Interpreting categorical X example

Population regression model

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i} + u_i$$

$Y =$ Reading test scores

$X_1 =$ 0/1 SES- 2nd quartile

$X_2 =$ 0/1 SES- 3rd quartile

$X_3 =$ 0/1 SES- 4th quartile

$\left.\begin{matrix} X_1 \\ X_2 \\ X_3 \end{matrix}\right\}$ Non reference groups

Reference category= 0/1 SES - 1st quartile

OLS prediction line without estimates

$$\hat{Y} = \hat{B}_0 + \hat{B}_1 x_{1i} + \hat{B}_2 x_{2i} + \hat{B}_3 x_{3i}$$

OLS prediction line with estimates

$$\hat{Y} = 45.1 + (3.5X_{1i}) + (6.4X_{2i}) + (10.8X_{3i})$$

$\hat{B}_1 = 3.5 =$ On average, being in the 2nd SES quartile as opposed to being in the 1st SES quartile is associated with a 3.5 point increase in reading test scores

$\hat{B}_2 = 6.4 =$ On average, being in the 3rd SES quartile as opposed the 1st SES quartile is associated with a 6.4 point increase in reading test scores

$\hat{\beta}_3 = 10.8$

On average, being in the 4th SES quartile as opposed to the 1st quartile is associated with a 10.8 point increase in reading test scores

$$\hat{Y} = 45.1 + (3.5\,x_{1i}) + (6.4\,x_{2i}) + (10.8\,x_{3i})$$

$$E(\hat{Y} \mid X = 4^{th} \text{ quartile})$$

$X_1 = \emptyset$
$X_2 = \emptyset$
$X_3 = 1$

$$\hat{Y} = 45.1 + (3.5 * \emptyset) + (6.4 * \emptyset) + (10.8 * 1)$$

$$\hat{Y} = 45.1 + \emptyset + \emptyset + 10.8$$

$$\hat{Y} = 55.9$$

$$\hat{Y} = 45.1 + 3.5 X_{1i} + 6.4 X_{2i} + 10.8 X_{3i}$$

$E(\hat{Y} \mid x = 3^{rd}\text{quartile})$

$x_1 = \emptyset$

$x_2 = 1$

$x_3 = \emptyset$

$$\hat{Y} = 45.1 + (3.5 * \emptyset) + (6.4 * 1) + (10.8 * \emptyset)$$

$$\hat{Y} = 45.1 + \emptyset + 6.4 + \emptyset$$

$$\hat{Y} = 51.5$$

$$\hat{Y} = 45.1 + 3.5X_{1i} + 6.4X_{2i} + 10.8X_{3i}$$

$$E(\hat{Y} \mid X = 1^{st} \text{ quartile})$$

$X_1 = \emptyset$

$X_2 = \emptyset$

$X_3 = \emptyset$

$$\hat{Y} = 45.1 + (3.5 * 0) + (6.4 * 0) + (10.8 * 0)$$

$$\hat{Y} = 45.1 + \emptyset + \emptyset + \emptyset$$

Beta zero is the predicted
Value of Y for reference group

$$\hat{Y} = 45.1 = \hat{B}_0$$