

Comprehension

The pharmaceutical company Sun Pharma is manufacturing a new batch of painkiller drugs, which are due for testing. Around 80,000 new products are created and need to be tested for their time of effect (which is measured as the time taken for the drug to completely cure the pain), as well as the quality assurance (which tells you whether the drug was able to do a satisfactory job or not).

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

b.) Calculate the required probability.

Solution:

Probability of drug producing satisfactory results: $P(A)$

Probability of drug not producing satisfactory results: $P(B)$

Given, $P(A) = 4P(B)$ and $P(A) + P(B) = 1$

Therefore,

$$P(B) = \frac{1}{5} P(A)$$

$$P(A) = \frac{4}{5} P(B)$$

Let X be the event out of 10 drugs, we select x number of drugs that do not give us the satisfactory result as the other drug. Then, X is the random variable with binomial distribution.

$$n = 10$$

$$p = \frac{1}{5}$$

$$P(X=x) = \binom{10}{x} \frac{1^x}{5} \frac{4^{10-x}}{5} \dots\dots\dots 0 \leq x \leq 10$$

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = \sum_{i=0}^3 \binom{10}{i} \frac{1^i}{5} \frac{4^{10-i}}{5}$$

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

b.) Find the required range.

Solution:

Given,

Sample size (n) = 100

Sample mean (\bar{X}) = 207 secs

Standard deviation (σ) = 65 secs

Significance level (α) = 5% (0.05)

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$207 \pm Z_{0.025} \frac{65}{\sqrt{100}}$$

$$207 \pm (1.96) \frac{65}{10}$$

$$207 \pm (1.96)(6.5)$$

$$207 \pm 12.74$$

Range : [219.74 , 194.26]

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure (with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having α and β as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both α and β values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your choice (Assume that only the values of α and β as mentioned above are provided to you and no other information is available).

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.