LINEAR ALGEBRA ASSIGNMENT

Submitted by:

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PROBLEM STATEMENT 1

Gaussian Elimination: Solve the system of equations 2x+4y+6z=4 x+5y+9z=2 2x+1y+3z=7 using Gaussian Elimination. Also, identify the pivots.

SOURCE CODE:

```
clc;clear;
A=[2,4,6; 1,5,9; 2,1,3]
b=[4;2;7]
n = 3
A_aug=[A b]
a=A_aug
disp('Augmented Matrix')
disp(a)
for i=2:n
  for j=2:n+1
    a(i,j)=a(i,j) - a(1,j) * a(i,1)/a(1,1);
      end
a(i,1) = 0;
end
for i=3:n
  for j=3:n + 1
    a(i,j)=a(i,j) - a(2,j)*a(i,2)/a(2,2);
end
a(i,2) = 0
disp('Upper triangular matrix')
disp(a)
end
x(n) = a(n,n+1)/a(n,n); for i=n-1:-1:1
  sumk=0;
  for k=i+1:n
    sumk=sumk + a(i,k) * x(k);
  end
  x(i)=(a(i,n+1) - sumk)/a(i,i);
end
disp(x(3),x(2),x(1), 'The values of x,y,z are');
disp(a(1,1),a(2,2),a(3,3), 'The pivots are');
```

OUTPUT:

```
Augmented Matrix

  2.   4.   6.   4.
  1.   5.   9.   2.
  2.   1.   3.   7.
```

Upper triangular matrix

2.	4.	6.	4.
0.	3.	6.	0.
0.	0.	3.	3.

The values of x,y,z are

3.

-2.

1.

The pivots are

3.

3.

2.

PROBLEM STATEMENT 2

LU Decomposition: Find the triangular factors L and U for the matrix A = [1, 1, 1; 4, 3, -1; 3, 5, 3]

SOURCE CODE:

```
clc;clear;
A=[1 1 1; 4 3 -1; 3 5 3];
num = 3;
U=A;
disp('Matrix A')
disp(A)
m=det (U ( 1,1) );
n=det (U (2,1) );
a=n/m;
U(2,:)=U(2,:)-U(1,:)/(m/n);
n=det(U(3,1));
b=n/m;
U(3,:)=U(3,:)-U(1,:)/(m/n);
m=det(U(2,2));
n=det(U(3,2));
c=n/m;
L=[1 ,0 ,0; a ,1 ,0; b ,c ,1];
disp(L,'Lower triangular matrix L')
U(3,:)=U(3,:)-U(2,:)/(m/n);
disp(U,'Upper triangular matrix U')
```

OUTPUT:

```
Matrix A

  1.   1.   1.
  4.   3.  -1.
  3.   5.   3.

Lower triangular matrix L

  1.   0.   0.
  4.   1.   0.
  3.  -2.   1.

Upper triangular matrix U

  1.   1.   1.
  0.  -1.  -5.
  0.   0.  -10.
```

PROBLEM STATEMENT 3

Inverse of a Matrix by the Gauss-Jordan Method: Find the inverse of the matrix A= [1, 2, 3; 0, 1, 4; 5, 6, 0]

SOURCE CODE:

```
clc;clear;
n=3;
A = [1 2 3;0 1 4;5 6 0];
disp(A, "Matrix A ");
Aug = [A,eye(n,n)];
for j=1:n-1
   for i=j+1:n
      Aug(i,j:2*n) = Aug(i,j:2*n)-Aug(i,j)/Aug(j,j)*Aug(j,j:2*n);
   end
end
for j=n:-1:2
   Aug(1:j-1,:)=Aug(1:j-1,:)-Aug(1:j-1,j)/Aug(j,j)*Aug(j,:);
end
for j=1:n
   Aug(j,:)=Aug(j,:)/Aug(j,j);
end
B=Aug(:,n+1:2*n);
disp(B,"Inverse of A ");
```

OUTPUT:

```
Matrix A

  1.   2.   3.
  0.   1.   4.
  5.   6.   0.

 Inverse of A

 -24.   18.   5.
  20.  -15.  -4.
 -5.    4.    1.
```

PROBLEM STATEMENT 4

Span of Column Space of a Matrix: Identify the columns that are in the column space of A where A = [2, 1, 0, 1; -1, 0, 2, 1; 3, 1, -1, 4]

SOURCE CODE:

```
clc;clear;
n=3;
a=[2 1 0 1;-1 0 2 1;3 1 -1 4];
disp('The given matrix is ');
disp(a);
a(2,:)=a(2,:)-(a(2,1)/a(1,1))*a(1,:);
a(3,:)=a(3,:)-(a(3,1)/a(1,1))*a(1,:);
a(3,:)=a(3,:)-(a(3,2)/a(2,2))*a(2,:);
a(1,:)=a(1,:)/a(1,1);
a(2,:)=a(2,:)/a(2,2);
disp('After Gaussian Elimination');
disp(a);
for i=1:n
   for j=i:n
      if(a(i,j)<>0)
         disp('is a pivot element',j,'column');
         break;
      end
   end
end
```

OUTPUT:

```
The given matrix is

  2.   1.   0.   1.
 -1.   0.   2.   1.
  3.   1.  -1.   4.

 After Gaussian Elimination

  1.   0.5   0.   0.5
  0.   1.    4.   3.
  0.   0.    1.   4.

 Column

  1.

 is a pivot element
```

column

2.

is a pivot element

column

3.

is a pivot element

PROBLEM STATEMENT 5

Four Fundamental Subspaces: Find the four fundamental spaces of A = [0, 2, 3, 4; 0, 6, 7, 8; 0, 10, 11, 12]

SOURCE CODE:

```
clc;clear;
n=3;
A=[0 2 3 4;0 6 7 8;0 10 11 12];
disp('The given matrix is ');
disp(A,'A=');
[m,n]=size(A);
disp(m,'m=');
disp(n,'n=');
[v,pivot]=rref(A);
disp(rref(A));
disp(v);
r=length(pivot);
disp(r,'rank=');
cs=A(:,pivot);
disp(cs,'Column Space = ');
ns=kernel(A);
disp(ns,'Null Space = ');
rs=v(1:r,:)';
disp(rs,'Row Space = ');
lns=kernel(A');
disp(lns,'Left Null Space = ');
```

OUTPUT:

```
The given matrix is

A=

  0.  2.   3.   4.
  0.  6.   7.   8.
  0.  10.  11.  12.

m=

  3.

n=

  4.

  0.  1.  0.  -1.
  0.  0.  1.  2.
  0.  0.  0.  0.
```

```
  0.   1.   0.  -1.
  0.   0.   1.   2.
  0.   0.   0.   0.
```

rank=

```
  2.
```

Column Space =

```
  2.    3.
  6.    7.
  10.   11.
```

Null Space =

```
  0.2903994   0.9569055
  0.390655   -0.1185551
 -0.7813101   0.2371101
  0.390655   -0.1185551
```

Row Space =

```
  0.   0.
  1.   0.
  0.   1.
 -1.   2.
```

Left Null Space =

```
  0.4082483
 -0.8164966
  0.4082483
```

PROBLEM STATEMENT 6

Projections by Least Squares: Find the solution x=(C,D) of the system Ax=b and the line of best fit C+Dt=b given by A=[1,1; 1,-1; -2,4] , b=[1,2,7]

SOURCE CODE:

```
clc;clear;
n=3;
A=[1 1;1 -1;-2 4];
disp('The given matrix ');
disp(A,'A=');
b=[1;2;7];
disp('The given matrix ');
disp(b,'b=');
x = (A'*A)\(A'*b);
disp(x, 'x=');
C = x(1,1);
D = x(2,1);
disp('The line of best fit is b=C + Dt');
disp(C, 'C=');
disp(D, 'D=');
```

OUTPUT:

```
The given matrix
A=
  1.   1.
  1.  -1.
 -2.   4.

The given matrix
b=
  1.
  2.
  4.

x=
  0.6818182
  1.1363636

The line of best fit is b=C + Dt

C=
  0.6818182

D=
  1.1363636
```

PROBLEM STATEMENT 7

Gram-Schmidt Orthogonalization: Apply the Gram-Schmidt process to the vectors (3,0,-1), (0,-2,2) and (-1,7,4) to produce a set of orthonormal vectors

SOURCE CODE:

```
clc;clear;
A = [3 0 -1; 0 -2 7; -1 2 4];
disp(A,'A = ');
[m,n] = size(A);
for k=1:n
   V(:,k) = A(:,k);
   for j=1:k-1
      R(j,k) = V(:,j)'*A(:,k);
      V(:,k) = V(:,k) - R(j,k)*V(:,j);
   end
   R(k,k) = norm(V(:,k));
   V(:,k) = V(:,k)/R(k,k);
end
disp(V,'Q = ');
```

OUTPUT:

```
A =

  3.   0.  -1.
  0.  -2.   7.
 -1.   2.   4.

 Q =

  0.9486833   0.2176429   0.2294157
  0.         -0.7254763   0.6882472
 -0.3162278   0.6529286   0.6882472
```

PROBLEM STATEMENT 8

Eigen values and Eigen Vectors of a Matrix: Find the Eigen values and the corresponding Eigen vectors of A=[4,1,-1;2,3,-1;-2,1,5]

SOURCE CODE:

```
clc; clear;
A = [4,1,-1;2,3,-1;-2,1,5];
disp(A,"The matrix A");
lam = poly(0,'lam');
lam = lam;
charMat = A-lam*eye(3,3);
disp(charMat,"The characteristic matrix is:");
charPoly = poly(A,'lam');
disp(charPoly,'The characteristic ploynomial is');
lam =spec(A);
disp(lam,'The eigen values of A are');
function [x, lam]=eigenvectors(A)
  [n,m]=size(A);
  lam=spec(A)';
  x=[];
  for k=1:3
    B=A-lam(k)*eye(3,3);
    C=B(1:n-1,1:n-1);
    b=-B(1:n-1,n);
    y=C\b;
    y=[y;1];
    y=y/norm(y);
    x=[x y];
  end
endfunction
get('eigenvectors');
[x,lam]= eigenvectors(A);
disp(x,'The eigen vectors of A are');
```

OUTPUT:

The matrix A

```
  4.   1.  -1.
  2.   3.  -1.
 -2.   1.   5.
```

The characteristic matrix is:

```
 4 -lam   1      -1

 2      3 -lam  -1

-2       1       5 -lam
```

The characteristic ploynomial is

$$-48 + 44lam - 12lam^{2} + lam^{3}$$

The eigen values of A are

```
 2.
 6.
 4.
```

The eigen vectors of A are

```
 0.5773503  -0.5773503   0.5773503
-0.5773503  -0.5773503   0.5773503
 0.5773503   0.5773503   0.5773503
```

PROBLEM STATEMENT 9

Largest Eigen Value of a Matrix by the Power Method: Find the largest Eigen value of A = [2,2,7; 3,0,1; 3,6,8] starting with x=[1,0,1]

SOURCE CODE:

```
clc;clear;
A = [2 2 7; 3 0 1; 3 6 8];
disp(A,'A = ');
u0 = [1 0 1]';
disp(u0,'The initial vector is');
v = A*u0;
a = max(u0);
disp(A,'First approximation to eigen value is ');
while abs(max(v)-a)>0.002
   disp(v,'current eigen vector is');
   a = max(v);
   disp(a,'Current eigen value is');
   u0 = v/max(v);
   v = A*u0;
end
format('v',4);
disp(max(v),'The largest eigen value is: ');
format('v',5);
disp(u0,'The corresponding eigen vector is: ');
```

OUTPUT:

```
 A =

   2.   2.   7.
   3.   0.   1.
   3.   6.   8.

 The initial vector is

   1.
   0.
   1.

 First approximation to eigen value is

   2.   2.   7.
   3.   0.   1.
   3.   6.   8.
```

current eigen vector is

9.
4.
11.

Current eigen value is

11.

current eigen vector is

9.36
3.45
12.6

Current eigen value is

12.6

current eigen vector is

9.03
3.22
11.9

Current eigen value is

11.9

current eigen vector is

9.07
3.28
11.9

Current eigen value is

11.9

current eigen vector is

9.07
3.28
11.9

Current eigen value is

11.9

current eigen vector is

9.07
3.28
11.9

Current eigen value is

11.9

The largest eigen value is:

12.

The corresponding eigen vector is:

0.76
0.27
1.