Component Contribution to Standard Deviation

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一、理论基础-马科维茨资产组合理论

假设资产组合有 n 种风险资产，它们的收益率是随机变量$r_1, r_2 \ldots \ldots r_n$。投资者投资此n种风险资产的权重为$\mathrm{W} = (W_1, W_2, \ldots \ldots W_N)$，其中实数$W_i$代表第 i 种证券的价值在总价值中所占的比重，因此，$W_1 + W_2 + \ldots \ldots W_n = 1$。两种资产收益率的协方差记为$\sigma_{ij} = COV(r_i, r_j), i, j = 1,2, \ldots \ldots n$，相应的协方差矩阵记为$\sum(\sigma_{i,j})_{m\times n}$：

$$\sum(\sigma_{i,j})_{m\times n} = \begin{bmatrix} \sigma_{1,1} & \cdots & \sigma_{1,n} \\ \vdots & \ddots & \vdots \\ \sigma_{n,1} & \cdots & \sigma_{n,n} \end{bmatrix} \quad (1)$$

则，组合收益率为：

$$r_p = \sum_{i=1}^{n} W_i r_i \qquad (2)$$

组合方差为：

$$\sigma_p^2 = W^T \sum W = \sum_{i=1}^{n} W_i^2\, \sigma_i^2 + \sum_{\substack{i,j=1 \\ i\neq j}}^{n} W_i\, W_j \sigma_{ij} \quad (3)$$

风险资产i对组合方差的边际贡献是：

$$\frac{\partial \sigma_p^2}{\partial W_i} = \frac{\partial \left(\sum_{i=1}^{n} W_i^2\, \sigma_i^2 + \sum_{\substack{i,j=1 \\ i\neq j}}^{n} W_i\, W_j \sigma_{ij} \right)}{\partial W_i} = 2W_i\sigma_i^2 + 2\sum_{j=1, j\neq i}^{n} W_j \sigma_{ij} = 2\sum_{j=1}^{n} W_j \sigma_{ij}$$

$$= 2COV(r_i, r_p) \quad (4)$$

风险资产 i 对组合标准差的边际贡献是：

$$\frac{\partial \sigma_p}{\partial W_i} = \frac{1}{2\sigma_p} \frac{\partial \sigma_p^2}{\partial W_i} = \frac{COV(r_i, r_p)}{\sigma_p} \quad (5)$$

结论：每种风险资产对组合标准差的边际贡献是其与组合的协方差与组合标准差之比。

进一步探讨风险资产对组合标准差的成分贡献：

$$conponent\_contribution_i = W_i \times \frac{\partial \sigma_p}{\partial W_i} \quad (6)$$

加总各风险资产对组合标准差的成分贡献可得：

$$\sum_{i=1}^{n} conponent\_contribution_i = \frac{\sum_{i=1}^{n} W_i\, COV(r_i, r_p)}{\sigma_p} = \frac{COV(r_P, r_p)}{\sigma_p} = \sigma_p \quad (7)$$

结论：各风险资产对组合标准差的成分贡献之和即为组合标准差。

二、程序设计

（一）收益率数据获取

1.从 yahoo 获取价格数据

```
#输入的股票代码
symbols <- c(input$stock1,input$stock2,
                input$stock3,input$stock4,input$stock5)
#输入始末时间带入
prices <-getSymbols(symbols,src = 'yahoo',from = input$date_start,
        to = input$date_end,auto.assign = TRUE,warnings = FALSE)%>%
        map(~Ad(get(.)))%>%
        reduce(merge)%>%
        `colnames<-`(symbols)
```

2.讲价格数据转换为 tibble 格式的月度收益率数据

```
asset_returns_dplyr_byhand <-
        prices%>%
        to.monthly(indexAt = "lastof",OHLC = FALSE)%>%
        tk_tbl(preserve_index = TRUE,rename_index = "date")%>%
        gather(asset,returns,-date)%>%
        group_by(asset)%>%
        mutate(returns = log(returns)- log(lag(returns) ))%>%
        spread(asset,returns)%>%
        select(date,symbols)%>%
```

```
slice(-1)
```

（二）构建指定期间的资产对组合标准差成分贡献方程

1.给定收益率和权重计算资产对组合标准差的成分贡献

```
component_contr_matrix_fun <- function(return,w){
    # create covariance matrix
    covariance_matrix = cov(return)
    # calculate portfolio standard deviation
    sd_portfolio<-sqrt(t(w)%*%covariance_matrix%*%w)
    # calculate marginal contribution of each asset
    margin_contribution<- w%*%covariance_matrix/sd_portfolio[1,1]
    # multiply marginal by weights vecoter
    conponent_contribution<-margin_contribution*w
    # divide by total standard deviation to get percentages
    conponent_percentages<-conponent_contribution/sd_portfolio[1,1]
    #convert the type
    conponent_percentages%>%
      as_tibble()%>%
      gather(asset,contribution)
```

2.计算给定区间的资产对组合标准差的成分贡献

```
  ##function for calculating interval sd
  interval_sd_by_hand<- function(return_df,start=1, window=24 ,weights){
    #funtion 参数赋值是默认值
    # start =1
    # window<-24
    # return_df<-asset_returns_dplyr_byhand
    start_date <-
      return_df$date[start]
    end_date <-
      return_df$date[c(start+window)]
    return_to_use <-
      filter(return_df,date>=start_date&
                     date<end_date)%>%
      select(-date)
    w<-weights
    component_percentages <- component_contr_matrix_fun(return_to_use,w)
    results_with_date <-
      component_percentages%>%
```

```
    mutate(date = ymd(end_date))%>%
    select(date,everything())%>%
    spread(asset,contribution)%>%
    mutate_if(is.numeric,function(x) x*100)
```

（三）计算风险资产对组合标准差的成分贡献

1.整个时间区间上资产对组合标准差的成分贡献

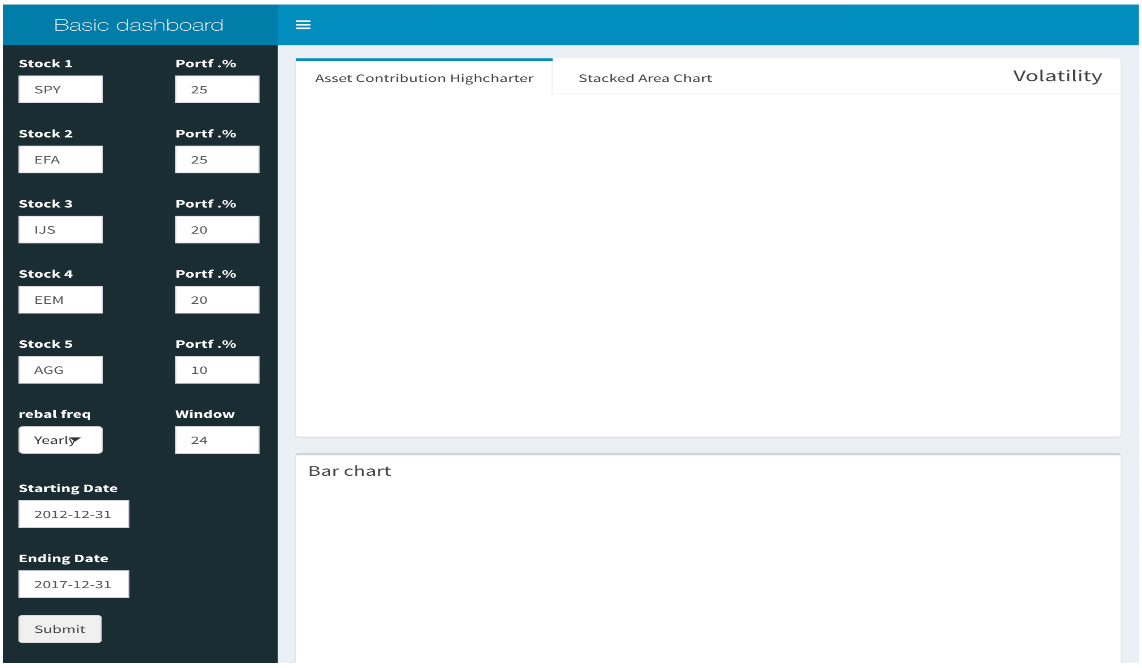
```
asset_return_xts <-asset_returns_dplyr_byhand()%>%
    tk_xts(date_var = date,
           silent = TRUE)
  w <- c(input$w1/100,input$w2/100, input$w3/100, input$w4/100, input$w5/100)
  port_vol_contr_total_builtin <-
    StdDev(asset_return_xts, weights = w, portfolio_method = "component")
  symbols <- c(input$stock1, input$stock2, input$stock3, input$stock4, input$stock5)
  percentages_tibble_pre_built <-
    port_vol_contr_total_builtin$pct_contrib_StdDev %>%
    tk_tbl(preserve_index = FALSE) %>%
    mutate(asset = symbols) %>%
    rename('risk contribution' = data) %>%
    mutate(`risk contribution` =
             round(`risk contribution`, 4) * 100,weights = w * 100) %>%
    select(asset, everything())
```

2.给定窗口条件下滚动时间期间的资产对组合标准差的成分贡献

```
asset_returns_dplyr_byhand <- asset_returns_dplyr_byhand()%>%
    tk_xts(date_var = date,
           silent = TRUE)
  asset_returns_dplyr_byhand<-asset_returns_dplyr_byhand%>%
    tk_tbl(preserve_index = TRUE,rename_index = "date")
  w <- c(input$w1/100,input$w2/100,
          input$w3/100, input$w4/100, input$w5/100)
  window <-input$window
  ##rolling function
  portfolio_vol_components_xts<-
    map_df(1:(nrow(asset_returns_dplyr_byhand)-window),
           interval_sd_by_hand,
           return_df = asset_returns_dplyr_byhand,
           weights=w,
           window= window)%>%
```

```
tk_xts(data_var = date,
       silent = TRUE)
```

（四）结果可视化



本小组提供了一个可交互的页面，可以根据用户需要获得相应的图表进行分析。

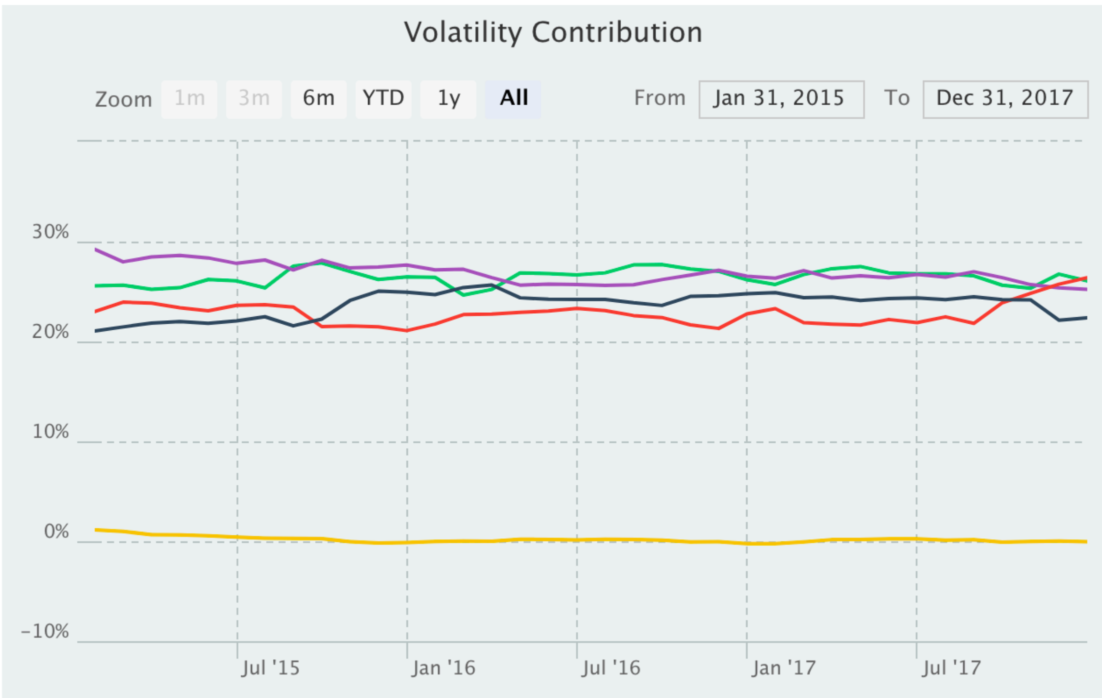
https://rucquant.shinyapps.io/10-Conponent-Contribution-to-Standard-Deviation/

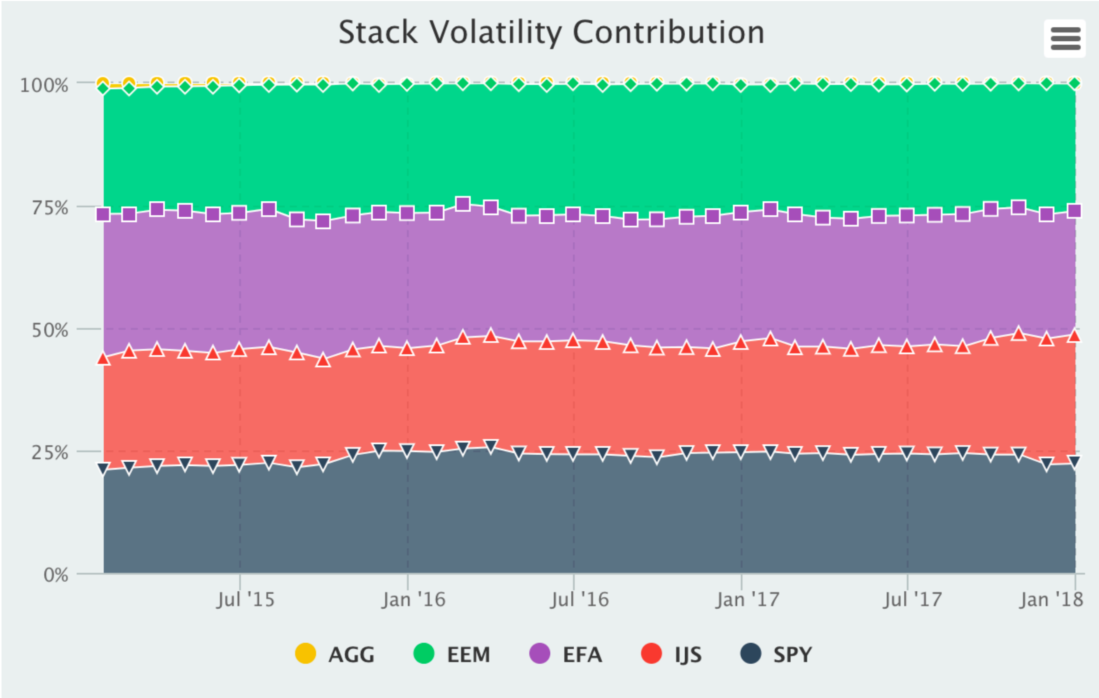
注：frequency 为 yearly 时，容易忽略的是，时间长度必须大于窗口长度，否则会报错。

代码文件已上传至本团队 github，链接如下：

https://github.com/Rucer-quant/10-Conponent-Contribution-to-Standard-Deviation/blob/master/componentcontribution.R

（1）24 个月为窗口的折线图和堆积面积图





（2）整个时间区间的资产对组合标准差成分贡献的柱状图