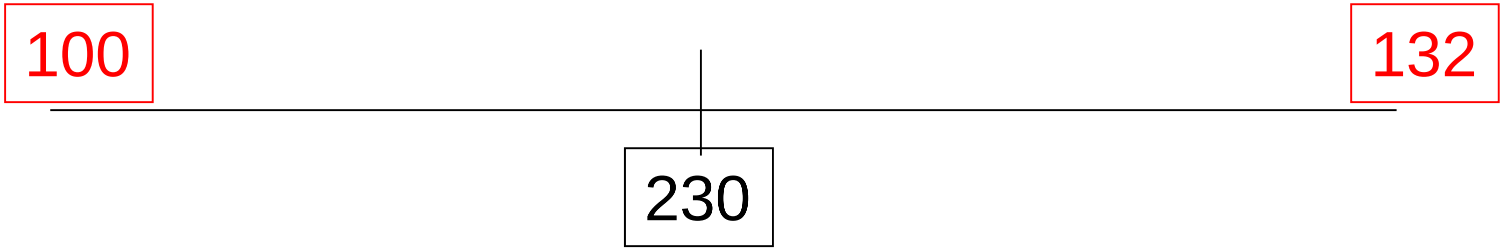
$$(1+i)^2 - 2.3(1+i) + 1.33 = 0$$

- 由于 $2.3^2 - 4 \times 1.33 = -0.03$，方程无实数解，不存在收益率（见下页图示）。

```
i=seq(-0.1,1,0.001)
v=(1+i)^(-1)
NPV=-100+230*v-133*v^2
plot(i,NPV,type='l',col=2,ylim=c(-10,5),lwd=2,xlab='利率',ylab='净现值',main='收益率不存在')
abline(h=0,col=3,lty=2)
```

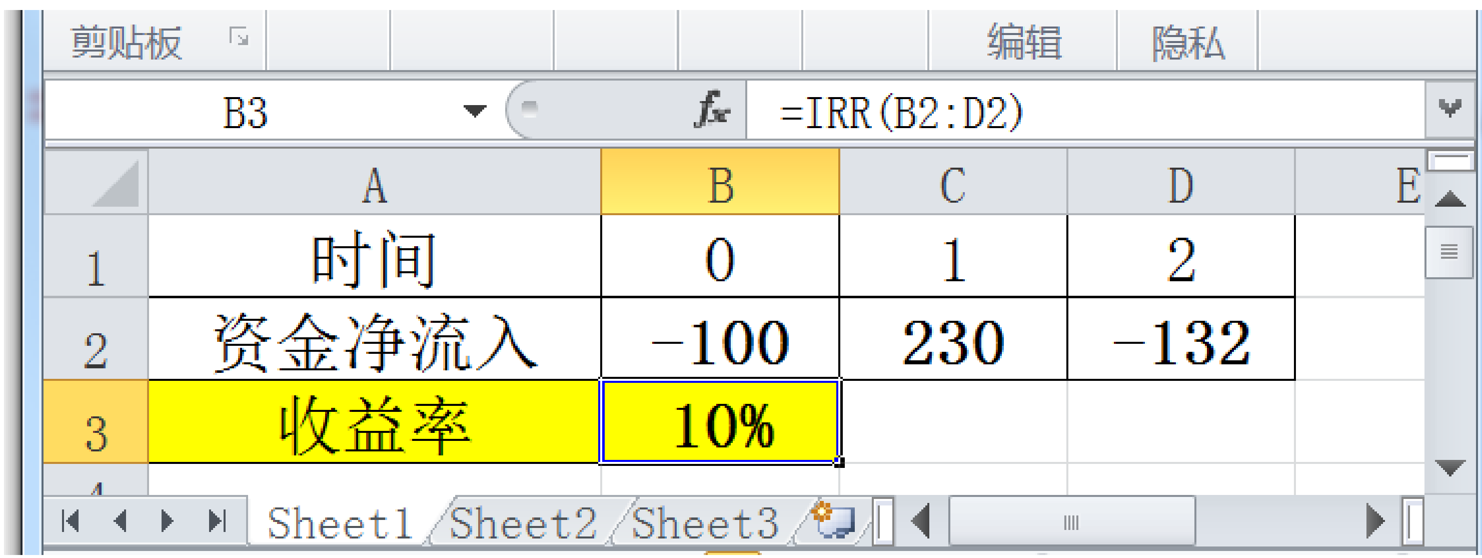






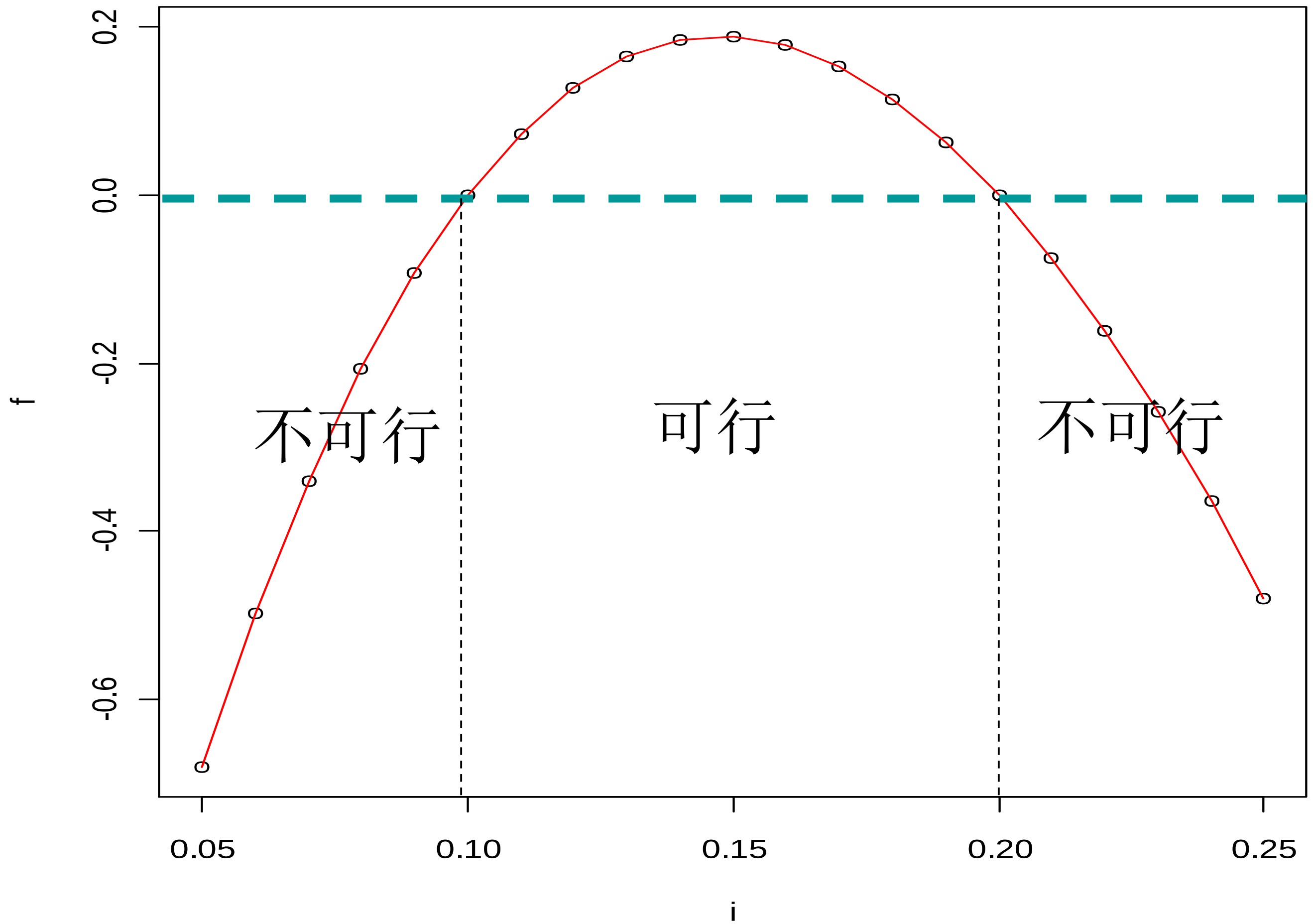
- **Example（不唯一）：** Consider a transaction in which a person makes payments of \$100 immediately and \$132 at the end of two years in exchange for a payment in return of \$230 at the end of one year.
- **解：** 价值方程为 $100+132v^2=230v$

$$\Rightarrow [(1+i)-1.1][(1+i)-1.2]=0 \quad \Rightarrow i=10\% \quad \text{or} \quad i=20\%$$



	A	B	C	D	E
1	时间	0	1	2	
2	资金净流入	−100	230	−132	
3	收益率	10%			

多重收益率下的净现值：$-100+230(1+i)^{-1}-132(1+i)^{-2}$



- 要求10%以下的收益率，不可行。
- 要求10%~20%的收益率，可行！
- 要求20%以上的收益率，不可行。

收益率唯一性的条件

- **准则一**：资金净流入只改变过一次符号。

- **准则二**：用收益率计算资金净流入的累积值，始终为负，直至最后一年末才等于零。

例：多重收益率（续前例）

#######多重收益率条件下的资金净流入net及其累积值cum

```r
t = 0:2
Out = c(100, 0, 132)   #流出
In = c(0, 230, 0)      #流入
net = In - Out         #净流入
i = 0.1                #多重收益率
cum = round(cumsum(net * (1 + i)^(-t)), 2)   #净流入的累积值
pander::pander(cbind(t, Out, In, net, cum))
```

t	Out	In	net	cum
0	100	0	-100	-100
1	0	230	230	109.1
2	132	0	-132	0

> #######多重收益率条件下的资金净流入net及其累积值cum

```
t = 0:2
Out = c(100, 0, 132)    #流出
In = c(0, 230, 0)       #流入
net = In - Out          #净流入
i = 0.2                 #多重收益率
cum = round(cumsum(net * (1 + i)^(-t)), 2)   #净流入的累积值
pander::pander(cbind(t, Out, In, net, cum))
```

t	Out	In	net	cum
0	100	0	-100	-100
1	0	230	230	91.67
2	132	0	-132	0

例：收益率是唯一的（计算过程见下页）

年度	资金流出	资金流入	资金净流入R_t	资金净流入的累积值
0	10		–10	–10.00
1	1		–1	–10.91
2	1	4	3	–8.43
3	1	4	3	–6.17
4	1	4	3	–4.12
5	1	4	3	–2.26
6		4	4	0
合计	15	20	5	

```r
t = 0:6
fout = c(10, rep(1, 5), 0)   #流出
fin = c(0, 0, rep(4, 5))     #流入
net = fin - fout             #净流入
f = function(i) {
    v = 1/(1 + i)
    sum(net * v^t)
}
i = uniroot(f, c(0, 1))$root
i  #收益率
## [1] 0.0998
cum = round(cumsum(net * (1 + i)^(-t)), 3)  #净流入的累积值
pander::pander(cbind(t, fout, fin, net, cum))
```

t	fout	fin	net	cum
0	10	0	-10	-10
1	1	0	-1	-10.91
2	1	4	3	-8.429
3	1	4	3	-6.174
4	1	4	3	-4.123
5	1	4	3	-2.259
6	0	4	4	0.001

基金的收益率：

基金的利息度量(Interest measurement of a fund)

- 币值加权收益率(dollar-weighted yield rate)：
  - 度量投资者的业绩
- 时间加权收益率(time-weighted yield rate)：
  - 度量基金经理人的业绩

币值加权收益率：精确计算

- 假设：期初的本金为A_0，在时刻t的新增投资为C_t，投资收益率为i，在期末的累积值可表示为（仅考虑一个时期）

$$A_0(1+i)+\sum_t C_t(1+i)^{(1-t)}$$

注：$C_t>0$表示增加投资；$C_t<0$表示减少投资。

用A_1表示期末的累积值，则有

$$A_0(1+i)+\sum_t C_t(1+i)^{(1-t)}=A_1$$

由此可求得收益率 i

币值加权收益率：近似计算

- 对于不足一个时期的新增投资，用单利近似复利:

$$A_0(1+i)+\sum_t C_t(1+i)^{(1-t)}$$

$$\approx A_0(1+i)+\sum_t C_t\left[1+(1-t)i\right]$$

$$=i\left[A_0+\sum_t C_t(1-t)\right]+(A_0+\sum_t C_t)\approx A_1$$

$$\Rightarrow \quad i\approx\frac{A_1-(A_0+\sum_t C_t)}{A_0+\sum_t C_t(1-t)}=\frac{I}{A_0+\sum_t C_t(1-t)}$$

对近似公式的解释： $$i \approx \frac{I}{A_0 + \sum_t C_t (1-t)}$$

（1）分母是加权计算的本金余额，以本金产生利息的时间长度为权数。

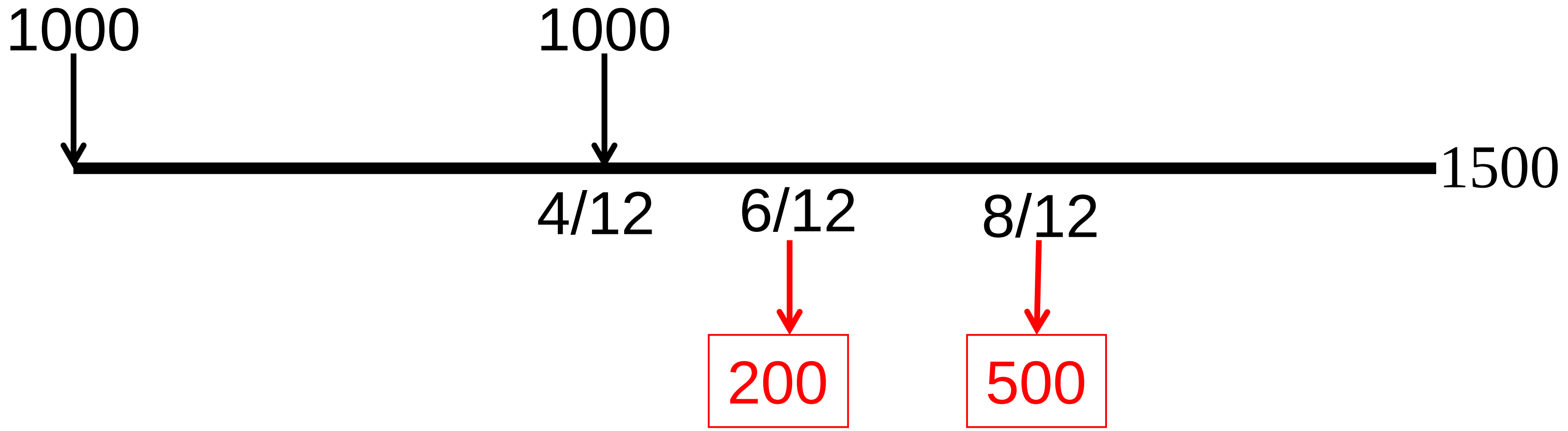
（2）如果假设新增投资发生在期中，即 $t = 0.5$，则

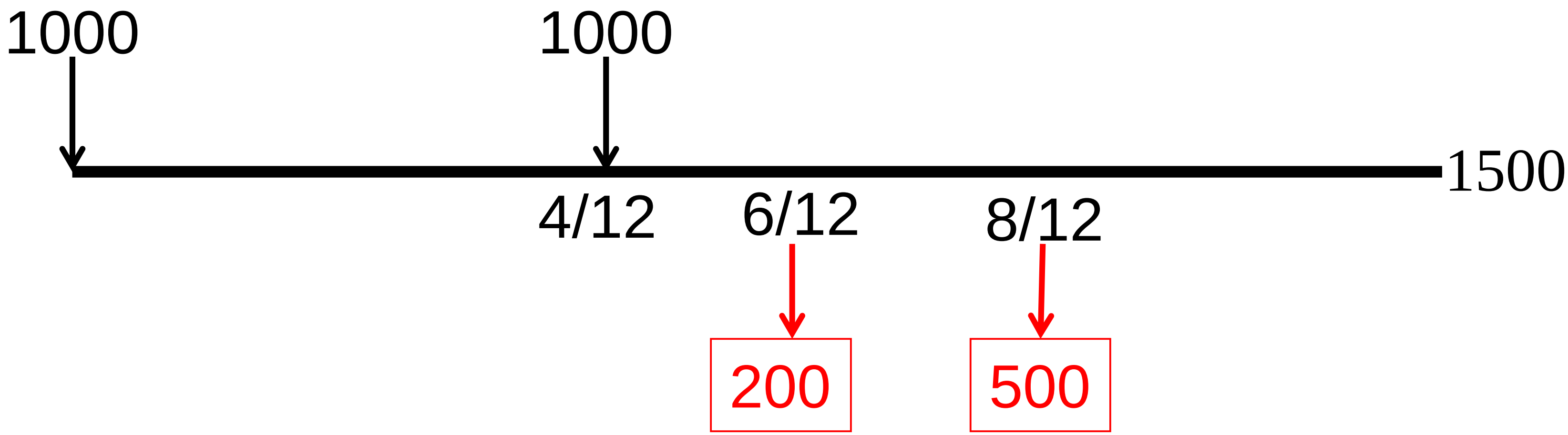
$$i \approx \frac{I}{A_0 + 0.5C_{0.5}} = \frac{I}{\left[A_0 + (A_0 + C_{0.5})\right]/2}$$

注：分母上是**期初本金**与**期末本金**的平均值

Example:

- At the beginning of the year, an investment fund was established with **an initial deposit** of 1000.
- A **new deposit** of 1000 was made at the end of 4 months.
- **Withdrawals** of 200 and 500 were made at the end of 6 months and 8 months, respectively.
- The amount in the fund at the end of the year is 1500.
- Calculate the dollar-weighted yield rate earned by the fund during the year.



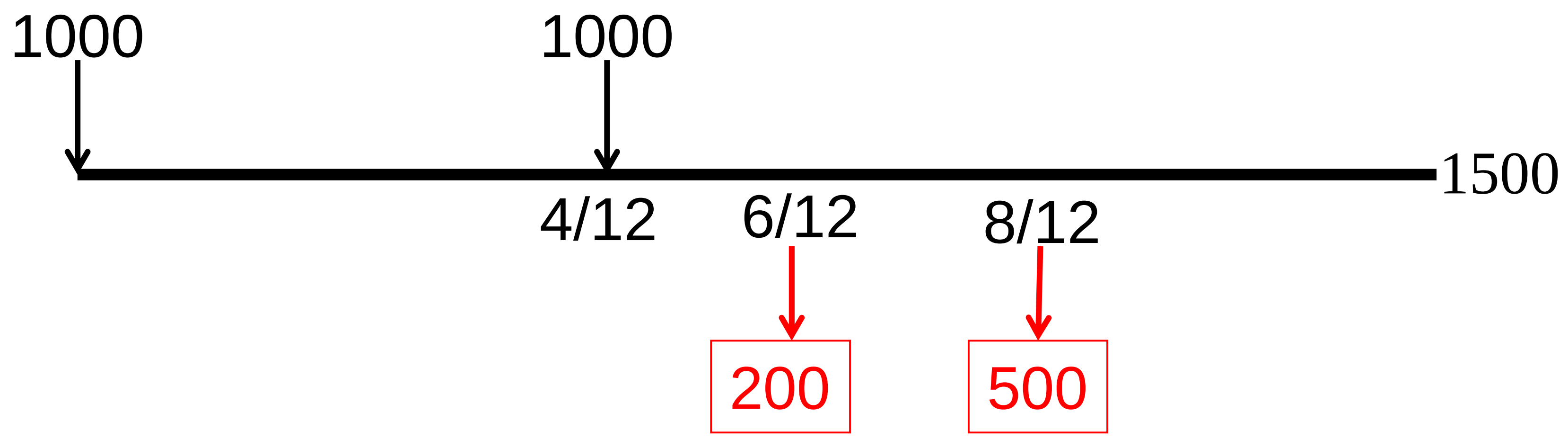


解：

- 年初本金A_0： 1000
- 新增投资 C_t： 1000，－200，－500   新增合计：C = 300
- 年末累积值 A_1：1500
- 当年利息： $I = A_1 - A_0 - C$ = 1500－1000－300 = 200

$$i = \frac{200}{1000 + 1000\left(1 - \frac{4}{12}\right) - 200\left(1 - \frac{6}{12}\right) - 500\left(1 - \frac{8}{12}\right)} = 14.29\%$$

精确计算？



$$1000(1+i)+1000(1+i)^{8/12}-200(1+i)^{6/12}-500(1+i)^{4/12}=1500$$

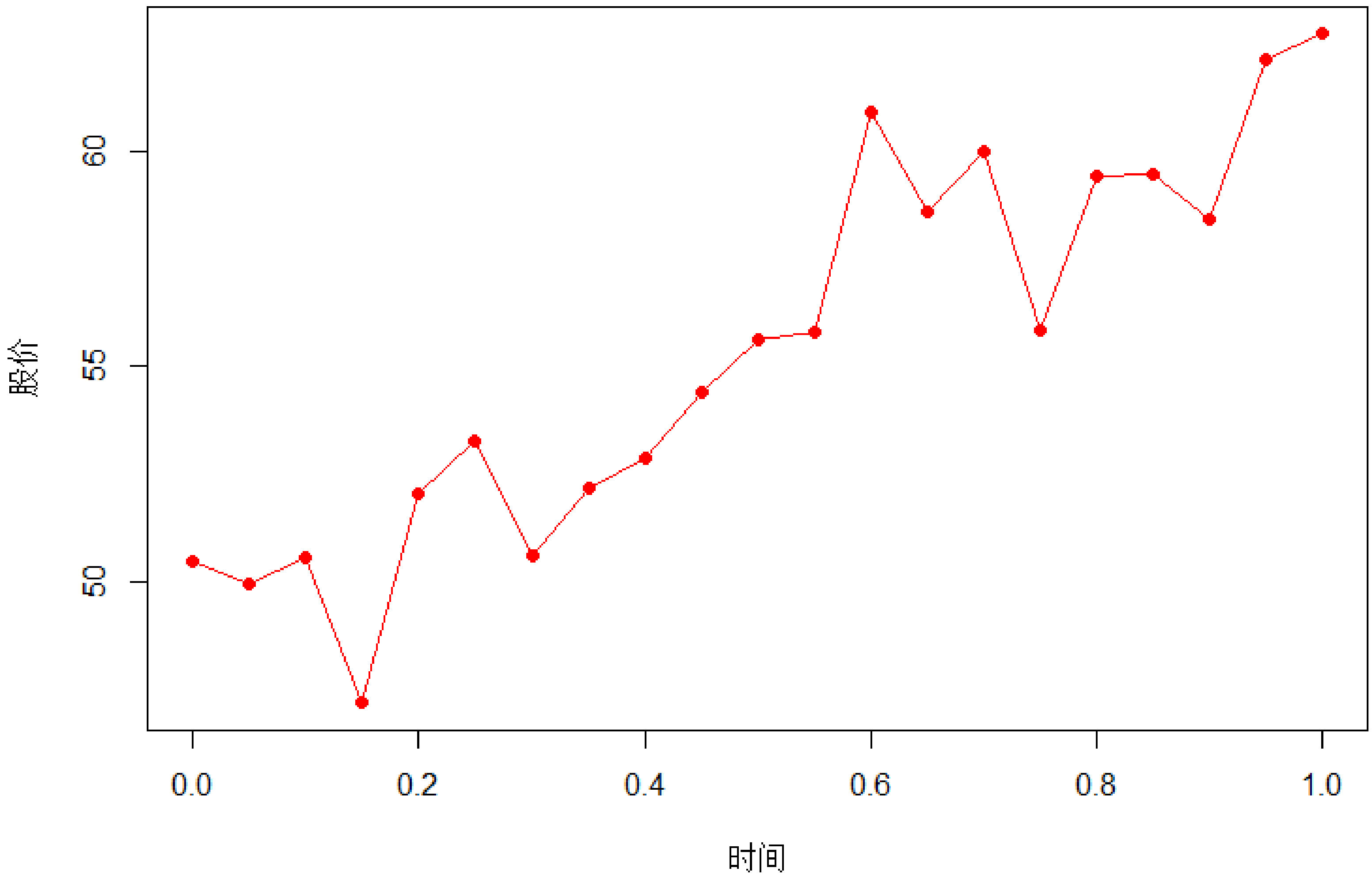
```r
f = function(i) {
    1000 * (1 + i) + 1000 * (1 + i)^(8/12) - 200 * (1 + i)^(6/12) -
        500 * (1 + i)^(4/12) - 1500
}
uniroot(f, c(0, 1))$root

## [1] 0.1433
```

精确计算结果14.33%，误差大约为0.4%

时间加权收益率（time-weighted rates of interest）

- 币值加权收益率：
  - 受本金增减变化的影响。而本金的增减由投资者决定。
  - 可以衡量投资者的收益，但不能衡量经理人的业绩。
- 时间加权收益率：扣除了本金增减变化的影响后所计算的收益率。 衡量经理人的业绩。

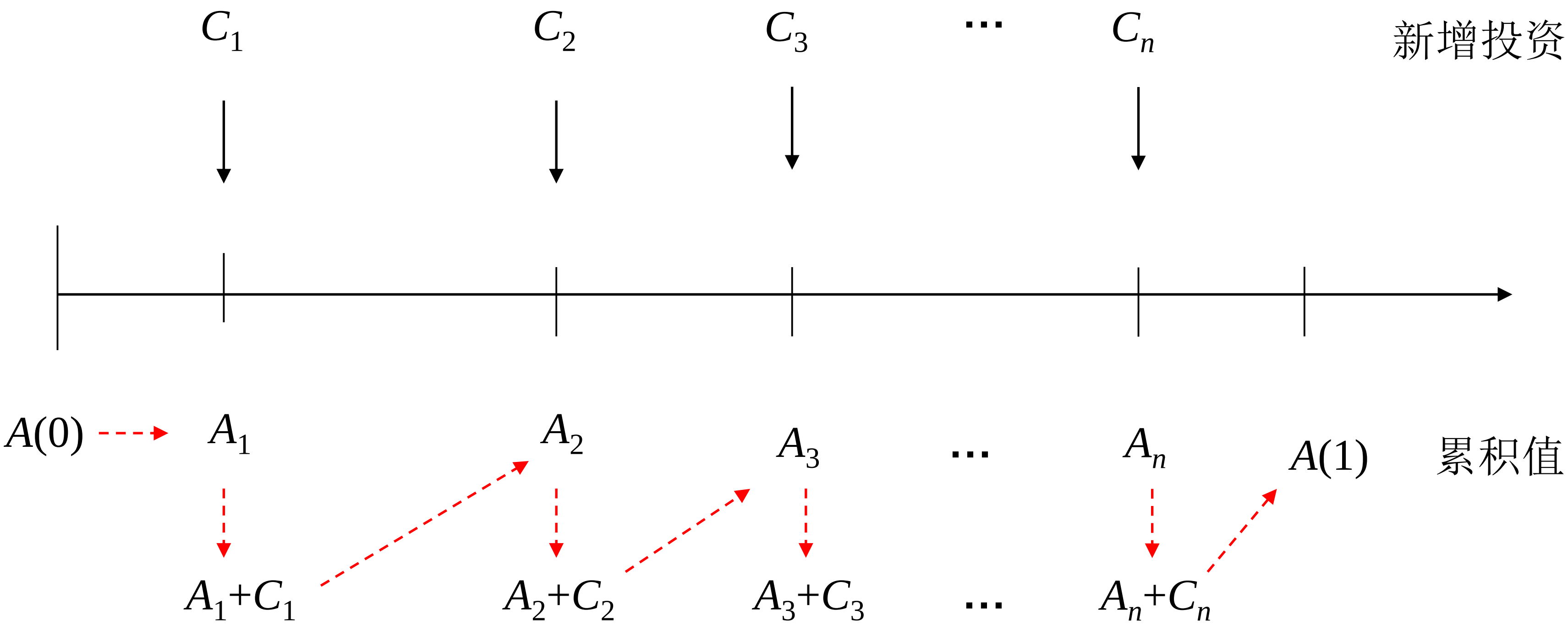
```
set.seed(111)
x=seq(0,1,0.05)
y=50+x*12+rnorm(length(x),0,2)
plot(x,y,type='o',xlab="时间",ylab='股价',col=2,pch=16)
```



时间加权收益率（*i*）的一般公式：

- 期初的本金为 $A(0)$
- 在第 k 个时间区间末的累积值为A_k
- 新增投资为 C_k（$k = 1, 2, \ldots, n$）
- 在年末 的累积值为 $A(1)$

仅考虑 1年的情形，$T = 1$



$$j_1 = \frac{A_1}{A(0)} - 1 \qquad j_2 = \frac{A_2}{A_1 + C_1} - 1 \qquad j_3 = \frac{A_3}{A_2 + C_2} - 1 \qquad j_{n+1} = \frac{A(1)}{A_n + C_n} - 1$$

收益率

$$(1+i) = (1+j_1)(1+j_2)\cdots(1+j_{n+1})$$

Example:

- On January 1 an investment account is worth \$100,000. On May 1 the value has increased to \$112,000 and \$30,000 of new principal is deposited. On November 1 the value has declined to \$125,000 and \$42,000 is withdrawn. On January 1 of the following year the investment account is again worth \$100,000. Compute the yield rate by:

  (1) the dollar-weighted method;

  (2) the time-weighted method.

Date:	1-1	5-1	11-1	1-1
Fund	100,000	112,000	125,000	100,000
C		30,000	-42,000	

- **解：** （1）币值加权收益率

由 $A_1 = A_0 + C + I$ 有

$$100000 = 100000 + (30000 - 42000) + I$$

$$I = 12000$$

故

$$i = \frac{12000}{100000 + \frac{8}{12} \times 30000 - \frac{2}{12} \times 42000} = 10.62\%$$

Date:	1-1	5-1	11-1	1-1
Fund	100,000	112,000	125,000	100,000
C		30,000	-42,000	

- （2）时间加权收益率

$$j=\left(\frac{112000}{100000}\right)\left(\frac{125000}{142000}\right)\left(\frac{100000}{83000}\right)-1 \quad =18.79\%$$

$\boxed{(1.12)}$ $\boxed{(0.88)}$ $\boxed{(1.20)}$

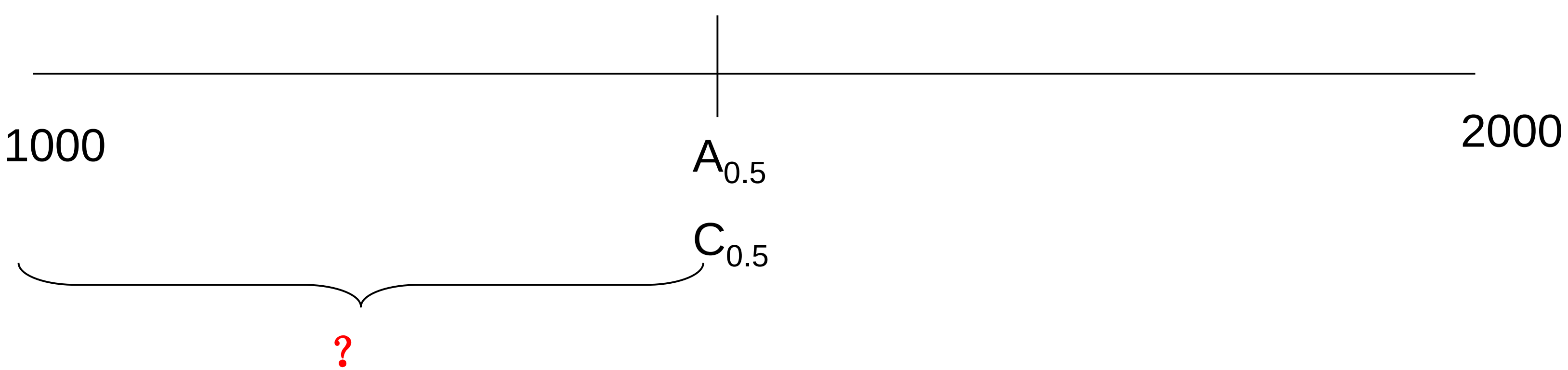
- **问题**：两种收益率差距为何这么大？10.62%与18.79%

Date:	1-1	5-1	11-1	1-1
Fund	100,000	112,000	125,000	100,000
C		30,000	-42,000	

1.12 0.88 1.20

Example:

- 1000 is deposited into a fund on January 1, 2013. Another deposit is made into the fund on July 1, 2013. On January 1, 2014, the balance in the fund is 2000.
- The time-weighted yield rate is 10% and the dollar-weighted yield rate is 9%.
- Calculate the annual effective interest rate earned on the fund during the first six month of 2013.



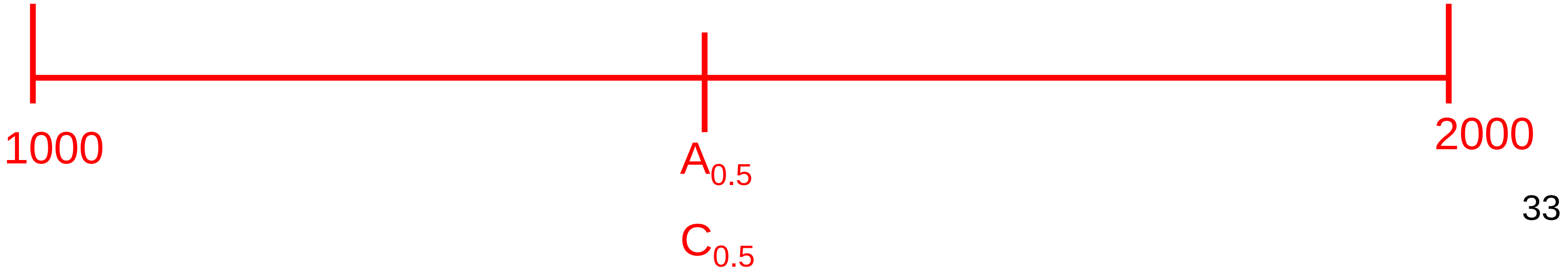
- **解**：币值加权收益率

$$0.09=\frac{2000-1000-C_{0.5}}{1000+0.5\times C_{0.5}} \Rightarrow C_{0.5}=870.81$$

时间加权收益率

$$1.10=\frac{A_{0.5}}{1000}\cdot\frac{2000}{A_{0.5}+C_{0.5}} \Rightarrow A_{0.5}=1064.32$$

故 $1000(1+i)^{0.5}=1064.32 \Rightarrow i=13.27\%.$



Exercise:

You are given the following information about the activity in two different investment accounts:

Account K			
Date	**Fund value before activity**	**Activity**	
		Deposit	**Withdrawal**
January 1, 2013	**100.0**		
July 1, 2013	**125.0**		x
October 1, 2013	**110.0**	$2x$	
December 31, 2013	**125.0**		

Account L			
Date	**Fund value before activity**	**Activity**	
		Deposit	**Withdrawal**
January 1, 20013	**100.0**		
July 1, 2013	**125.0**		x
December 31, 20013	**105.8**		

During 2013, the dollar-weighted return for investment account *K* equals the time-weighted return for investment account *L*, which equals *i*. Calculate *i*.

For account *K*:

Amount of interest $I = 125 - 100 - 2x + x = 25 - x$

$$i = \frac{25-x}{100-x\left(\frac{6}{12}\right)+2x\left(\frac{3}{12}\right)} = \frac{25-x}{100} \Rightarrow (1+i) = \frac{125-x}{100}$$

For account *L*:

$$(1+i) = \frac{125}{100} \times \frac{105.8}{125-x} = \frac{132.5}{125-x}$$

So $$\frac{125-x}{100} = \frac{132.5}{125-x} \Rightarrow x = 10$$

$i = 15\%$

再投资与修正收益率

- 再投资：前期投资的收入按新的利率再次进行投资。
- **例：**考虑两种可选的投资项目

  A）投资5年，每年的利率为9%

  B）投资10年，每年的利率为8%
- 如果两种投资在10年期间的收益无差异，项目A在5年后的再投资收益率应为多少？

$$(1+0.09)^5(1+i)^5=(1+0.08)^{10} \quad \Rightarrow \quad i=7.01\%$$

例：有一笔1000万元的贷款，期限为10年，年实际利率为 9%，有下面三种还款方式：

- 本金和利息在第10年末一次还清；
- 每年末偿还当年的利息，本金在第10年末归还。
- 在10年内每年末偿还相同的金额。

- 假设偿还给银行的款项可按7%的利率再投资，请比较在这三种还款方式下银行的年收益率。

解:

（1）无需再投资，收益率为 i＝9％

（2）所有付款在第10年末的累积值为

价值方程： $1000+90s_{\overline{10}|0.07}=2243.48$

$$1000(1+i)^{10}=2243.48 \Rightarrow i=8.42\%$$

（3）所有付款在第10年末的累积值为

$$\left[\frac{1000}{a_{\overline{10}|0.09}}\right]s_{\overline{10}|0.07}=2152.88$$

价值方程：

$$1000(1+i)^{10}=2152.85 \Rightarrow i=7.97\%$$

Exercise

- Mary invests 1000 at the end of each year for 5 years at an annual effective rate of 9%, and reinvests the interest at an annual effective rate of 9%. At the end of 5 years, her investment has value of X.
- John invests 1000 at the beginning of each year for 5 years at an annual effective rate of **10%** and reinvests the interest at an annual effective rate of **8%**. At the end of 5 years, his investment has value Y .
- Calculate $Y - X$ and yield rate of John's investment.

Solution:

- The future value at time 5 of Mary's investments is

$$X = 1000 s_{\overline{5}|9\%} = 5984.71$$

Balance in John's account	1000	2000	3000	4000	5000	$\boxed{5000}$
Time	0	1	2	3	4	5

Hence, the interest payments John gets are:

John's interest payments	0	100	200	300	400	500
Time	0	1	2	3	4	5

The accumulation of these interest payments at time 5 is:

$$Y = 100(Is)_{\overline{5}|8\%} = 100 \times 1.08^5 (Ia)_{\overline{5}|8\%} = 100 \times 1.08^5 \frac{\ddot{a}_{\overline{5}|0.08} - 5(1.08)^{-5}}{0.05} = 1669.91$$

The total accumulation of John's investments is 6669.91.

yield rate of John's investment:

$$1000\ddot{s}_{\overline{5}|} = 6669.91 \Rightarrow i = 9.7643\%$$

Exercise:

- At time $t = 0$, John invests 2000 in a fund earning annual effective rate of 8%.
- He reinvests each interest payment in individual separate funds each earning annual effective rate of 9%.
- The interest payments from the separate funds are accumulated in a side fund that guarantees an annual effective rate of 7%.
- Determine the total value of all funds at $t = 10$.

时间	0	1	2	3		9	10	10年末的累积值
fund	2000							2000
separate funds		160						
			160					
				160				
					…			
						160		
							160	160*10
side fund			160*9%	2*160*9%			9*160*9%	$160*9\%*(Is)_9$

Solution:

- The interest earned each year is (2000)(0.08) = 160.
- The interest obtained from 160 invested at an effective annual rate of 9 % is (160)(0.09) =15.4.
- So, the cash flow going into the account paying an annual effective rate of 7% is

time	0	1	2	3	…	10
contributions	0	0	15.4	2(15.4)	…	9(15.4)

The future value of this cash flow is

$$15.4(Is)_{\overline{9}|7\%} = 15.4(1+7\%)^9 (Ia)_{\overline{9}|7\%} = 839.62$$

The total value of all funds is

$$2000 + (10)(160) + 839.62 = 4439.62$$

修正收益率:

- 如果再投资的利率与筹集资金的利率不同时，则评价项目应该使用修正收益率（modified rate of internal rate）。

- 在计算修正收益率时，对资金流出使用筹资的利率计算其现值，而对资金流入使用再投资的利率计算其累积值。

例：投资者以8%的利率借入资金进行投资，在时点0的投资额为10000元，在第2年末的投资额为11550元。在第1年末获得了21500元的收益。投资者对收益进行再投资的利率为5%。请计算该项投资的修正收益率。

【解】筹资利率为8%，资金流出的现值为

$$10000+11550(1+8\%)^{-2}=19902.26$$

投资收入以5% 的利率再投资，故在第2年末的累积值为

$$21500\times(1+5\%)=22575$$

令修正收益率为 i，则有

$$19902.26\times(1+i)^2=22575 \Rightarrow i=6.5\%$$

计算修正收益率的EXCEL函数：

=MIRR({-10000, 21500, -11550}, 8%, 5%)

Exercise（课外）:

- Payments of 1000 are invested at the end of each year for 5 years.
- The payments earn interest at an annual effective rate of 10%.
- The interest can be reinvested at an annual effective rate of 6% in the first 4 years and at an annual effective rate of k there after.
- The amount in the fund at the end of 5 years is 6090.
- Calculate k.

Solution :

Balance in the 1st account	1000	2000	3000	4000	5000
Time	1	2	3	4	5

Hence, the interest payments are:

Cashflow of interest payments	0	100	200	300	400
Time	1	2	3	4	5

The total accumulation for both accounts at time 5 is

$$6090 = 5000 + 100\,(Is)_{\overline{3}|0.06}\,(1+k) + 400.$$

k = 10.51%

$$\boxed{(Is)_{\overline{n}|} = (1+i)^n (Ia)_{\overline{n}|} = (1+i)^n \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}}$$

Exercise（课外）

- Eric deposits 12 into a fund at time 0 and an additional 12 into the same fund at time 10.
- The fund credits interest at an annual effective rate of i.
- Interest is payable annually and reinvested at an annual effective rate of $0.75i$.
- At time 20, the accumulated amount of the reinvested interest payments is equal to 64.
- Calculate i, $i > 0$.

Solution: *The cahsflow of interest is*

Payments	$12i$	$12i$	$12i$	$\cdots$	$12i$	$24i$	$24i$	$\cdots$	$24i$
Time	1	2	3	$\cdots$	*10*	11	12	$\cdots$	*20*

The future value at time 20 of this cash flow is

$$12i \times s_{\overline{20}|0.75i} + 12i \times s_{\overline{10}|0.75i} = 64 \Rightarrow i = 9.57\%$$

Exercise:

- Victor invests 300 into a bank account at the beginning of each year for 20 years.
- The account pays out interest at the end of every year at an annual effective interest rate of i .
- The interest is reinvested at an annual effective rate of $(i/2)$.
- The yield rate on the entire investment over the 20 year period is 8% annual effective.
- Determine i .

Solution:

Since the yield rate on the entire investment over the 20 year period is 8% annual effective, the accumulated value at the end of 20 years is $300\ddot{s}_{\overline{20|}0.08} = 14826.87.$

The cash flow of the interests is

$Payments$	$300i$	$2(300i)$	$3(300i)$	$\cdots$	$20(300i)$
$Time$	1	2	3	$\cdots$	20

So, the future value at time 20 of this cashflow is

$$300i(Is)_{\overline{20|}i/2} = 300i(1+i)^{20}(Ia)_{\overline{20|}i/2} = 14826.87 \Rightarrow i = 10\%$$

Exercise:

- Susan invests Z at the end of each year for seven years at an annual effective interest rate of 5%. The interest credited at the end of each year is reinvested at an annual effective rate of 6%. The accumulated value at the end of seven years is X .
- Lori invests Z at the end of each year for 14 years at an annual effective interest rate of 2.5%. The interest credited at the end of each year is reinvested at an annual effective rate of 3%. The accumulated value at the end of 14 years is Y .
- Calculate Y/X .

Solution:

We may assume that $Z = 1$. Susan's accumulated value consists of her \$7 invested plus the cashflow for her interest. The cashflow of her interest payments are

Payments	0	0.05	2(0.05)	$\cdots$	6(0.05)
Time	1	2	3	$\cdots$	7

The accumulated value of this cash flow is

$$0.05(Is)_{\overline{6}|6\%} = 0.05(1+6\%)^6(Ia)_{\overline{6}|6\%} = 1.1653$$

So, Susan's accumulated value is $X = 8.1653$.

Lori’s accumulated value consists of her \$14 invested plus the cash flow for her interest. The cash flow of her interest payments are

$Payments$	0	0.025	2(0.025)	$\cdots$	13(0.025)
$Time$	1	2	3	$\cdots$	14

The accumulated value of this cash flow is

$$0.025(Is)_{\overline{13}|3\%} = 0.025(1+3\%)^{13}(Ia)_{\overline{13}|3\%} = 2.5719$$

So, Susan’s accumulated value is Y = 16.5719.
We have that Y/X = 2.0296.

收益分配(Allocating investment income)

- 基金（Fund）：从广义上说，基金是指为了某种目的而设立的具有一定数量的资金。如：信托投资基金、公积金、保险基金、退休基金，各种基金会的基金。
- 狭义：证券投资基金。
  - 分类：
    - 开放型基金，封闭型基金
    - 股票基金，债券基金，货币市场基金，混合基金

- **基金的收益：**
  - 收入：
    - 利息（债券，银行存款）
    - 股息
    - 资本利得（股票价格上涨）
  - 收入减去费用后是基金的**净收益**，可供基金持有人分配。
- **问题：**
  - 基金包括不同时期的投资。
  - 如何把基金的收入分配给不同时期的投资？
- **分配方法：**
  - 投资组合法(portfolio method)
  - 投资年度法(investment method)

投资组合法(portfolio methods)

- **适用情况：**基金的收益率水平一直保持恒定
- **方法：**按照基金的平均收益率分配投资收入。
- **例：**如果基金的收益率一直保持在6%的水平，某个投资者的投资额是10000元，存入基金的时间是9个月，那么应该分配给他的利息收入应该是

复利法：$10000(1 + 0.06)^{9/12} - 10000 = 446.71$

单利法：$10000 \times 6\% \times 9/12 = 450$

- **问题**：投资期限较长时，可能不公平。
- **例**：
  - **假设**：基金去年的收益率为8％，今年的收益率为10％，平均为8.5%。
  - 如果对今年的新投资按8.5％分配收益，对新投资者不公平，也不能吸引新投资。
- **如何解决？**用**投资年度**方法分配收益。如：
  - 今年初的新增投资，按10%分配收益。
  - 去年初的投资，本金按照8%分配收益，但其利息在今年再投资按照10%分配收益，相当于平均按照8.148%分配收益，即(0.08+0.08*0.1)/1.08=8.148%

投资年度法(investment year method，new year method)

- **适用情况：**收益率波动较大。
- **方法：**新投入的资金在若干年内按投资年度利率分配收益，超过一定年数以后，再按组合利率分配收益。
  - **投资年度利率：**投资的年度不同，利率不同。
  - **组合利率：**不同年度的投资享有的平均利率。

投资年度利率：例

投资发生的日历年度	投资年度利率(%)					组合利率(%)	组合利率的日历年度
x	i_0^x	i_1^x	i_2^x	i_3^x	i_4^x	i_{x+5}	$x+5$
2010	6.50	6.53	6.56	6.62	***6.72***	7.45	2015
2011	7.00	7.00	7.03	***7.09***	7.24	7.47	2016
2012	7.00	7.01	***7.06***	7.18	7.12	7.47	2017
2013	7.20	***7.22***	7.30	7.26	7.22		
2014	***7.50***	7.53	7.50	7.46			
2015	8.00	7.96	7.90				
2016	7.50	7.49					
2017	7.30						

Exercise：

- You are given the following table of interest rates.
- A person deposits 1000 on January 1, 2007. Let the following be the accumulated value of the 1000 on January 1, 2010:
  - *P*: under the investment year method
  - *Q*: under the portfolio yield method
  - *R*: where the balance is withdrawn at the end of every year and is reinvested at the new money rate.
- Calculate *P*, *Q*, and *R*.

Calendar Year of Original Investment	Investment Year Rates (in %)					Calendar year of portfolio rate	Portfolio Rates (in %)
2002	8.25	8.25	8.4	8.5	8.5	2007	8.35
2003	8.5	8.7	8.75	8.9	9.0	2008	8.6
2004	9.0	9.0	9.1	9.1	9.2	2009	8.85
2005	9.0	9.1	9.2	9.3	9.4	2010	9.1
2006	9.25	9.35	9.5	9.55	9.6	2011	9.35
2007	9.5	9.5	9.6	9.7	9.7		
2008	10.0	10.0	9.9	9.8			
2009	10.0	9.8	9.7				
2010	9.5	9.5					
2011	9.0						

- **Solution:**
  - P = 1000(1.095)(1.095) (1.096) =1314.13
  - Q =1000 (1.0835) (1.086) (1.0885) = 1280.82
  - R = 1000 (1.095)(1.10) (1.10) =1324.95

Exercise : An investment of 100 is made at the beginning of years 2000, 2001, and 2002. The total amount of investment interest credited by the fund during the year 2003 is equal to 28.40. Find *r*.

Calendar year of investment	Investment year rates			Calendar year of portfolio rate	Portfolio rate
	i_1	i_2	i_3		
2000	10%	10%	r	2003	8%
2001	12%	5%	10%	2004	$r-0.01$
2002	8%	$r-0.02$	12%	2005	6%
2003	9%	11%	6%	2006	9%
2004	7%	7%	10%	2007	10%

- 解：

- 2000年初的投资到2003年初的积累值为：
  $100(1.10)(1.10)(1+r)=121(1+r)$
  在2003年的利息收入： $121(1+r)(0.08)$
- 2001年初的投资到2003年初的积累值为$100(1.12)(1.05)=117.6$
  在2003年的利息收入为 $117.6(0.1)=11.76$
- 2002年初的投资到2003年初的积累值为：$100(1.08)=108$
  在2003年的利息收入为 $108(r-0.02)$
- 总利息收入：$121(1+r)(0.08)+11.76+108(r-0.02)=28.4$
  故 $r=7.75\%$