Game Theory - AKQ Poker

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1 Rules of AKQ Poker

1.1 Initial Setup (Ante)

Both Player 1 and Player 2 place an initial bet of \$1 into the pot, making the starting pot \$2. The game uses a deck of exactly three cards: Ace (A), King (K), and Queen (Q).

1.2 Card Priority

The cards follow the standard poker hierarchy:

$$\text{Ace (A)} > \text{King (K)} > \text{Queen (Q)}$$

1.3 The Deal

Each player is dealt one card privately, and the third card is discarded without showing. For example, if Player 1 has the Ace, then Player 2 must hold either the King or the Queen.

Now the game begins.

1.4 Action (Single Betting Round)

Player 1 starts the betting round. The pot is currently \$2.

1.4.1 Player 1 can Bet \$1 (Pot becomes \$3)

Player 2 can either:

- **Call \$1** - Pot becomes \$4. Game goes to a **showdown**; higher card wins the entire pot.
- **Fold** - Player 1 wins the pot of \$3.

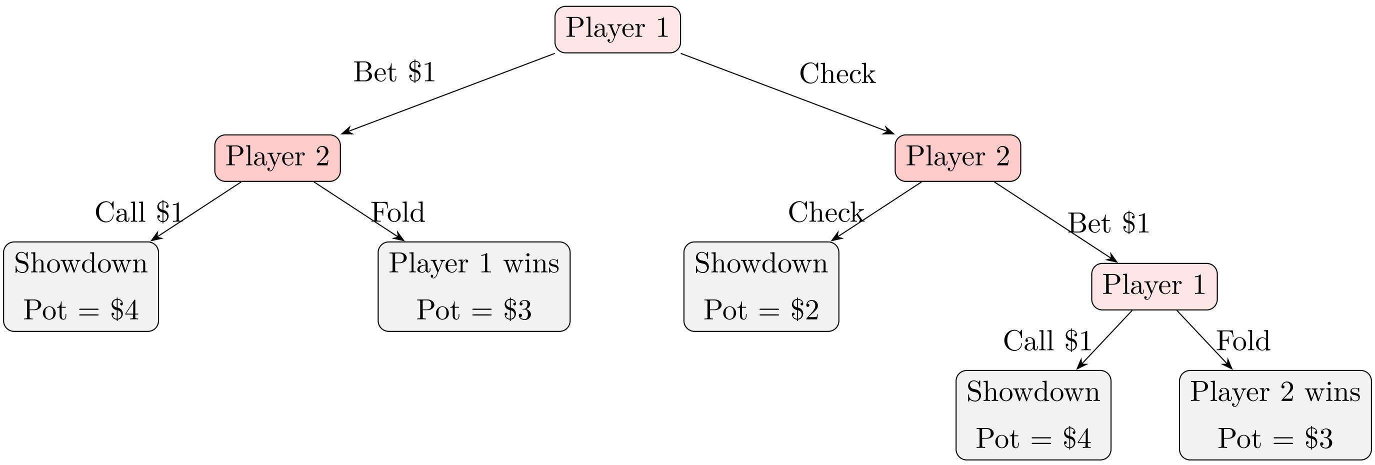
1.4.2 Player 1 can Check (Pot remains at \$2)

Player 2 can either:

- **Check** - Game goes to a **showdown**; higher card wins the pot of \$2.
- **Bet \$1** (Pot becomes \$3)
  Now Player 1 must decide:
  - **Call \$1** - Pot becomes \$4. Game goes to a **showdown**; higher card wins.

– **Fold** - Player 2 wins the pot of $3.

Game Tree for AKQ Poker



2 Strategic Analysis

Here, we break down the strategic considerations for both players.

2.1 Player 1's Strategic Dilemmas

Player 1 acts first, so they have less information. This makes their first move very important for the rest of the game.

2.1.1 Holding an Ace

With the unbeatable hand, Player 1's objective is simple: maximize profit. The clear, optimal play is to **Bet**.

- **Why bet?** A bet forces Player 2, who holds either a King or a Queen, to make a decision for more money. If Player 2 folds, P1 wins the pot immediately. If Player 2 calls, P1 wins a larger pot.

- **Why not check?** Checking is a major error here. If P1 checks, Player 2 might check behind with their King or Queen, and P1 would only win the $2 ante. By checking, P1 gives Player 2 a chance to a cheap showdown. The bet is essential to increase the value in the pot.

2.1.2 Holding a King

The King is the most complex hand for Player 1.

- **The Problem with Betting:** If Player 1 bets with a King, they put themselves in a difficult position. Player 2, holding an Ace, will always call, and Player 1 will lose $2. Player 2, holding a Queen, will almost always fold, as their hand is too weak to call.

- **The Problem with Checking:** Checking helps control the pot size, when Player 1 has a weaker hand. But it also shows weakness. A smart Player 2 might take advantage by betting , whether they have a strong hand (Ace) or are bluffing (Queen). This puts Player 1 in a tough spot. In the optimal strategy, Player 1 should check with weak hands and fold if Player 2 bets, to avoid losing even more.

2.1.3 Holding a Queen

With the worst possible hand, Player 1 cannot win a showdown. The only path to victory is to make Player 2 fold.

- **Betting (Bluffing):** The target of a bluff with a Queen is specifically Player 2's King. By betting, Player 1 fakes an Ace. If Player 2 is convinced or unwilling to risk their money, they may fold their King, awarding the pot to Player 1.

- **The Risk of Bluffing:** This is a high-risk, high-reward. If Player 2 holds the Ace, they will call and Player 1 loses even more. Even if Player 2 has a King, they might call the bluff and still win. So the bluff only works if it convinces King to fold often enough to make up for those times it fails.

- **The Rationale for Checking:** Checking seems safe, but it is almost guaranteed to lose the $1 ante. As check signals weakness, and Player 2 will bet with both their Kings and Aces.

2.2 Player 2's Strategic Dilemmas (The Responder)

Player 2 has an advantage because they get to see what Player 1 does first. This helps him better guess what kind of card Player 1 might have.

2.2.1 Responding to a Bet from Player 1

- **Holding an Ace:** This is the easiest decision.They should always **Call** to win the now larger pot.

- **Holding a King:** Whether to call or fold depends on how often Player 1 is bluffing. If P1 bluffs a lot, calling works. If not, folding is safer. Since P2 can't know for sure, the best play is to **mix** between calling and folding.
- **Holding a Queen:** Player 2's Queen loses to both an Ace and a King. Since Player 1's bet implies he holds an Ace, this is a **Fold**.

2.2.2 Responding to a Check from Player 1

A check from Player 1 strongly suggests Player 1 does not hold an Ace.

- **Holding an Ace:** Player 1 is showing weakness. This is the perfect chance to **bet** and try to get value.
- **Holding a King:** Player 1 checked, so they probably have either an Ace or a Queen. If Player 2 **bets**, they might make a Queen fold but if Player 1 has an Ace, they'll likely call and win. So, **checking is the safer choice** it lets the King win against weaker without risking more money.
- **Holding a Queen:** Player 1 has checked, indicating they likely hold a King. Player 2 has the worst hand. The only way to win is to bet and bluff to have an Ace. The goal is to scare Player 1 into folding a King.

2.3 The Necessity of Bluffing

Bluffing keeps the game interesting and balanced. If Player 1 bluffs with a Queen, Player 2 has to think carefully about whether to call with a King. But if Player 2 starts calling more, bluffing becomes risky for Player 1. Bluffing creates doubt and gives weak card a chance to win. Without bluffing, the game would be boring and predictable.

3 Payoff Table

P1 Card	P2 Card	P1 Action	P2 Action	P1 Reaction	Pot Outcome	P1 Payoff	P2 Payoff
A	K	Bet	Call	–	P1 wins \$4	+2	-2
A	Q	Bet	Fold	–	P1 wins \$2	+1	-1
A	K	Check	Check	–	P1 wins \$2	+1	-1
A	Q	Check	Check	–	P1 wins \$2	+1	-1
K	A	Bet	Call	–	P2 wins \$4	-2	+2
K	Q	Bet	Fold	–	P1 wins \$2	+1	-1
K	A	Check	Bet	Fold	P1 folds	-1	+1
K	A	Check	Bet	Call	P2 wins \$4	-2	+2
K	Q	Check	Check	–	P1 wins \$2	+1	-1
K	Q	Check	Bet	Fold	P1 folds	-1	+1
K	Q	Check	Bet	Call	P1 wins \$4	+2	-2
Q	A	Bet	Call	–	P2 wins \$4	-2	+2
Q	K	Bet	Fold	–	P1 wins \$2	+1	-1
Q	A	Check	Bet	Fold	P1 folds	-1	+1
Q	A	Check	Bet	Call	P2 wins \$4	-2	+2
Q	K	Check	Check	–	P2 wins \$2	-1	+1
Q	K	Check	Bet	Fold	P1 folds	-1	+1
Q	K	Check	Bet	Call	P2 wins \$4	-2	+2

3.1 Payoff Analysis and Expected Value Calculations

To move from strategic intuition to an optimal strategy, we must quantify the decisions players face. We use the concept of Expected Value (EV), which calculates the average outcome of a decision given the probabilities of different results. A rational player will choose the action with the higher EV.

3.1.1 Decision 1: Player 1, Holding a Queen, Considers Bluffing

Here we analyze the game from the perspective of Player 1, who holds a Queen and must decide whether to bet (bluff) or check. Player 1 knows that Player 2 must hold either an Ace or a King, each with a 50% probability.

Action: Bet \$1 (The Bluff) - When Player 1 bluffs, the outcome depends entirely on Player 2's hand and subsequent action. Let p_{call} be the probability that Player 2 calls with a King.

- If P2 has an Ace (50% chance), P2 will always call. P1 loses the \$1 ante and the \$1 bet, for a total loss of \$2.
- If P2 has a King (50% chance), P1's outcome depends on P2's decision:
  - If P2 calls (with probability p_{call}), P1's bluff is caught. P1 loses \$2.
  - If P2 folds (with probability $1 - p_{\text{call}}$), the bluff succeeds. P1 wins the \$2 pot (the antes), for a net profit of \$1.

The Expected Value of bluffing is therefore a weighted average of these outcomes:

$$\begin{aligned}
EV_{P1}(\text{Bluff with Q}) &= \underbrace{P(\text{P2 has A}) \times (-\$2)}_{\text{Loss against Ace}} + \underbrace{P(\text{P2 has K}) \times \Big[p_{\text{call}}(-\$2) + (1 - p_{\text{call}})(+\$1)\Big]}_{\text{Outcome against King}} \\
&= 0.5(-2) + 0.5\Big[-2p_{\text{call}} + 1 - p_{\text{call}}\Big] \\
&= -1 + 0.5(1 - 3p_{\text{call}}) \\
&= -0.5 - 1.5p_{\text{call}}
\end{aligned}$$

Action: Check - If Player 1 checks with a Queen, they signal weakness. A rational Player 2 will always bet with either an Ace or a King to claim the pot. Player 1, holding the worst hand, will have no choice but to fold to this bet, thereby losing their initial \$1 ante.

$$EV_{P1}(\text{Check with Q}) = -\$1$$

The Decision Threshold - Player 1 should choose to bluff if the EV of bluffing is greater than the EV of checking.

$$\begin{aligned}
EV(\text{Bluff}) &> EV(\text{Check}) \\
-0.5 - 1.5p_{\text{call}} &> -1 \\
0.5 &> 1.5p_{\text{call}} \\
\frac{0.5}{1.5} &> p_{\text{call}} \\
\implies p_{\text{call}} &< \frac{1}{3}
\end{aligned}$$

Conclusion: This calculation reveals that Player 1 should only bluff if they believe Player 2 is cautious and will call with a King less than one-third of the time.

3.1.2 Decision 2: Player 2, Holding a King, Considers Calling

Now we analyze the pivotal decision for Player 2, who holds a King and faces a bet from Player 1. P2 knows that P1 must hold either an Ace or a Queen (a bluff). P2's decision depends on the probability that P1 is bluffing. Let p_{bluff} be the frequency with which P1 bluffs with a Queen.

Conditional Probabilities - Given that Player 1 has bet, we must update our probabilities. We assume P1 always bets with an Ace (100% of the time) and bets with a Queen with frequency p_{bluff}.

- $P(\text{P1 has A} \mid \text{P1 bets}) = \frac{P(\text{bet}|\text{A})P(\text{A})}{P(\text{bet})} = \frac{1\times P(\text{A})}{1\times P(\text{A})+p_{\text{bluff}}\times P(\text{Q})} = \frac{1}{1+p_{\text{bluff}}}$
- $P(\text{P1 has Q} \mid \text{P1 bets}) = \frac{P(\text{bet}|\text{Q})P(\text{Q})}{P(\text{bet})} = \frac{p_{\text{bluff}}\times P(\text{Q})}{1\times P(\text{A})+p_{\text{bluff}}\times P(\text{Q})} = \frac{p_{\text{bluff}}}{1+p_{\text{bluff}}}$

*Note: $P(A) = P(Q)$

Action: Call - If Player 2 calls, they risk \$2 (their \$1 ante + \$1 call) to win a pot of \$4.

- If P1 has an Ace, P2 calls and loses \$2.
- If P1 has a Queen, P2 calls and wins the \$4 pot, for a net profit of \$2.

The Expected Value of calling is:

$$\begin{aligned} EV_{P2}(\text{Call with K}) &= P(\text{A} \mid \text{bet}) \times (-\$2) + P(\text{Q} \mid \text{bet}) \times (+\$2) \\ &= \left(\frac{1}{1+p_{\text{bluff}}}\right)(-2) + \left(\frac{p_{\text{bluff}}}{1+p_{\text{bluff}}}\right)(2) \\ &= \frac{-2+2p_{\text{bluff}}}{1+p_{\text{bluff}}} \end{aligned}$$

Action: Fold If Player 2 folds, they surrender their \$1 ante. The outcome is certain.

$$EV_{P2}(\text{Fold with K}) = -\$1$$

3.1.3 The Decision Threshold

Player 2 should choose to call if the EV of calling is greater than the EV of folding.

$$\begin{aligned}
EV(\text{Call}) &> EV(\text{Fold}) \\
\frac{2p_{\text{bluff}} - 2}{1 + p_{\text{bluff}}} &> -1 \\
2p_{\text{bluff}} - 2 &> -1(1 + p_{\text{bluff}}) \\
2p_{\text{bluff}} - 2 &> -1 - p_{\text{bluff}} \\
3p_{\text{bluff}} &> 1 \\
\implies p_{\text{bluff}} &> \frac{1}{3}
\end{aligned}$$

Conclusion: This shows that Player 2 should only call with their King if they believe Player 1 is bluffing more than one-third of the time. These two calculations are the two sides of the same strategic coin and lead directly to the mixed strategy Nash Equilibrium, where both players choose a frequency that makes their opponent indifferent to their decision.

4 Nash Equilibrium and Mixed Strategies

Earlier, we saw that each player's best choice depends on what they think the other player will do. Game theory helps us figure out a stable outcome in such situations using the **Nash Equilibrium**.

4.1 Defining Nash Equilibrium

In the context of AKQ Poker, a **Nash Equilibrium** is a pair of strategies, one for Player 1 and one for Player 2, in which neither player can gain an advantage by unilaterally changing their own strategy while the other player's strategy remains constant. It is a state of mutual best response where both players are executing a perfect counter-strategy to their opponent. If Player 1 is playing their equilibrium strategy, any deviation by Player 2 will, at best, break even or, more likely, decrease Player 2's overall profits, and vice-versa.

4.2 The Necessity of Mixed Strategies

In AKQ poker, it is impossible to find a Nash Equilibrium using only **pure strategies** (where a player always chooses the same action in a given situation). Any pure strategy is predictable and therefore exploitable, leading to a logical loop:

- **If P1 never bluffs with a Queen**, P2's best response is to always fold a King to a bet (knowing it must be an Ace).
- **But, if P2 always folds a King**, P1's best response is to start bluffing with a Queen to steal the pot.
- **But, if P1 always bluffs with a Queen**, P2's best response is to always call with a King to catch the bluff.
- **But, if P2 always calls with a King**, P1's best response is to stop bluffing with a Queen, as it's no longer profitable.

Therefore there is no set of stable pure strategies and the solution is a **mixed strategy**

4.3 The Nash Equilibrium Solution for AKQ Poker

The established, unexploitable Nash Equilibrium for this game involves a mix of pure and randomized actions:

Player 1's Strategy

Player 1's strategy is aggressive with the strongest hand, cautious with the middle hand, and deceptive with the weakest hand.

- **With an Ace:** Always Bet.
- **With a King:** Always Check. If Player 2 then bets, Player 1 should fold.
- **With a Queen:** Bet (bluff) with probability $\frac{1}{3}$. Check with probability $\frac{2}{3}$.

Player 2's Strategy

Player 2's strategy uses the information from P1's action to make calculated decisions.

- **If Player 1 Bets:**
  - **With an Ace:** Always Call.
  - **With a King:** Call with probability $\frac{1}{3}$. Fold with probability $\frac{2}{3}$.
  - **With a Queen:** Always Fold.
- **If Player 1 Checks:**
  - **With an Ace:** Always Bet.

– **With a King:** Always Check.
– **With a Queen:** Always Bet (as a bluff).

5 Reflection and Application

5.1 Why Bluffing Matters in AKQ Poker

Bluffing is an essential part of optimal strategy in the AKQ Poker game. If Player 1 only bets when holding an Ace, Player 2 will quickly recognize the pattern and fold weaker hands like King or Queen. This predictability reduces Player 1's potential gains.

To avoid this, Player 1 must occasionally bet with a weaker card, such as a Queen — this is called *bluffing*.

Bluffing serves two main purposes:

- Sometimes the bluff works, and Player 1 wins the pot despite holding a weaker hand.
- Bluffing makes Player 1's actions less predictable, leading Player 2 to call with weaker hands and increasing Player 1's profit when holding an Ace.

5.2 Strategic Lessons from the Game

- **Think Ahead:** Always consider what the opponent knows and how they might respond.
- **Stay Unpredictable:** Mixed strategies reduce the risk of being exploited.
- **Position Matters:** Playing first (Player 1), means having less information, so the strategy must account for that disadvantage.
- **Best Moves Aren't Always Obvious:** Counterintuitive plays, like bluffing with a weak hand, can be optimal.

5.3 Real-World Connection: Pricing Strategy

The ideas from AKQ Poker closely relate to how firms behave in competitive markets, especially in pricing decisions.

- **Thinking Ahead Like in Poker:** Just as Player 1 must anticipate Player 2's reaction, businesses must consider how competitors will respond to pricing changes.

- **Strategic Use of Low Prices:** Firms may set prices low not for immediate gain, but to introduce uncertainty. This is similar to bluffing with a Queen — it keeps competitors guessing.
- **Pressuring Competitors:** Aggressive pricing can push rivals to match bad discounts, like calling a bluff and losing.
- **Maintaining a Balanced Strategy:** A successful long-term pricing strategy blends both aggressive and conservative approaches, just like mixing strong plays with occasional bluffs in poker.

Example: Amazon uses loss-leader pricing by selling items like Kindle devices below cost to attract customers. This short-term loss leads to long-term advantage, much like a poker bluff that pressures rivals and shapes their strategy.
