Some Aspects of Modelling Wind Turbines and Aggregated Wind Turbines

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Abstract—**In this paper we consider some fundamental aspects of wind park modelling. At first, main modelling steps with respect to nonlinear electromechanical models of wind turbines are presented where some recent mathematical results will be included. Based on this conceptual considerations about complex wind turbine modelling, the problem of modelling aggregated wind turbines (wind parks) using order reduction methods will be addressed.**

I. Introduction

The increasing interest in renewable energy during the last few decades is related to fact that the corresponding energy resources such as sunlight, geothermal heat and wind are available almost without any restrictions. However, if renewable energy is converted into electrical energy, the conversion processes performed by high efficiency are not a simple task. At time, electrical power production from wind and solar energy are the most prominent examples of renewable energy systems. In the following, we consider aggregated wind turbines where such systems, consist of a few tens until several hundred wind turbines, are called wind farms (e.g. Gasch et al. [4]). Since wind energy has its origin in mechanical movements of the air, a wind turbine is an electromechanical converter such that mechanical as well as electromagnetic aspects have to be described. In order to construct different kinds of electromagnetic generators the induction law is used for the conversion process from mechanical to electrical energy. However, for high efficient wind turbines a generator is needed which is well adapted to offshore and onshore wind fields and optimal connected to the electric power grid by suitable power electronics. In order to study the dynamical behavior of power systems including wind farms, a detailed modelling process of all components results in a high complex model of the system. Due to its complexity simulation results are hardly computable. Therefore, the complexity of such models has to be reduced. In the following sections, we consider electromechanical models of wind turbines and their potential for applying order reduction methods. Thereby, we discuss the wind farm modelling and some corresponding order reduction concepts.

II. Models of Wind Turbines

Wind turbines, for the conversion of wind energy into electrical energy, are based on induction generators. Here the rotor of the wind turbine is connected over a gear box with the rotor of the generator. Hence, tailed study see Perdana [15]. It turns out, that in contrast to other applications of induction generators, the influence of the mechanical part (drive train) of wind mills is more essential due to the lower stiffness of the drive train shaft. Therefore, a suitable mechanical modelling is necessary in order to obtain a model of the wind mills that characterize all essential dynamical aspects of the electromechanical system.

Based on an aero-dynamical model of the wind mill, the mechanical load on the wind turbine is reduced by some kind of blade angle control. The standard approach of blade angle control in the case of fixed speed wind turbines is a (mechanical) stall control whereas variable speed wind turbines use a pitch control. Thereby, the pitch control operations are performed by power electronic components.

With respect to the grid connection, fixed speed and variable speed wind turbines differ by a direct transformer coupling and a partial-scale or a full-scale frequency power converter, respectively, of generator and grid. It is well-known that the direct grid connected and fixed speed wind turbines dominated the wind energy systems for many years where squirrel cage induction generators (SCIG) are included; for this statement as well as the following considerations see e.g. Michalke [9]. Another type of direct grid connected wind turbines are based on wound rotor induction generators (WRIG) such that small variations of the speed are possible.

A disadvantage of both concepts is that the reactive power has to be compensated by a capacitor bank. If a doubly-fed induction generator (DFIG) and a synchronous generator (SG), respectively, are combined with a power converter, we obtain variable speed wind turbines. Thereby, the controllability is improved and compensation techniques can be omitted. The SG is equipped with a permanent magnet (PMSG) or it can be electrical exited by a DC source (DCSG). Today, DFIG wind turbines with a partial-size frequency converter are the most attractive turbine concepts. One of the main reasons is that only a part of the total wind turbine power is processed in the power converter such that size, costs and losses are much smaller than in turbines concepts with a full-size frequency converter; see e.g. Muller et al. [6].

Whereas the development of efficient electrical machines started already in the second half of the nineties century (see e.g. [5]), first systematic analysis methods of electrical machines date back to the beginning of the twenties century [1]. Thus, many monographs about this subject are available; see e.g. [20]. Although it is known that the electromechanical equations of electrical machines are nonlinear, in most of the monographs and journal articles the linearized descriptive equations are studied. However, several papers are available where also the mathematical structure of the different mathematical models of generators and their analysis concepts are discussed. In the following, we will consider some of these analysis concepts for electric generators.

Following Youla and Bongiogno [22], from a conceptual point of view, an electromagnetic rotating machine is a collection of n windings threaded either in air or around ferromagnetic material or both. The windings are disposed into two groups, the stator and the rotor. The stator contains s windings and is stationary while the rotor contains the remaining $n-s=r$ windings and rotates as a rigid body about an axis, fixed in the laboratory coordinate system. In general, eddy currents, hysteretic effects, and creep of the material lead to a strong nonlinear system. Let ψ_k denote the magnetic flux linkage and i_k the current of the k-th winding. If linear magnetic materials are assumed, we obtain nonlinear electro-mechanic generator equations. In order to prove this statement, an affine constitutive model for the n windings is used ($\Psi = (\psi_1, \dots, \psi_n)$, $\mathbf{i} = (i_1, \dots, i_n)$)

$$\Psi = \mathbf{L}(\varphi)\,\mathbf{i} + \mu(\varphi), \tag{1}$$

where $\mathbf{L}(\varphi)$ is the $n \times n$ multiport inductance matrix of the n windings, φ is the mechanical position of the rotor and $\mu(\varphi)$ is the flux linkage of a permanent magnet. Using (1) the state-space equations for the electrical state Ψ and the two mechanical states ω and φ can be derived (Liu et al. [7]):

$$\begin{aligned} \dot{\Psi} &= -\mathbf{R}\mathbf{L}^{-1}(\varphi)\Psi - \omega\mathbf{k}(\varphi) + \mathbf{v}(t), && (2)\\ \dot{\omega} &= \frac{1}{T}\Big(-\frac{1}{2}\Big\langle \Psi, \frac{d\mathbf{L}^{-1}(\varphi)}{d\varphi}\Psi \Big\rangle \\ &\quad + \big\langle \Psi, \mathbf{L}^{-1}(\varphi)\mathbf{k}(\varphi)\big\rangle \\ &\quad + \eta(\varphi) + \tau(\omega,\varphi,t)\Big). && (3)\\ \dot{\varphi} &= \omega. && (4) \end{aligned}$$

Here, ω is the angular speed of the rotor, $\mathbf{v}$ is the vector of the source voltages, τ is the net external torque on the rotor, the constant diagonal matrix $\mathbf{R}$ includes the winding resistances, T is the rotational inertia of the rotor and η is the torque of the permanent magnets alone.

Since $\mathbf{L}$, $\mathbf{k}$ and η depend on φ, the state-space equations (2)-(4) are nonlinear. Thus, it is not an easy task to determine the conditions for the existence of a steady state of constant speed operation for the generator. Furthermore, the computation of this steady state and its stability can be done only by numerical methods. However, in some cases a nonlinear transformation $\phi := \mathbf{P}(\varphi)\Psi$ with $\phi = (\phi_1, \dots, \phi_n)$ exists, such that the transformed version of the state-space equations has the following form

$$
\begin{aligned}
\dot{\phi} &= -(\mathbf{C}_0 - \omega \mathbf{C}_1)\phi - \omega \mathbf{d}_1 + \mathbf{u}, && (5)\\
\dot{\omega} &= T^{-1}\left(-\frac{1}{2}\langle \phi, \mathbf{C}_2\phi\rangle + \phi \mathbf{d}_2 + \eta + \tau\right), && (6)\\
\dot{\varphi} &= \omega, && (7)
\end{aligned}
$$

with φ-dependent $\mathbf{C}_0$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{d}_1$, and $\mathbf{d}_2$.

If $\eta(\varphi) = 0$ and if there exists a matrix $\mathbf{P}$ such that the matrices $\mathbf{C}_i$ $(i = 0, 1, 2)$ and vectors $\mathbf{d}_j$ $(j = 1, 2)$ are constant, then the analysis of the generator equations is obviously much easier. We calculate the steady state $(\phi_s, \omega_s, \varphi_s)$ by means of $\dot{\phi} = 0$, $\dot{\omega} = 0$ and $\dot{\varphi} = 0$ (for constant u and τ). Then, (5) is solved with respect to $\phi_s(\omega_s)$, substitute it in (6) and obtain a system of nonlinear equations where real solutions ω_s^i have to be calculated. Finally, a stability analysis of these steady states $(\phi_s, \omega_s, \varphi_s)$ $(i = 1, 2, \ldots)$ can be performed by a standard linearization process. For this purpose, the linearized equations (5)-(7) have to be derived. However, in many textbooks on electrical machines the nonlinear generator equations (2)-(4) will be linearized and a linear system of differential equations arises where the coefficients are φ-dependent. Subsequently, the nonlinear transformation $\phi := \mathbf{P}(\varphi)\Psi$ is applied in order to obtain linear differential equations with constant coefficients.

In any case, it is useful for the analysis of electrical generators to have necessary and sufficient conditions such that a nonlinear transformation $\phi := \mathbf{P}(\varphi)\Psi$ exists. A first result in this direction was presented by Park [14], based on the work of Blondel [1]. Therefore, these transformations are denoted as Blondel-Park (BP) transformations. There are several other results with respect to the existence of BP transformations but one of the most interesting theorems was proven by Liu et al. [7].

Theorem:

The state-space equations (2)-(4) of electrical machines with affine magnetics $(\mathbf{L}(\varphi), \mathbf{k}(\varphi), \mathbf{R})$, with can be transformed by a BP transformation into state-space equations (5)-(7) with φ-independent $\mathbf{C}_0$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{d}_1$, $\mathbf{d}_2$ if and only if a constant $n \times n$ matrix $\mathbf{V}$ exists and the following conditions are satisfied:

$$
\begin{aligned}
\mathbf{V}\mathbf{L}(\varphi) + \mathbf{L}(\varphi)\mathbf{V}^T &= -(d\mathbf{L}(\varphi)/\varphi), && (8)\\
\mathbf{V}\mathbf{R} + \mathbf{R}\mathbf{V}^T &= \mathbf{0}, && (9)\\
\mathbf{V}\mathbf{k}(\varphi) &= -(d\mathbf{k}(\varphi)/d\varphi). && (10)
\end{aligned}
$$

A simple form of such a BP transformation has the form

$$\mathbf{P}(\varphi) = \mathbf{P}_0\, e^{\mathbf{V}\varphi}, \tag{11}$$

where $\mathbf{P}_0$ denotes a nonsingular constant $n \times n$ matrix.

If we consider a two-axis uniform-air-gap machine with the inductance matrix

$$\mathbf{L}(\varphi) = \begin{pmatrix} L_s\mathbf{I} & Me^{\mathbf{J}\varphi} \\ Me^{-\mathbf{J}\varphi} & L_r\mathbf{I} \end{pmatrix}, \tag{12}$$

where L_s is the stator and rotor inductance, respectively, M is the mutual inductance, $\mathbf{I}$ is a 2×2 identity matrix, and $\mathbf{J}$ is the symplectic 2×2 matrix

$$\mathbf{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{13}$$

In the case of identical resistances in stator and rotor, we obtain from the solution of (8)-(10) the following well-known BP transformation

$$\mathbf{P}(\varphi) = \mathbf{P}_0\, e^{\mathbf{V}\varphi} = \mathbf{P}_0 \begin{pmatrix} e^{\mathbf{J}a\varphi} & \mathbf{0} \\ \mathbf{0} & e^{\mathbf{J}(1+a)\varphi} \end{pmatrix} \tag{14}$$

if $a \in \mathbb{R} \setminus \{-1, 0\}$.

Other examples can be found in Liu et al. [7] and in the classical literature of electrical machines (e.g. Müller and Ponick [11]). If the electrical machine is not ideal in the sense of Blondel-Park, a rotating coordinate system and the corresponding Blondel-Park transformation does not exist. Under certain conditions the Floquet theory can be used to build up a generalized theory for linear rotating generators; for further details see Youla and Bongiorno [22] and Verghese et al. [21]).

However, certain types of wind turbines include electronic components just like the frequency converters in DFIGs where the individual frequency of each turbine will be converted into the synchronous frequency of the grid. Since these power electronic components based on electronic switching processes modelling of these components becomes even more involved and complicate nonlinear behavior is possible. In order to combine time continuous and switching parts of a hybrid system, the so-called state-space averaging techniques can be used. Thereby, that the switching system can be modelled entirely by a time continuous system; see e.g. Chayawatto et al. [2].

III. Modelling of Wind Farms

In section III we consider different types of induction generators for wind turbines where rather complex models are available. Furthermore, several aspects of modelling the mechanical and electronic part of wind turbines are addressed. However, a wind farm consists of a few tens or hundreds of wind turbines, so that an extremely complex model for a wind farm occurs. A detailed discussion about wind farms can be found in the monograph of Muyeen et al. [13]. Obviously, order reduction techniques are needed to obtain a wind farm model that can be simulated in suitable manner. A wind farm can be interpreted as a network of wind turbines where the interconnections are high power transmission lines with some kind of tree structure. Therefore, the order reduction methods are related to the turbines and interconnections.

A study about the modelling of wind farms with fixed-speed wind turbines was published by Saad-Saoud et al. [19]. In this publication a third order model for each induction generator was suggested and a comparison with the behavior of high order models were presented. A standard approach to derive a reduced-order model of wind turbine generators starts from the system DAEs (differential-algebraic equations). In first step we calculate a equilibrium point and then the DAE is linearized around this equilibrium point. In a second step we use a MOR (model order reduction) technique to obtain a simplified linear generator model. Pulgar-Painemal [17] eliminates only the algebraic variables of the DAE, but there are more sophisticated MOR methods available; e.g. Mehrmann and Stykel [8]. In a more recent paper the stability analysis of DFIG-based wind farms and conventional synchronous generators were published (Munoz et al. [12]). However, there are several other publications about this subject.

In addition to the wind turbine models, we have to consider the power transmission lines from the wind mills to the grid. Moreover, we have to study the interactions between the wind turbines. In order to study the internal behavior of a wind farm, a detailed model of the aggregated wind turbines has to be constructed. Corresponding studies are available for wind farms with SCIG and DFIG, respectively; for further references see Fernandez et al. [3]. Due to the complexity of detailed wind farm models, simulation results are not computable in an efficient manner. Thus, a suitable order reduction method should be applied to develop a simplified wind farm model that can be used in a simulation concept for power systems. If a wind farm model consists of linear components, several model order reduction techniques are available. A computer-aided simulation system for the aggregation of wind parks was presented by Rudion et al. [18]. However, if some components of a wind farm are nonlinear it is difficult to find applicable order reduction methods.

An alternative concept to standard order reduction methods is well-known in circuit theory, where a sub-circuit is replaced by an equivalent simplified circuit. In the case of a 2-pole sub-circuit, it can be represented by an equivalent voltage or current source with resistor. In the same manner, an aggregated wind turbine model can be replaced by an equivalent wind turbine model representing the entire wind farm. However, there are different strategies known in the literature to use the equivalent wind turbine model; see e.g. see Fernandez et al. [3]. In some papers the generated mechanical power (SCIG) and the active and reactive power (DFIG), respectively, of a collection of wind turbine models are calculated, where the turbines are driven by individual wind fields. Thus, an equivalent electrical power model of the wind farm can be parametrized. In other papers the mechanical generator torques of a collection of wind turbines are calculated, where again the turbines are driven by individual wind fields. With these data the fluctuating total mechanical torque of the wind farm can be determined. This torque can be used as an input quantity for an equivalent wind turbine that is connected to the grid. Fernandez et al. [3] generalize this concept and represent the wind farm by an equivalent wind turbine that replaces the aggregated wind turbines with respect to the mechanical as well as the electromagnetic part. Their equivalent model is essentially a linearized model, which achieves a good approximation of the quantitative behavior of wind farms.

In a more general concept of wind farm modelling not only the quantitative properties of reduced models but also their qualitative behavior with respect to parameter variations and its influence to the grid should be included. Thereby, an aggregated nonlinear wind turbine model can be replaced by an equivalent nonlinear wind turbine model. For example, different kinds of stability conflicts can appear in an aggregated wind turbine system. In this case not only the stability of equilibrium points but also the stability of limit cycles of are interest. Furthermore, the asymptotic solutions can change with respect to at least one parameter. Therefore bifurcation methods have to be included. For this purpose, nonlinear dynamic models would be useful that possess equilibrium as well as limit cycle solution which already discussed with respect to stability behavior of electrical machines; see e.g. [10]. Furthermore, with respect to the fluctuations in the wind field, a stochastic modelling is needed. Until now, only linearized stochastic models for electric turbines are considered; see Pidre et al. [16].

IV. Conclusion

In this paper some fundamental aspects of modelling of wind farms are presented. Therefore we discussed the main steps for the construction of nonlinear electromechanical models of wind turbines and discuss possible applications of order reduction methods. Using these reduced models better structural insights into such systems as well as more efficient simulation processes can be obtained.

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