Using Chernoff Bound in the Proof of Lemma 6 of Approximate Similarity Search Under Edit Distance using Locality-Sensitive Hashing

Lemma 6. For any string x of length d, $Pr_\rho[\tau(x,\rho) is complete] \geq 1 - 1/n^2$.

A transcript $\tau(x,\rho)$ is complete if $|\tau(x,\rho)| < 8d/(1-p_a) + 6\log n$.
Let the transcript contain l insert operations, as the maximum length of a string is d, $|\tau(x,\rho)| \leq d + l$

We check the bounds for the probability of $l > 7d/(1-p_a) + 6\log n$.
Consider success to be when we do not have hash-insert operations. This behaves like a geometric progression with probability $= (1-p_a)$. The Expected number of hash-inserts we need to get an operation which is not a hash-insert would be $\frac{1}{(1-p_a)}$
Hence, $E[hash\ inserts\ for\ each\ character] = 1/(1-p_a)$.
$\therefore$ E[l] $= d/(1-p_a)$.

One of the Chernoff bounds is:
$Pr(X \geq (1+\delta)E[X]) \leq (\frac{e^\delta}{(1+\delta)^{(1+\delta)}})^{E[X]}$
$\implies Pr(X \geq (1+\delta)E[X]) \leq e^{(\delta-(1+\delta)ln(1+\delta))*E[X]}$ ——(1)
To calculate $Pr(l > 7d/(1-p_a) + 6\log n)$ with $E[X] = d/(1-p_a)$, $\delta = 6 + \frac{6(1-p_a)\log n}{d}$

Referring to "A guided tour of Chernoff bounds":
$\epsilon - (1+\epsilon)ln(1+\epsilon) \leq -\epsilon^2/3$, when $0 \leq \epsilon \leq 1$

We have the value of $\delta = 6 + \frac{6(1-p_a)\log n}{d} > 6$, hence we cannot use the above bound, instead we use:
$\delta - (1+\delta)ln(1+\delta) \leq -\delta/3$, when $\delta > 1$

Substituting in (1), we get:
$Pr(l \geq (1+\delta)E[X]) \leq e^{-\delta E[X]/3}$
$Pr(l > 7d/(1-p_a) + 6\log n) < e^{-(6d/(1-p_a)+6\log n)/3}$

As $\frac{6d}{(1-p_a)} > 0$,

$\implies Pr(l > 7d/(1-p_a) + 6\log n) < e^{-(6\log n)/3}$
$\implies Pr(l > 7d/(1-p_a) + 6\log n) < 1/n^2$
Hence, $Pr(l < 7d/(1-p_a) + 6\log n) > 1 - 1/n^2$