Lecture 20

Binomial Theorem

12 October | Thota Rudhra Kumar

Lectures on Quantitative Methods by Rakesh Nigam

20.0.1 Introduction

The algebraic expression which contains only two terms (+ or -) is called binomial.It is a two-term polynomial.Also,it is called a sum or difference between two or more monomials.It is a method of expanding an expression which has been raised to any finite power.A binomial-Theorem is a powerful tool of expansion,which has application in Algebra,probability,etc. For example , x+2 is a binomial,where x and 2 are two separate terms.Also,the coefficient of x is 1, the exponent of x is 1 and 2 is the constant here.Therefore, A binomial is a two-term algebraic expression that contains variable,coefficient,exponents and constant.
hspace2cm

$$(x+1)^2 = (x+1)(x+1)$$

$$(a+b)^1 = x^2 + x + x + 1$$

$$(x+1)^2 = x^2 + 2x + 1$$

A binomial expression is the sum, or difference of two terms. For example, x+1 , 3x+2y , a-b , are all binomial expressions.To raise a binomial expression to a power higher than 2,

$$(a+b)^0 = 1a^0b^0$$

$$(a+b)^1 = 1a^2 + 1a^0b^1$$

$$(a+b)^2 = 1a^2b^0 + 2a^1b^1 + 1a^0b^2$$

$$(a+b)^3 = (a+b)(a+b)^2 = a(a+b)^2 + b(a+b)$$

$$(a+b)^3 = a(a^2 + 2ab + b^2) + b(ab2 + 2ab + b^2)$$

$$(a+b)^3 = 1a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + 1b^3$$

$$(a+b)^3 = 1a^3b^0 + (2+1)a^2b^1 + (1+2)a^1b^2 + 1a^0b^3$$

Some important observations:

a) The total number of terms in the binomial expansion

$$(a+b)^n.$$

b) In the expansion, the first term is raised to the power of the binomial and in each subsequent terms the power of a reduces by one with simultaneous increase in the power of b by one, till power of b becomes equal to the power of binomial, i.e., the power of a is n in the first term, (n – 1) in the second term and so on ending with zero in the last term. At the same time power of b is 0 in the first term, 1 in the second term and 2 in the third term and so on, ending with n in the last term.

c) In any term the sum of the indices (exponents) of ‘a’ and ‘b’ is equal to n.

d) The coefficients in the expansion follow a certain pattern known as pascal’s triangle.

It is very cumbersome to do this by repeatedly multiplying x+1. In this unit will learn how a triangular pattern of numbers, know as Pascal’s triangle.

20.1 (i) As we move through each term from left to right,the power of "a" decreases from 2 down to zero.

20.1(ii) The power of "b" increases from zero up to 2.

20.1(iii) The coefficients of each term, (1 , 2 , 1) are the numbers which appears in the row of Pascal's triangle beginning 1, 2.

20.1(iv) The term 2ab arises from contributions of 1ab and 1ba , i.e. 1ab+ 1ba = 2ab .

20.0.2 Pascal's Triangle

Pascal's triangle is a triangular arrangement of numbers that gives the coefficients in the expansion of any binomial expression.Pascal's triangle is used widely in probability theory,combinatorics,and algebra.

The sum of the numbers on a diagonal of Pascal's triangle equals the number below the last sum. For example, $1 + 2 = 3$, $1 + 2 + 3 = 6$,$1 + 3 = 4$, $1 + 3 + 6 = 10$, etc.

$n = 0$: 1

$n = 1$: 1 1

$n = 2$: 1 2 1

$n = 3$: 1 3 3 1

$n = 4$: 1 4 6 4 1

Compare the coefficients in these polynomials to the number in Pascal triangle.
Each row is a Palindrome number, that remains the same when it's digits are reversed.

(a+b)(c+d) = ac=ad+bc+bd
(a+b)(c+d) = a(c+d) + b(c+d)
(a+b)(c+d) = (ac+ad) + (bc+bd)

20.2 (i) Eight PGDM students are participating for a race. In how many ways can the first four prizes be won?

solution:- 8 students in a race first four places out of 8.

$$\frac{8!}{4!}$$

n = 8 and r = 4,
It is possible 1680 ways.

20.2(ii) From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

Solution:- we have a (3men and 2 women) or (4men and 1 women) or (5 men only.

= (525 + 210 + 21)
= 756

$(a+b)^n = \sum_{k+0}^{n} \binom{n}{k} a^k b^{n-k}$

- Number of ways of choosing k and b's
- Number of ways of choosing (n - k) a's

$$\binom{n}{n-k}$$

Middle terms:
The middle term depends upon the value of n.

(a) If n is even: then the total number of terms in the expansion of the

$$(a+b)^n$$

is

$$n+1$$

(odd).

(b) If n is odd: then the total number of terms in the expansion of the

$$(a+b)^n$$

is n + 1 (even).

NOTE: 1

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots\ldots, \binom{n}{k} \cdots \binom{n}{n}$$

NOTE: 2

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

Theorem:

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

1. Subsets of "S" contain" a"

2. Subsets of " S" not contain "a"

$$n = 6$$

$$r = 4$$

$$S = a, b, c, d, e, f = \binom{6}{4}$$

Subset of "S" contain "a"[a,1,2,3] =

$$\binom{5}{3}$$

Do not contain

$$"a"[1, 2, 3, 4]$$

=

$$\binom{5}{3}$$

$$\binom{6}{4} = \binom{5}{3} + \binom{5}{4}$$

Theorem :

$$\binom{n}{k} = \binom{n}{n-k}$$

Since choosing K objects from ”n” objects leave (n - k) objects and then choosing the remaining (n - k) 0objects from ”n” leaves K objects.

Answers for Practice Problems :

1) Expand

$$(u - v)^5.$$

2) Expand

$$(2t + 3/t)^4.$$

3) Find the number of terms in

$$(1 + 2x + x^2)^5 0.$$

4) Find the middle term of $(1 - 3x + 3x^2 - x^3)^{2n}$.

5) Find the numerically greatest term in

$$(1 - 3x)^{10}$$

when

$$x = (1/2).$$

6) Find the remainder when 7^{103} is divided by 25.